Info

- Homework and EMCFs are posted.
- The quiz in lab/workshop on Friday will cover area and u-substitution.
- Visit the Alpha Lambda Delta Bake Sale in the PGH Breezeway starting at 10am.

Example:

\[ u = 3x^2 + 2 \]
\[ \frac{du}{dx} = 6x \]
\[ \int_1^2 \frac{x}{\sqrt{3x^2 + 2}} \, dx = \frac{2}{6x} \int_{u(1)}^{u(2)} \frac{u}{\sqrt{u}} \, du = \frac{2}{6} \left( \sqrt{2} - \sqrt{1} \right) = \frac{1}{3} (\sqrt{2} - 1) \]

Chain rule:
\[ \frac{d}{dx} F(u) = \frac{d}{du} F(u) \cdot \frac{du}{dx} \]

Example:

\[ \int_0^{\pi/4} \sin(2x)(\cos(2x) + 2)^3 \, dx = \]
\[ = -\frac{1}{2} \int_0^{\pi/4} \frac{(\cos(2x) + 2)^3}{u} \cdot (-\sin(2x) \, dx) \]
\[ = -\frac{1}{2} \int_2^{2} u^3 \, du = -\frac{1}{2} \cdot \frac{1}{4} u^4 \Bigg|_2^3 = -\frac{1}{8} (16 - 81) = \frac{65}{8} \]
2. \( \int_0^1 \frac{1}{\sqrt{3x+1}} \, dx = \)

Example: Suppose \( f \) is a continuous function and 
\( \int_{-1}^{2} f(x) \, dx = 5 \) and \( \int_{-1}^{2} f(x) \, dx = 6 \). Give 
the value for \( \int_{-1}^{2} f(x) \, dx \).

Example in the video

Question: The graph of \( f \) is shown below. Is there a value of \( x \) so 
that \( g(x) \) is smaller than the net area bounded by 
the graph of \( f \) over the interval \([a,b]?)

\[
\text{Value of } x \Rightarrow \frac{1}{b} \int_a^b f(x) \, dx = \frac{1}{b} (b-a)
\]

\[
\int_a^x f(x) \, dx \leq \int_b^x f(x) \, dx
\]

Yes, if \( x \) is such that \( f(x) < g(x) \) for all \( a \leq x \leq b \).
**Question:** The graph of $f$ is shown below. Is there a value of $y$ so that $y(b-a)$ is larger than the net area bounded by the graph of $f$ over the interval $[a,b]$?

![Graph of $f$](image)

**Theorem:** The mean value theorem for integrals. Suppose $f$ is a continuous function on the interval $[a,b]$. Then there is a value $c$ in the interval $[a,b]$ such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx$$

**Example:** Give the average value of $f(x) = \sin(x)$ on the interval $[0,\pi]$.

$$f(x) = \sin(x) \text{ on } [0,\pi]$$

$$\text{Average Value} = \frac{1}{\pi - 0} \int_0^\pi \sin(x) \, dx$$

$$= \left[ -\cos(x) \right]_0^\pi$$

$$= \frac{1}{\pi} (1 - (-1)) = \frac{2}{\pi}$$

**Note:** The value $f(c)$ is called the mean value or average value of the function over the interval $[a,b]$. The formula for determining $c$ is:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

**Example:** The graph of $f$ is shown below. Is there a value of $y$ so that $y(b-a)$ is equal to the net area bounded by the graph of $f$ over the interval $[a,b]$?

![Graph of $f$](image)
Example:  Given the average value of \( f(x) = 3x^2 - x \) on the interval \([-1, 3]\), find the value of \( c \) where \( f(c) \) has this value on the interval \([-1, 3]\).

\[
\frac{3}{3 - (-1)} \int_{-1}^{3} (3x^2 - x) \, dx = 6
\]

Find \( a \), \( b \), \( c \) such that \(-1 < c < 5\) and \( a^2 - b^2 = 0\).

\[ a = \frac{1 + \sqrt{1 + 7b}}{2} = \frac{1 + \sqrt{8}}{2} \]
\[ b = \frac{\sqrt{2}}{2} \]
\[ c = \frac{1 + \sqrt{1 + 7b}}{2} = \frac{1 + \sqrt{8}}{2} \]

Note: \(-1 < \frac{1 + \sqrt{8}}{2} < 3\)