Today: *More Section 6.2/6.3*

- **Test 4**, Dec. 6 - 8
- **Final Exam**, Dec. 17 - 19
- Dates are subject to change...
- **Practice Test 4** (counts as a quiz) and **Practice Final Exam** (counts as 2 quizzes) will be posted soon.
- **EMCFs** are posted for the rest of the week, and **Homework** is posted for Monday.
- **Practice Problems** will be posted soon for Test 4.
- **Friday's Quiz** will cover volumes of revolution.
Example: Create a right circular cone with height $h$ and base radius $r$ as a volume of revolution. Then use your creation to derive the formula for the volume of a cone.

\[ y = \frac{h}{r} \quad x + h \]

**Cylindrical Shell**

\[ \text{Thickness} = dx \]

\[ \text{Surface Area (shell)} = 2\pi x \left( \frac{-h}{r} x + h \right) \]

\[ \text{Inf. Volume (shell)} = 2\pi x \left( \frac{-h}{r} x + h \right) dx \]

\[ \text{Volume (cone)} = \int_0^r 2\pi x \left( \frac{-h}{r} x + h \right) dx \]

\[ = \int_0^r \left( -\frac{2\pi h}{r} x^2 + 2\pi h x \right) dx \]

\[ = \left. \left( -\frac{2\pi h}{r} \frac{x^3}{3} + \pi h x^2 \right) \right|_0^r \]

\[ = -\frac{2}{3} \frac{2\pi h}{r} r^2 + \pi h r^2 \]

\[ = \frac{1}{3} \pi r^2 h \]
Popper P32

1. Give the volume of the solid generated when the region bounded between \( y = x^3 \) and the \( x \)-axis on the interval \( [0,1] \) is rotated around the \( x \)-axis.

2. Give the volume of the solid generated when the region bounded between \( y = x^3 \) and the \( x \)-axis on the interval \( [0,1] \) is rotated around the \( y \)-axis.
Volumes by Slicing: Main Setting

A solid $S$ intersects the $xy$-plane in the region $R$ shown below. Every cross section of the solid $S$ taken perpendicular to a given line can be described. Find the volume of $S.$
Typically, the solid object is aligned so that the cross sections perpendicular to one of the axes is known.
**Example:** A solid object is aligned so that its base is the region in the \( xy \)-plane bounded between the \( x \)-axis and the curve \( y = 1 - x^2 \). Cross sections taken perpendicular to the \( x \)-axis are squares. Find the volume of the region.

\[
\text{Thickness} = dx \\
\text{Area} = (1-x^2)^2 \\
\int_{-1}^{1} \text{Volume} = (1-x^2)^2 \, dx \\
\text{Full Volume} = \int_{-1}^{1} (1-x^2)^2 \, dx
\]
Example: A solid object is aligned so that its base is the region in the $xy$-plane bounded between the $x$-axis and the curve $y = 1 - x^2$. Cross sections taken perpendicular to the $y$-axis are semi-circles with their diagonals lying in the $xy$-plane. Find the volume of the region.

$$\begin{align*}
y &= 1 - x^2 \Rightarrow x^2 &= 1 - y \\
x &= \pm \sqrt{1 - y}
\end{align*}$$

4. Give the volume.

Thickness = $dy$

Area = $\frac{1}{2} \pi r^2 = \frac{1}{2} \pi (1 - y)$

Inf Volume (cross section) = $\frac{1}{2} \pi (1 - y) dy$

Full Volume = $\int_0^1 \frac{1}{2} \pi (1 - y) dy =$