Today: Section 6.1

- Test 4: Dec. 6 - 8
- Final Exam: Dec. 17 - 19
- Dates are subject to slight modification.
- Homework and an EMCF are Due on Monday.
- An EMCF is due on Wednesday (even though we do not have class).
- Homework and an EMCF are due on the Monday following the break.

Happy Birthday Christine!!

Review

Theorem: (The mean value theorem for integrals.) Suppose $f$ is a continuous function on the interval $[a,b]$. Then there is a value $c$ so that $a < c < b$, and

$$f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx$$

the average value of $f$ on $[a,b]$

Free Friday!!
Popper P29
1. $3 + 4 =

Review Example: Give the average value of the function $f(x) = x^2$ on the interval $[-1,2]$, and determine the number of values where $f$ achieves this average value on this interval.

$$= \frac{1}{2-(-1)} \int_{-1}^{2} x^2 \, dx = \frac{1}{3} \cdot \frac{1}{3} x^3 \bigg|_{-1}^{2}$$

$$= \frac{1}{3} (8 - (-1)) = \frac{9}{3} = 3$$

Solve $x^2 = 1$ for $-1 \leq x \leq 2$.

$$x = \pm 1$$

both are in this interval.

Review Example: Suppose you know $\int (f(x) - 2x) \, dx = 3$. Give the average value of $f$ on the interval $[0,2]$.

$$= \frac{1}{2-0} \int_0^2 f(x) \, dx = \frac{1}{2} \cdot 3 = \frac{3}{2}$$

$$\int_0^2 (f(x) - 2x) \, dx = 3$$

$$\int_0^2 f(x) \, dx - \int_0^2 2x \, dx = 3$$

$$\int_0^2 f(x) \, dx = x^2 \bigg|_0^2 + 3 = 4 - 0 + 3 = 7$$
New

**Question:** How do we find the area of the region shown below?

\[
\text{Area} = \int_{a}^{b} \left( f(y) - g(y) \right) \, dy
\]

**Example:** Give the area of the region bounded by the graphs of \( x = y^2 \) and \( y = x - 2 \).

\[
\begin{align*}
\text{Area} &= \int_{-1}^{2} \left( \text{right} - \text{left} \right) \, dy \\
&= \int_{-1}^{2} \left( y + 2 - y^2 \right) \, dy \\
&= \left. \left( \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \right) \right|_{-1}^{2} \\
&= \left( \frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3 \right) - \left( \frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3 \right) \\
&= \frac{8}{3} - \frac{1}{2} = \frac{17}{6}
\end{align*}
\]

**Example:** Rewrite the solution to the previous area problem using integrals in terms of \( x \).

\[
\begin{align*}
\text{Area} &= \int_{0}^{2} \left( x^2 - (x-2) \right) \, dx \\
&= \int_{0}^{2} (x^2 - x + 2) \, dx \\
&= \left. \left( \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \right) \right|_{0}^{2} \\
&= \frac{8}{3} - \frac{1}{2} = \frac{17}{6}
\end{align*}
\]