Homework Help - 4.4, 4.5

4.4: 2, 3, 5, 6, 8, 9, 11, 12, 14, 15, 17, 19, 21, 22, 23, 24

4.5: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 23, 25, 27, 43

4.6: 1, 2, 5, 6, 9,10, 13, 14, 17, 18, 21, 22, 25, 26, 29, 30, 31, 32

#6 4.4 $f(x) = x + \frac{1}{x^2}$ ← Domain: All x except 0.

Get $\boxed{\text{C. N.}}$

Classify extrema vals.

→ value in the domain where $f'(x) = 0$ or $f'(x)$ dne.

→ $f'(x) = 1 - 2x^{-3} = 1 - \frac{2}{x^3}$

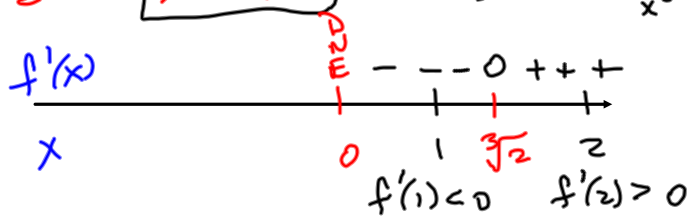
Note: $f'(x)$ exists at all x in the domain.

C.N. Set $f'(x) = 0$.

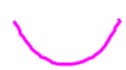
$1 - \frac{2}{x^3} = 0$. $1 = \frac{2}{x^3}$

$x^3 = 2 \Rightarrow \boxed{x = \sqrt[3]{2}}$. $f'(x) = 1 - \frac{2}{x^3}$

Slope chart



Shape



$\therefore$ loc. min at $x = \sqrt[3]{2}$.

14 4.4 $f(x) = x\sqrt{4-x^2}$

C.N. and classify extrema.

Domain: Need $4 - x^2 \geq 0$

$4 \geq x^2$. i.e. $-2 \leq x \leq 2$.

C.N. $f'(x) = \sqrt{4-x^2} + x \cdot \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$

$= \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}}$

Domain of $f'(x)$: $4 - x^2 > 0$

$-2 < x < 2$.

$\therefore$ $x = -2, 2$ are c.n. $\boxed{f'(x) \text{ d.n.e.}}$

check $f'(x) = 0$.

$$\sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}} = 0$$

$$\frac{4 - x^2 - x^2}{\sqrt{4-x^2}} = 0$$

$\Leftrightarrow$ $4 - 2x^2 = 0$ $\quad x = \pm\sqrt{2}$.

C.N.: $-2, -\sqrt{2}, \sqrt{2}, 2$

slope chart

$f'(x)$: DNE $- - - -$ 0 $+ + + + +$ 0 $- - - -$ DNE

x: $\boxed{-2}$ $\quad -\frac{3}{2}$ $\quad \boxed{-\sqrt{2}}$ $\quad 0$ $\quad \boxed{\sqrt{2}}$ $\quad \frac{3}{2}$ $\quad \boxed{2}$

$f'(-\frac{3}{2}) < 0$ $\quad f'(0) > 0$ $\quad f'(\frac{3}{2}) < 0$

Shape

End pt. local max at $x = -2$

loc. min at $x = -\sqrt{2}$

loc. max at $x = \sqrt{2}$

End pt. loc. min at $x = 2$.

#22 4.4: $f(x) = \sin(2x) - x$

$0 \le x \le \pi$

C.N. and classify extrema.

$f'(x) = 2\cos(2x) - 1$ exists for all x.

Only C.N. are ones where $f'(x) = 0$. $(0 \le x \le \pi)$

$2\cos(2x) - 1 = 0 \iff \cos(2x) = \frac{1}{2}$

$2x = \frac{\pi}{3}$ or $2x = \frac{5\pi}{3}$

$x = \pi/6$ or $x = 5\pi/6$

C.N.

Classify!

$f''(x) = -4\sin(2x)$.

$f''(\pi/6) = -4\sin(\pi/3) < 0$, $f''\left(\frac{5\pi}{6}\right) > 0$

f has a loc. max at $x = \pi/6$

f has a loc. min at $x = 5\pi/6$.

#24. 4.4

$f(x) = \sin^4(x) - \sin^2(x)$

$0 \le x \le \frac{2\pi}{3}$

C.N. and classify extrema.

$f'(x) = 4\sin^3(x)\cos(x) - 2\sin(x)\cos(x)$

(exists for all x)

$\therefore$ C.N. occur where $f'(x) = 0$.

$f'(x) = 0 \iff 4\sin^3(x)\cos(x) - 2\sin(x)\cos(x) = 0$

$f'(x) \to \underbrace{2\sin(x)\cos(x)}_{\sin(2x)}\left(2\sin^2(x) - 1\right) = 0$

$\sin(2x) = 0$ or $2\sin^2(x) - 1 = 0$

$\left(0 \le x \le \frac{2\pi}{3}\right)$

$\Updownarrow$ $\qquad$ $\Updownarrow$

$2x = 0$ or $2x = \pi$ $\qquad$ $\sin(x) = \pm\frac{\sqrt{2}}{2}$

$x = 0$ or $x = \frac{\pi}{2}$ $\qquad$ $x = \pi/4$

C.N.

Note: $f'(x) = \sin(2x)\left(2\sin^2(x) - 1\right)$

$f''(x) = 2\cos(2x)\left(2\sin^2(x) - 1\right) + \sin(2x)\cdot 2 \cdot \underbrace{2\sin(x)\cos(x)}_{\sin(2x)}$

$= 2\cos(2x)\left(2\sin^2(x) - 1\right) + 2\sin^2(2x).$

$f''(0) = -2 < 0$. $\quad f''(\pi/2) = 2\cdot(-1)(2\cdot 1 - 1) + 0 = -2 < 0$

$f''(\pi/4) = 0 + 2 > 0$

loc. max. at $x = 0$.

loc. min. at $x = \pi/4$.

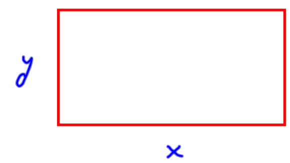
loc. max at $x = \pi/2$

4.5

6. Find the dimensions of the rectangle of area A square units that has the smallest perimeter.

← fixed area they called A.

↓ minimize perimeter.



$$P = 2x + 2y$$

$$xy = A$$

$$y = \frac{A}{x}$$

$$P(x) = 2x + \frac{2A}{x}$$

$$x > 0.$$

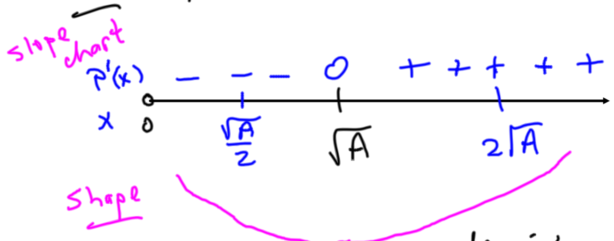
$$P'(x) = 2 - \frac{2A}{x^2}$$

Set $P'(x) = 0 \iff 2 - \frac{2A}{x^2} = 0$

$$\frac{x^2}{A} = 1 \iff x^2 = A$$

$$x = \pm\sqrt{A}$$

C.N. $x = \sqrt{A}$



$$P'(x) = 2 - \frac{2A}{x^2}$$

$$P'\left(\frac{\sqrt{A}}{2}\right) = 2 - \frac{2A}{A/4} = 2 - 8 < 0$$

$$P'(2\sqrt{A}) = 2 - \frac{2A}{4A} = 2 - \frac{1}{2} > 0$$

Perimeter is minimized when

$$x = \sqrt{A}.$$

$$y = \frac{A}{x} = \sqrt{A}.$$

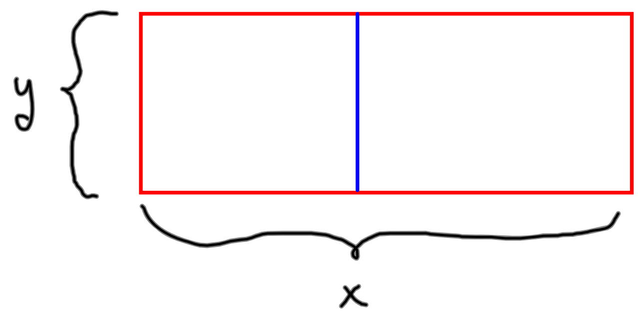
SQUARE!

4.5

8. A rectangular warehouse will have 5000 square feet of floor space and will be separated into two rectangular rooms by an interior wall. The cost of the exterior walls is \$150 per linear foot and the cost of the interior wall is \$100 per linear foot. Find the dimensions that will minimize the cost of building the warehouse.

$$xy = 5000$$

$$y = \frac{5000}{x}$$



$$\text{Cost} = 150(2x + 2y) + 100y$$

$$C(x) = 150\left(2x + \frac{10000}{x}\right) + \frac{500{,}000}{x}$$

$$x > 0.$$

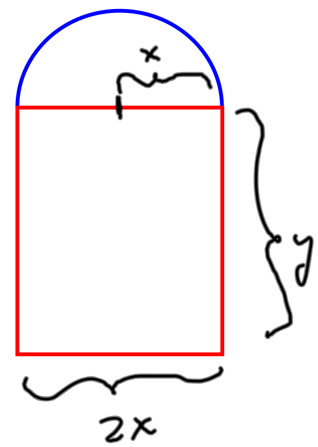
Minimize $C(x)$.

$$C(x) = 300x + \frac{2000000}{x}, \quad x > 0.$$

you do it.

#9. 4.5

Fixed perimeter P.



$$P = \pi x + 2x + 2y$$

fixed

$$y = \frac{P - \pi x - 2x}{2}$$

max

$$\text{Light} = \text{Area of Rect} + \frac{1}{3}\text{Area Semi}$$
$$= 2xy + \frac{1}{3}\cdot\frac{1}{2}\pi x^2$$

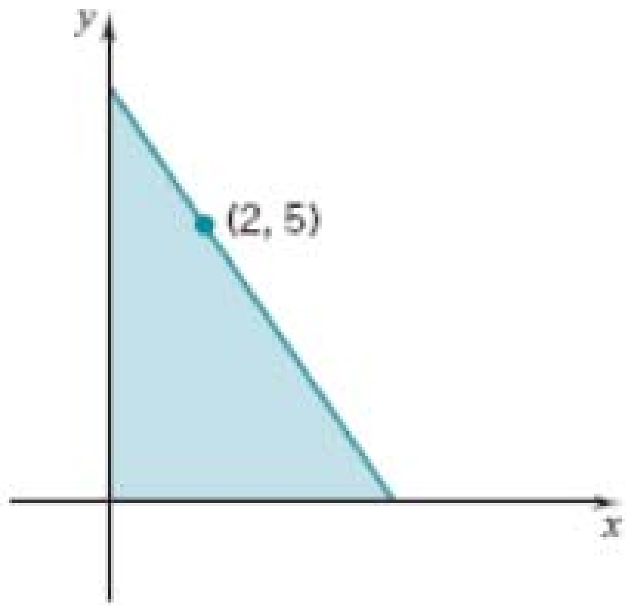
$$L(x) = 2x\left(\frac{P-(\pi+2)x}{2}\right) + \frac{1}{6}\pi x^2$$
$$= Px - (\pi+2)x^2 + \frac{1}{6}\pi x^2$$

$$L(x) = Px - \left(\pi + 2 - \frac{\pi}{6}\right)x^2$$

Maximize $\qquad 0 \le x \le \frac{P}{\pi+2}$

you.

13. A triangle is formed by the coordinate axes and a line through the point (2, 5) as in the figure. Determine the slope of this line if the area of the triangle is to be a minimum.



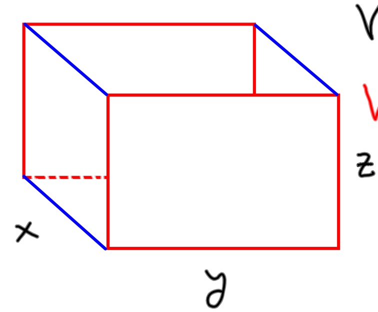
Done in class.

15. What are the dimensions of the base of the rectangular box of greatest volume that can be constructed from 100 square inches of cardboard if the base is to be twice as long as it is wide? Assume that the box has top.

$y = 2x$

$V = xyz$

$V(x) = x \cdot (2x) \frac{100 - 4x^2}{6x}$



100 = bottom + top + sides

$= xy + xy + 2xz + 2yz$

$= x(2x) + x(2x) + 2xz + 2(2x)z$

$\boxed{100 = 4x^2 + 6xz}$

$z = \frac{100 - 4x^2}{6x}$

$V(x) = \frac{100x}{3} - \frac{4}{3}x^3$ maximize

$0 \le x \le 5$

x is largest when $z = 0$.

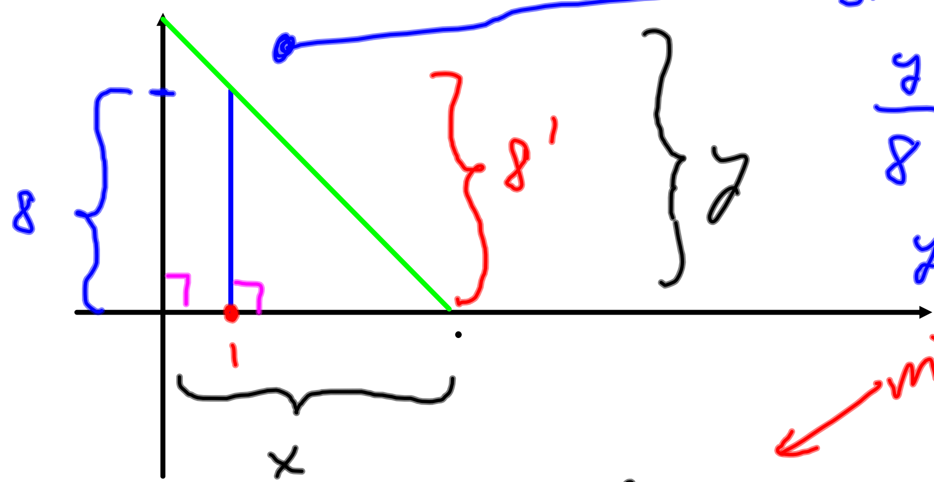
1. $V(0)$, $V(5)$
2. Set $V'(x) = 0$ and solve for $0 \le x \le 5$. $x = \frac{5}{\sqrt{3}}$
   Get $V(5/\sqrt{3})$.
3. Compare.

Note: $V''(x) = -8x$

Answer Question.

27. An 8-foot-high fence is located 1 foot from a building. Determine the length of the shortest ladder that can be leaned against the building and touch the top of the fence.

similar Δ's



$$\frac{y}{8} = \frac{x}{x-1}$$

$$y = \frac{8x}{x-1}$$

minimize.

$$\text{Length of Ladder} = \sqrt{x^2 + y^2}$$

$$L(x) = \sqrt{x^2 + \left(\frac{8x}{x-1}\right)^2}, \quad 1 < x < \infty$$

minimize

you