Info

The video and notes from last night's review for Test 4 are posted.

There will be a quiz in lab/workshop on Friday!!

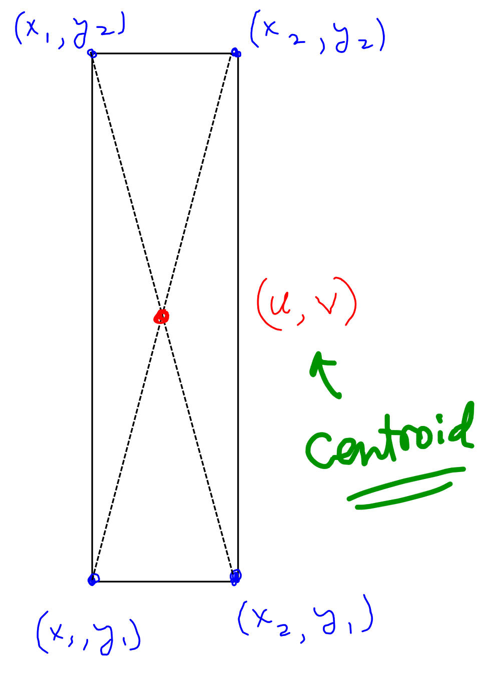
Centroid of a Region

Question: Suppose the rectangle on the right is a thin sheet of rigid material. If you were going to place it horizontally flat, and try to balance it on a pin, where should the tip of the pin contact the rectangle?

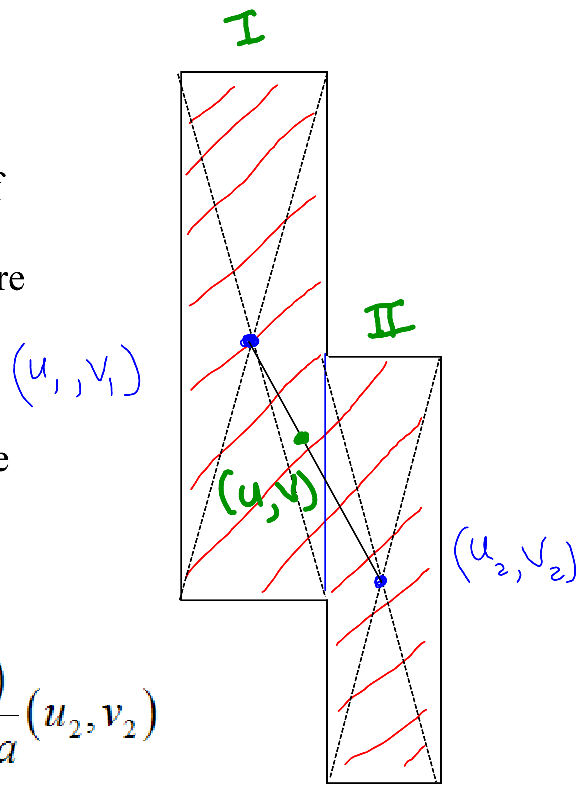
Note: This point is the **centroid** of the rectangle.

$$u = \frac{1}{2}(x_1 + x_2)$$

$$v = \frac{1}{2}(y_1 + y_2)$$



Question: Suppose the region on the right is a thin sheet of rigid material. If you were going to place it horizontally flat, and try to balance it on a pin, where should the tip of the pin contact the region?



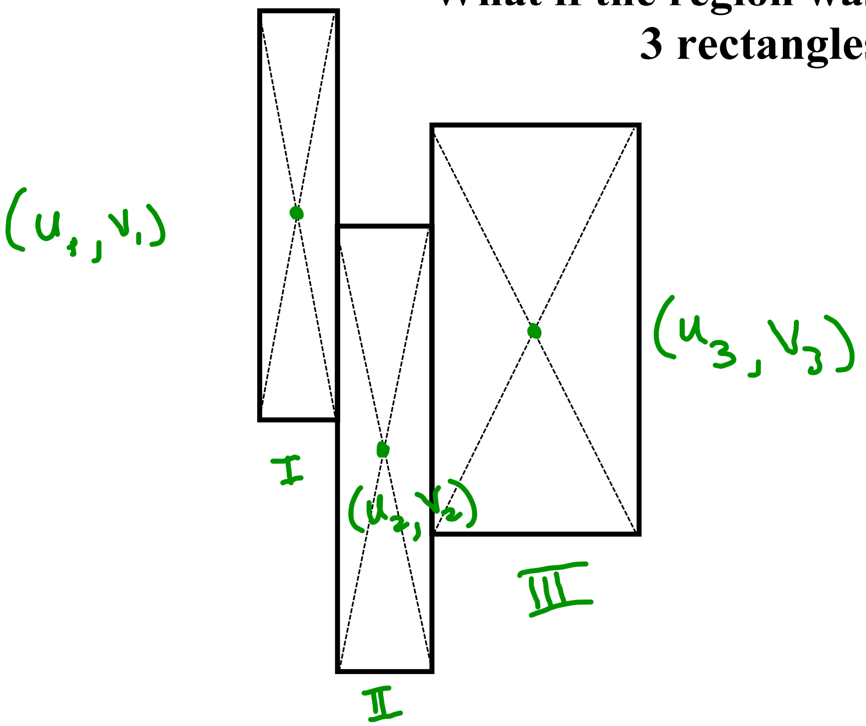
Note: This point is the **centroid** of the region.

$$(u, v) = \frac{Area(I)}{Total\ Area}(u_1, v_1) + \frac{Area(II)}{Total\ Area}(u_2, v_2)$$

$$u = \frac{Area(I)u_1 + Area(II)u_2}{Total\ Area}$$

$$v = \frac{Area(I)v_1 + Area(II)v_2}{Total\ Area}$$

What if the region was made up of 3 rectangles?



$$\frac{Area(I)}{Total\ Area}(u_1,v_1)+\frac{Area(II)}{Total\ Area}(u_2,v_2)+\frac{Area(III)}{Total\ Area}(u_3,v_3)$$

(u, v) Centroid of combined 3.

$$u = \frac{Area(I)u_1 + Area(II)u_2 + Area(III)u_3}{Total\ Area}$$

$$v = \frac{Area(I)v_1 + Area(II)v_2 + Area(III)v_3}{Total\ Area}$$

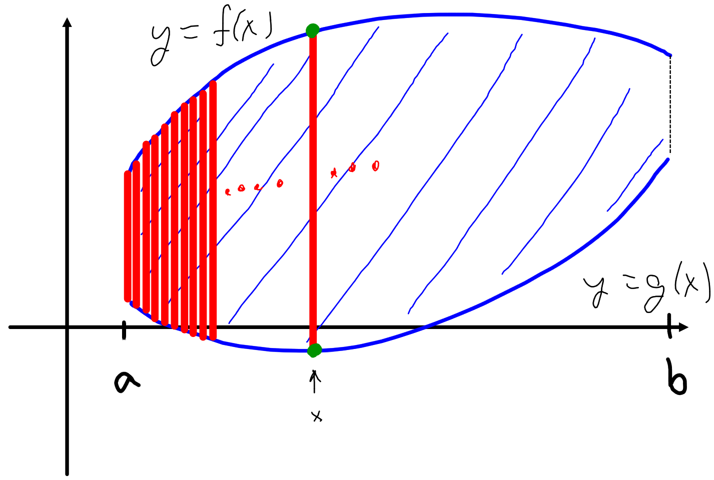
* What if the region was made up of n rectangles?

$$\frac{Area(\text{Rect }1)}{Total\ Area}(u_1,v_1)+\frac{Area(\text{Rect }2)}{Total\ Area}(u_2,v_2)+\cdots+\frac{Area(\text{Rect }n)}{Total\ Area}(u_n,v_n)$$

Where (u_i,v_i) is the centroid of the i^{th} rectangle.

(u,v) centroid of the total region made up of the n rectangles.

Centroids of Regions in the xy Plane



Idea: Fill the region with very thin vertical rectangles.

x-coord of centroid of the rectangle at $x = x$

y-coord of centroid of the rectangle at $x = \frac{1}{2}(f(x)+g(x))$

$$\text{weight factor} = \frac{\text{Area of rectangle at } x}{\text{Total Area}}$$

$$= \frac{(f(x)-g(x))dx}{\int_a^b (f(x)-g(x))dx}$$

Sum weight factors times centroids $= \displaystyle\int_a^b \frac{\left(x, \frac{1}{2}(f(x)+g(x))\right)(f(x)-g(x))dx}{\boxed{\int_a^b (f(x)-g(x))dx}}$

$$\text{Centroid} = \frac{\left(\int_a^b x(f(x)-g(x))dx, \int_a^b \frac{1}{2}(f(x)^2-g(x)^2)dx\right)}{\int_a^b (f(x)-g(x))dx}$$

Centroid of a Region

The centroid of the region bounded above by $y = f(x)$ and below by $y = g(x)$ on the interval $[a,b]$ is given by the point $(\bar{x},\bar{y})$ where

$$\bar{x} = \frac{\int_a^b x\left(f(x)-g(x)\right)dx}{\int_a^b \left(f(x)-g(x)\right)dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b \frac{1}{2}\left([f(x)]^2-[g(x)]^2\right)dx}{\int_a^b \left(f(x)-g(x)\right)dx}$$

Special Case

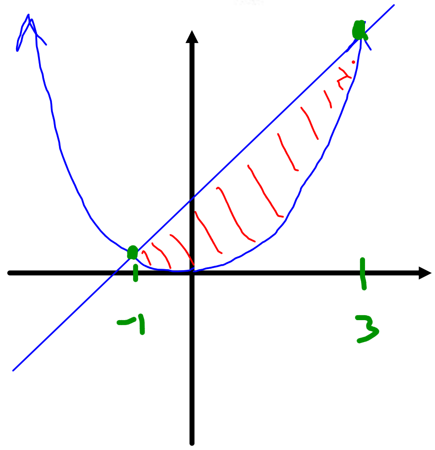
$g(x) = 0$

The centroid of the region bounded above by $y = f(x)$ and below by the x-axis on the interval $[a,b]$ is given by the point $(\bar{x},\bar{y})$ where

$$\bar{x} = \frac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx} \quad \text{and} \quad \bar{y} = \frac{\int_a^b \frac{1}{2}[f(x)]^2\,dx}{\int_a^b f(x)\,dx}$$

Find the centroid of the region bounded by the curves $y = x^2$ and $y = 2x + 3$.

$$\bar{x} = \frac{\int_a^b x(f(x)-g(x))dx}{\int_a^b (f(x)-g(x))dx} \text{ and } \bar{y} = \frac{\int_a^b \frac{1}{2}\left([f(x)]^2 - [g(x)]^2\right)dx}{\int_a^b (f(x)-g(x))dx}$$



$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, \; x = 3$

Centroid $= (\bar{x}, \bar{y})$

$$\bar{x} = \frac{\int_{-1}^{3} x(2x+3-x^2)\,dx}{\int_{-1}^{3} (2x+3-x^2)\,dx}$$

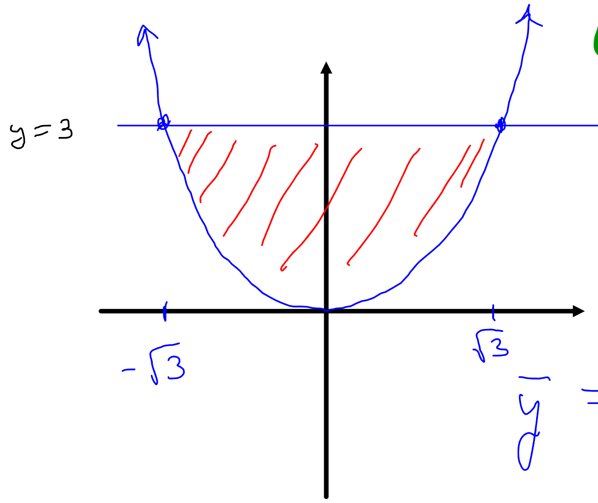
$$\bar{y} = \frac{\int_{-1}^{3} \frac{1}{2}\left((2x+3)^2 - (x^2)^2\right)dx}{\int_{-1}^{3} (2x+3-x^2)\,dx}$$

3 separate integrations <u>MUST</u> <u>be</u> <u>done</u>.

you do it.

Find the centroid of the region bounded by $y = x^2$ and $y = 3$.

You are allowed to be smart!!!!



From symmetry,

$\bar{x} = 0$.

$$\bar{y} = \frac{\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{2}(9 - x^4)\,dx}{\int_{-\sqrt{3}}^{\sqrt{3}} (3 - x^2)\,dx}$$

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1. Numerator
2. Denominator
3. y-coordinate of the centroid

$$= \frac{\frac{36}{5}\sqrt{3}}{4\sqrt{3}} = \frac{9}{5}$$

The region is shown on the right along with its centroid at (0,9/5).