Exam Blueprint

12 Multiple Choice
8 Free Response

Topics:

Integration and/or Differentiation: Exponential
Logarithmic (natural and other bases)
Inverse trig
Trig substitution
By parts
Improper
Hyperbolic sine and cosine

Inverse functions: one-to-one?
Derivatives
Find inverse if possible

Polar: Area enclosed

Parametric: derivatives, equations from/to parametric / in x and y
Slope of tangent lines

Sequences: Converge/Diverge

Limits: Sequences
Indeterminate forms

Infinite Series: all the "tests" – know which one to use when
Absolute/conditional convergence
Sum

Taylor Polynomials/Series: Form in x and in (x – a)
Error – know the formula, find the error, find "n"
Coefficients

T2 - T4

Taylor/Power Series: Radius of convergence
Interval of convergence
Integration/Differentiation

New

Review your old tests, old quizzes, class notes and the review. And by "review" – I mean work the problems, not look at them.

Math 1432

Final Exam Review

1. Give the equation of the tangent line to the given graph at the point where x = 0
   a. $f(x)=\ln(6x+1)+e^{2x}$
   b. $f(x)=\ln(2x+1)-3e^{-4x}$
   c. $f(x)=\sqrt{9-x^2}$

   you

2. Find the inverse of the following:
   a. $f(x)=\dfrac{2}{3-x}$
   b. $f(x)=\dfrac{x+1}{x+2}$

   you

3. Find the derivative of the inverse for the following:
   a. $f(x)=x^3+1,\quad f(2)=9,\quad (f^{-1})'(9)=$
   b. $f(-3)=1,\ f(1)=2,\ f'(-3)=3,\ f'(1)=-2,\quad (f^{-1})'(1)=$
   c. $f(x)$ passes through the points (3 , -2) and (-2 , 1). The slope of the tangent line to the graph of $f(x)$ at ***x*** = 3 is -1/4. Evaluate the derivative of the inverse of f at -2.
4. Find the equation of the tangent and the normal lines to the parametric curves at the given points:
   a. $x(t)=-2\cos 2t,\ y(t)=4+2t,\ (-2,4)$
   b. $x(t)=3\cos(3t)+2t,\ y(t)=1+5t,\ (3,1)$
5. Give an equation relating *x* and *y* for the curve given parametrically by
   a. $x(t)=-1+3\cos t \quad y(t)=1+2\sin t$
   b. $x(t)=-1+3\cosh t \quad y(t)=1+2\sinh t$
   c. $x(t)=-1+4e^t \quad y(t)=2+3e^{-t}$ $\rightarrow\ y=2+\dfrac{3}{e^t}=2+\dfrac{3}{\frac{x+1}{4}}$

   $x=-1+4e^t$

   $\Rightarrow \dfrac{x+1}{4}=e^t$

   $\therefore\ y=2+\dfrac{12}{x+1}$

6. Differentiate the function:
   a. $f(x)=3^{x^2}$
   b. $f(x)=\tan(\log_5 x)$
   c. $f(x)=x^{\sin x}$
   d. $f(x)=\sinh(3x)$
   e. $f(x)=\dfrac{\cosh x}{x}$

   you

use $\cos^2(t)+\sin^2(t)=1$

$\cosh^2(t)-\sinh^2(t)=1$

3. Find the derivative of the inverse for the following:

a. $f(x) = x^3 + 1, \quad f(2) = 9, \quad (f^{-1})'(9) =$

b. $f(-3) = 1, f(1) = 2, f'(-3) = 3, f'(1) = -2, \quad (f^{-1})'(1) =$

c. $f(x)$ passes through the points $\boxed{(3, -2)}$ and $\boxed{(-2, 1)}$. The slope of the tangent line to the graph of $f(x)$ at $\boldsymbol{x} = 3$ is -1/4. Evaluate the derivative of the inverse of f at -2.

$\boxed{f(3) = -2}$

$f(-2) = 1$

$f'(3)$

$$(f^{-1})'(-2) = \frac{1}{f'(3)} = \frac{1}{-1/4} = -4$$

$= -\frac{1}{4}$

$$(f^{-1})'(9) = \frac{1}{f'(a)} \quad \text{where } f(a) = 9$$

Note: $f(2) = 9$

so $a = 2$

$$= \frac{1}{f'(2)}$$

$f(x) = x^3 + 1$

$f'(x) = 3x^2 \Rightarrow f'(2) = 12$

$$= \frac{1}{12}.$$

$$(f^{-1})'(1) = \frac{1}{f'(a)} \quad \text{where } f(a) = 1 \Rightarrow a = -3$$

$$= \frac{1}{f'(-3)} = \frac{1}{3}.$$

4. Find the equation of the tangent and the normal lines to the parametric curves at the given points:

a. $x(t) = -2\cos 2t,\ y(t) = 4 + 2t,\ (-2,4)$

b. $x(t) = 3\cos(3t) + 2t,\ y(t) = 1 + 5t,\ (3,1)$

$x(?) = -2$

$y(?) = 4$

T.L.: Point: $(-2,4)$

Slope: $\dfrac{y'(?)}{x'(?)}$

$-2\cos(2t) = -2$

$t = 0$ works.

$x'(t) = 4\sin(2t)$

$y'(t) = 2$

Note: $y(0) = 4$ ✓

$= \dfrac{y'(0)}{x'(0)}$

$= \dfrac{2}{0}$ ← Vertical Line!!

thm. vertical line through $(-2,4)$ is

$$\boxed{x = -2}$$

∴ Normal line is horizontal. i.e. slope $= 0$.

$$\boxed{y = 4}$$

Parametrically: T.L.

$$\left.\begin{aligned} x(t) &= x(0) + t\,x'(0) = -2 \\ y(t) &= y(0) + t\,y'(0) = 4 + 2t \end{aligned}\right\}$$

Says $x = -2$ and y is anything!!

7. Integrate:

a. $\int (\cosh(3x)+\sinh(2x))dx$ — you

b. $\int 4^{3x}\,dx$ — you

c. $\int \frac{\log_2(x^3)}{x}dx$ — you

$$= \int \frac{\ln(x^3)/\ln(2)}{x}dx$$

$$= \frac{1}{\ln(2)}\int \frac{3\ln(x)}{x}dx$$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$$= \frac{3}{\ln(2)}\cdot\frac{(\ln(x))^2}{2} + C$$

$\int a^u du = \frac{a^u}{\ln(a)} + C$

$\int \sinh(u)du = \cosh(u) + C$

d. $\int (2^{7x} - \sinh(5x))dx$

e. $\int \frac{\sin(3x)}{16+\cos^2(3x)}dx$

f. $\int \frac{6x}{4+x^4}dx$

g. $\int \tan(3x)dx$

h. $\int \frac{\arctan(3x)}{1+9x^2}dx$

i. $\int \frac{1}{\sqrt{4+x^2}}dx$

j. $\int \sqrt{9-x^2}\,dx$

k. $\int 3\ln(4x)dx$

l. $\int x^2 e^x dx$

m. $\int \frac{5x+14}{(x+1)(x^2-4)}dx$

n. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}dx$

o. $\int \frac{2x^2}{\sqrt{9-x^2}}dx$

p. $\int 2\arctan(10x)dx$

q. $\int 3x\cos(2x)dx$

8. Write an expression for the nth term of the sequence:

a. 1, 4, 7, 10, ...

b. 2, -1, $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$

e. $-\frac{1}{3}\int \frac{-3\sin(3x)}{16+\cos^2(3x)}\,dx$

f. $\int \frac{6x}{4+x^4}\,dx$

g. $\int \tan(3x)\,dx$

h. $\int \frac{\arctan(3x)}{1+9x^2}\,dx$

i. $\int \frac{1}{\sqrt{4+x^2}}\,dx$

j. trig sub $\int \sqrt{9-x^2}\,dx$

k. $\int 3\ln(4x)\,dx$ parts

l. $\int x^2e^x\,dx$

m. $\int \frac{5x+14}{(x+1)(x^2-4)}\,dx$

n. $\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$

o. trig sub $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

$u=\cos(3x)$

$du=-3\sin(3x)\,dx$

$$\underbrace{-\frac{1}{3}\int \frac{1}{16+u^2}\,du}$$

↳ arctan integral

$$-\frac{1}{3}\cdot\frac{1}{4}\arctan\left(\frac{u}{4}\right)+C$$

$$-\frac{1}{12}\arctan\left(\frac{\cos(3x)}{4}\right)+C$$

$$6\cdot\frac{1}{2}\int \frac{2x}{4+(x^2)^2}\,dx$$

$u=x^2$

$du=2x\,dx$

$$=3\int \frac{1}{4+u^2}\,du$$

$$=\frac{3}{2}\arctan\left(\frac{u}{2}\right)+C$$

$$=\frac{3}{2}\arctan\left(\frac{x^2}{2}\right)+C$$

$$\int \frac{\sin(3x)}{\cos(3x)}\,dx$$

Nearly $\int \frac{1}{u}\,du$

$u=\arctan(3x)$

$$du=\frac{1}{1+(3x)^2}\cdot 3\,dx$$

$$\frac{1}{3}\int u\,du=\frac{1}{3}\cdot\frac{1}{2}u^2+C$$

$$=\frac{1}{6}(\arctan(3x))^2+C$$

trig subst.

$\boxed{x=2\tan(\theta)}$ $dx=2\sec^2(\theta)\,d\theta$

$$\int \frac{1}{\sqrt{4+4\tan^2(\theta)}}\,2\sec^2(\theta)\,d\theta$$

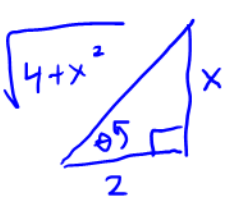
$$=\int \frac{\sec^2(\theta)}{\sqrt{1+\tan^2(\theta)}}\,d\theta=\int \frac{\sec^2(\theta)}{\sqrt{\sec^2(\theta)}}\,d\theta$$

$$=\int \sec(\theta)\,d\theta$$

$$=\ln|\sec(\theta)+\tan(\theta)|+C$$

$x=2\tan(\theta)$

$\Rightarrow \tan(\theta)=\frac{x}{2}$



$$\Rightarrow \sec(\theta)=\frac{\sqrt{4+x^2}}{2}$$

$$=\ln\left|\frac{\sqrt{4+x^2}}{2}+\frac{x}{2}\right|+C$$

$$\int 3\ln(4x)\,dx = 3\int \ln(4x)\,dx$$

parts

$u = \ln(4x)$ $\quad du = \frac{1}{x}\,dx$

$dv = dx$ $\quad v = x$

$$= 3\left[x\ln(4x) - \int dx\right]$$

$$= 3x\ln(4x) - 3x + C$$

$$\int \frac{x^2+5x+2}{(x+1)(x^2+1)}\,dx$$

pfd

$2 < 3 \Rightarrow$ No division

$$\frac{\overbrace{x^2+5x+2}^{\text{degree } 2}}{\underbrace{(x+1)}_{\text{Linear}}\underbrace{(x^2+1)}_{\text{irred. quad.}}} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

degree 3

$$\Leftrightarrow \quad x^2+5x+2 = A(x^2+1) + (Bx+C)(x+1)$$

$x=-1$: $\quad -2 = 2A \quad \Rightarrow A = -1$

Groc. Shop

$$x^2+5x+2 = -x^2-1 + Bx^2 + Cx + Bx + C$$

x^2: $\quad 1 = -1 + B \quad \Rightarrow B = 2$

x: $\quad 5 = C + B = C + 2 \quad \Rightarrow C = 3.$

$$\frac{x^2+5x+2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$A = -1$

$B = 2$

$C = 3$

$$\therefore \int \frac{x^2+5x+2}{(x+1)(x^2+1)}dx = \int \left[\frac{-1}{x+1} + \frac{2x+3}{x^2+1} \right] dx$$

$$= -\ln|x+1| + \int \frac{2x+3}{x^2+1}dx$$

$$= -\ln|x+1| + \int \frac{2x}{x^2+1}dx + \int \frac{3}{x^2+1}dx$$

$$= -\ln|x+1| + \ln|x^2+1| + 3\arctan(x) + C$$

$a>0$

$$\int \frac{1}{a^2+u^2}\,du = \int \frac{1}{a^2\left(1+\frac{u^2}{a^2}\right)}\,du$$

$$= \boxed{\frac{1}{a^2}\int \frac{1}{1+\left(\frac{u}{a}\right)^2}\,du}$$

$v = \frac{u}{a}$

$dv = \frac{1}{a}\,du$

$$= \frac{1}{a}\int \frac{1}{1+v^2}\,dv$$

$$= \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \int \frac{1}{\sqrt{a^2\left(1-u^2/a^2\right)}}\,du = \boxed{\frac{1}{a}\int \frac{1}{\sqrt{1-\left(\frac{u}{a}\right)^2}}\,du}$$

$$= \int \frac{1}{\sqrt{1-v^2}}\,dv$$

$$= \arcsin\left(\frac{u}{a}\right)+C$$

use derivatives

9. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

Limit ⇒ bounded

a. $a_n = \dfrac{2n}{1+n}$

b. $a_n = \dfrac{\cos n}{n}$

10. Determine if the following sequences converge or diverge. If they converge, give the limit.

bounces btwn -1 and 1

a. $\left\{(-1)^n\left(\underbrace{\dfrac{n}{n+1}}_{1}\right)\right\} \rightarrow$ No limit

b. $\left\{\dfrac{6n^2-2n+1}{4n^2-1}\right\} \rightarrow \dfrac{3}{2}$

c. $\left\{\dfrac{(n+2)!}{n!}\right\} = \{(n+2)(n+1)\} \rightarrow \infty$ diverges.

d. $\left\{\dfrac{3}{e^n}\right\} \rightarrow 0$

e. $\left\{\dfrac{4n+1}{n^2-3n}\right\} \rightarrow 0$

f. $\left\{\dfrac{e^n}{n^3}\right\} \rightarrow \infty$ diverges

11. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

$\cos(\pi n) = (-1)^n$

\+ a. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sqrt{n}}{n+3}$ Cond.

\+ b. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos \pi n}{n^2}$ Abs.

c. $\displaystyle\sum_{n=0}^{\infty}\frac{4n(-1)^n}{3n^2+2n+1}$ Cond.

\+ d. $\displaystyle\sum_{n=0}^{\infty}\frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ Cond.

e. $\displaystyle\sum_{n=0}^{\infty}\frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ diverges. Terms do not go to 0.

\+ f. $\displaystyle\sum_{n=0}^{\infty}\left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$ diverges. " " " " " "

use derivatives

9. Determine if the following sequences are monotonic. Also indicate if the sequence is bounded and if it is give the least upper bound and/or greatest lower bound.

Limit ⇒ bounded

a. $a_n = \dfrac{2n}{1+n}$

b. $a_n = \dfrac{\cos n}{n}$

$\lim\limits_{n\to\infty} \dfrac{2n}{1+n} = 2$ $\therefore$ the sequence is bounded (b/c if a sequence has a limit, it is bounded)

Monotone: $f(x) = \dfrac{2x}{1+x}$, $x \geq 1$.

$$f'(x) = \frac{(1+x)\cdot 2 - 2x}{(1+x)^2} = \frac{2}{(1+x)^2} > 0$$

$\therefore$ $f(x)$ is increasing ⇒ the sequence is increasing.

→ i.e. monotone.

$\dfrac{2n}{1+n}$. increasing. limit is 2

LUB = 2

(assume n starts at 1)

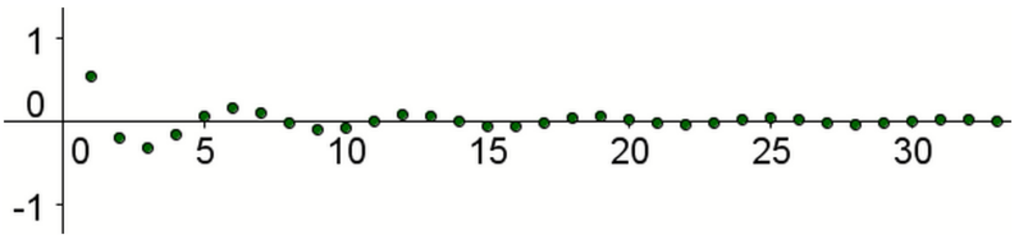
GLB = First term. = 1.

$\dfrac{\cos(n)}{n}$.

$\lim_{n \to \infty} \dfrac{\cos(n)}{n} = 0$

$\therefore$ the sequence is bounded.

<u>Monotone</u>? <u>No</u>. Terms go back and forth btwn + and − b/c of cos(n).



<u>LUB</u> : occurs when $n=1 \rightarrow \dfrac{\cos(1)}{1} = \cos(1)$

<u>GLB</u> : occurs when $n=3 \rightarrow \dfrac{\cos(3)}{3}$.

+ a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$ Cond.

+ b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$ Abs.

c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$ Cond.

+ d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$ Cond.

e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$ diverges.

+ f. $\sum_{n=0}^{\infty} \left(4(-1)^n\left(\frac{n}{n+3}\right)^n\right)$ diverges.

a. $\sum \frac{(-1)^{n+1}\sqrt{n}}{n+3}$. $\boxed{\text{Check ABS.}}$ NO

$\sum \frac{\sqrt{n}}{n+3}$.

Do LCT with $\sum \frac{1}{\sqrt{n}}$

which diverges.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n+3}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{n}{n+3} = 1 > 0$$

$\Rightarrow \sum \frac{\sqrt{n}}{n+3}$ diverges.

Conditional? Use the A.S.T.

$$\sum (-1)^n \boxed{\frac{\sqrt{n}}{n+3}}$$

1. $\frac{\sqrt{n}}{n+3} > 0$ (eventually) ✓

2. $\lim_{n\to\infty} \frac{\sqrt{n}}{n+3} = 0$ ✓

3. Terms eventually decrease.

$$f(x) = \frac{\sqrt{x}}{x+3} \qquad \underline{\underline{x > 0}}$$

$$f'(x) = \frac{(x+3)\cdot\frac{1}{2\sqrt{x}} - \sqrt{x}}{(x+3)^2} = \frac{\frac{1}{2}\sqrt{x} + \frac{3}{2\sqrt{x}} - \sqrt{x}}{(x+3)^2}$$

dec. inc

$$= \frac{\frac{3}{2\sqrt{x}} - \frac{1}{2}\sqrt{x}}{(x+3)^2}$$

<u>Note</u>: at $x=4$

$\frac{3}{4} - 1 < 0$

$\Rightarrow f'(x) < 0$ for $x \geq 4$

$\Rightarrow f$ eventually decreasing

$\Rightarrow \frac{\sqrt{n}}{n+3}$ are eventually decreasing

$\therefore$ by the AST $\sum (-1)^n \frac{\sqrt{n}}{n+3}$ converges

$\Longrightarrow$ the series converges conditionally.
(b/c it converges, but not absolutely)

$$\sum \frac{\cos(\pi n)}{n^2} = \sum \frac{(-1)^n}{n^2}$$

ABS? Look at $\sum \frac{1}{n^2}$

This is a convergent p-series.

$\therefore \sum \frac{\cos(\pi n)}{n^2}$ converges Absolutely.

$$\sum \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$$

ABS? Look at $\sum \frac{3}{\sqrt{3n^2+2n+1}}$

Do a LCT with $\sum \frac{1}{n}$.
(which diverges)

$$\lim_{n\to\infty} \left[\frac{\frac{3}{\sqrt{3n^2+2n+1}}}{\frac{1}{n}} \right] = \lim_{n\to\infty} \frac{3n}{\sqrt{3n^2+2n+1}} = \sqrt{3} > 0$$

$\therefore \sum \frac{3}{\sqrt{3n^2+2n+1}}$ diverges.

So our series does not converge absolutely.

Conditional? $\sum (-1)^n \frac{3}{\sqrt{3n^2+2n+1}}$

1. positive? ✓
2. go to zero? ✓
3. decreasing? ✓

$\therefore$ A.S.T. $\Rightarrow$ the series converges.

$\Rightarrow$ we get cond. convergence.
(as is, but not abs)

$$\sum 4(-1)^n \overbrace{\left(\frac{n}{n+3}\right)^n}$$

Diverges b/c terms do not go to zero!

<u>Note</u>:

$$\lim_{n\to\infty}\left(\frac{n}{n+3}\right)^n = \lim_{n\to\infty}\left(\frac{n+3-3}{n+3}\right)^n$$

$$= \lim_{n\to\infty}\left(1-\frac{3}{n+3}\right)^{(n+3)\frac{n}{(n+3)}}$$

$$= \lim_{m\to\infty}\left[\underbrace{\left(1-\frac{3}{m}\right)^m}_{\to e^{-3}}\right]^{\frac{n}{n+3}\to 1}$$

$$=(e^{-3})^1 = e^{-3}.$$

Note: $\arctan(n) \to \frac{\pi}{2}$

g. $\sum_{n=0}^{\infty}\left(\frac{2(-1)^n \arctan n}{3+n^2+n^3}\right)$ ABS

h. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3^n}{4^n+3n}\right)$ ABS (next page)

i. $\sum_{n=0}^{\infty}\left(\frac{(-1)^n 3}{(n+2)\ln(n+2)}\right)$ Cond.

12. Find the sum of the following convergent series:

a. $\sum_{n=0}^{\infty}2\left(-\frac{4}{9}\right)^n$

b. $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{5}{6^n}\right)$

$$\sum_{n=m}^{\infty} r^n = \begin{cases} r^m \cdot \frac{1}{1-r}, & |r|<1 \\ \text{diverges if } & |r|\geq 1 \end{cases}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n - 5\sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n$$

$$= \frac{1}{1-1/3} - 5\cdot\frac{1}{1-1/6}$$

$$= \frac{3}{2} - 6 = -\frac{9}{2}.$$

13. State the indeterminate form and compute the following limits :

$\frac{\infty}{\infty}$ ← a. $\lim_{n\to\infty}\frac{\ln(n+4)}{n+2} = 0$

∞^0 ← b. $\lim_{n\to\infty}(3n)^{\frac{2}{n}} = 1$

1^∞ ← c. $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^{2n} = e^6$

$\frac{0}{0}$ ← d. $\lim_{x\to 0}\frac{x-\sin(2x)}{x+\sin(2x)} = -\frac{1}{3}$

$\frac{0}{0}$ ← e. $\lim_{x\to 0}\frac{e^{x^2}-1}{2x^2} = \frac{1}{2}$

∞^0 ← f. $\lim_{x\to 0+}\left(\frac{1}{x}\right)^x = 1$

$\frac{0}{0}$ ← g. $\lim_{x\to 0}\frac{3e^{x/3}-(3+x)}{x^2} = \frac{1}{6}$

$\frac{\infty}{\infty}$ ← h. $\lim_{x\to\infty}\frac{x^2}{\ln x} = \infty$ d.n.e

$\frac{0}{0}$ ← i. $\lim_{x\to 0}\frac{1+x-e^x}{x(e^x-1)} = ??$

$\frac{0}{0}$ ← j. $\lim_{x\to 0}\frac{\arctan(4x)}{x} = 4$

$$e^{\frac{x}{3}} = 1 + \frac{x}{3} + \frac{1}{2}\left(\frac{x}{3}\right)^2 + \cdots$$

$$3e^{x/3} = 3 + x + \frac{3}{2}\cdot\frac{1}{9}\cdot x^2 + \cdots$$

$$\lim_{u\to 0}\frac{\sin(u)}{u} = 1$$

$\frac{0}{0}$ ← g. $\lim_{x\to 0}\frac{3e^{x/3}-(3+x)}{x^2} = \frac{1}{6}$ *

$\frac{\infty}{\infty}$ ← h. $\lim_{x\to\infty}\frac{x^2}{\ln x} = \infty$ d.n.e

$\frac{0}{0}$ ← i. $\lim_{x\to 0}\frac{1+x-e^x}{x(e^x-1)} = ??$ *

$\frac{0}{0}$ $\lim_{x\to 0}\frac{3e^{x/3}-(3+x)}{x^2} = \frac{1}{6}$ Try L.H.

$\lim_{x\to 0}\frac{e^{x/3}-1}{2x} = \frac{1}{6}$ $\frac{0}{0}$

use L.H. again

$\lim_{x\to 0}\frac{\frac{1}{3}e^{x/3}}{2} = \frac{1}{6}$

$\frac{0}{0}$ $\lim_{x\to 0}\frac{1+x-e^x}{xe^x-x} = -\frac{1}{2}$ Try L.H.

$\lim_{x\to 0}\frac{1-e^x}{xe^x+e^x-1} = -\frac{1}{2}$ $\frac{0}{0}$

use L.H. again.

$\lim_{x\to 0}\frac{-e^x}{xe^x+e^x+e^x} = -\frac{1}{2}$

$$\sum \frac{(-1)^n 3^n}{4^n+3n}$$

ABS? $\sum \frac{3^n}{4^n+3n}$

LCT with $\sum \left(\frac{3}{4}\right)^n$

which is a convergent geom series.

$$\lim_{n\to\infty} \left[\frac{\frac{3^n}{4^n+3n}}{\frac{3^n}{4^n}} \right]$$

$$= \lim_{n\to\infty} \frac{4^n \cdot 3^n}{3^n(4^n+3n)} = 1 < \infty$$

$\therefore \sum \frac{3^n}{4^n+3n}$ converges

$\Rightarrow$ Our series converges absolutely.

14. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$

b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$

15. For each of the problems in number 14, give the antiderivate F of the power series so that F(0)=0.

16. Evaluate each improper integral:

a. $\int_0^{27} x^{-2/3}$

b. $\int_0^{4} \frac{1}{\sqrt{4-x}}$

17. Find the formula for the area of $r = 1 + 2\sin\theta$

a. Inside inner loop

b. Inside outer loop but outside inner loop

c. Inside outer loop and below x-axis

See video for reviews T3 + T4.

yes

18. Find the smallest value of n so that the nth degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at x = 0 approximates $\ln(2)$ with an error of no more than 0.001 (also be able to do this with some of the other Taylor Polynomials)

19. Find the radius of convergence and interval of convergence for the following Power series:

center is 2

$[-1, 5)$ a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$ $R = 3$

$\sum \left|\frac{x-2}{3}\right|^{n+1} \cdot \frac{1}{n+1}$

ratio or root

$|x-2| < 3$

centered at 1

$(-2, 4)$ b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$ $R = 3$ $|x-1| < 3$

center is 0

$(-4, 4)$ c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$ $R = 4$ $|x| < 4$

d. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n n!}{n^n}$ $R = e$ Harder! Do ratio test. $|x| < e$

20. Use logarithmic differentiation to find the derivative of:

a. $y = (3x-1)^{\sin(x)}$

b. $y = (x+1)^{\ln(x)}$

c. $y = (x^2+2)^{(\frac{1}{\ln x})}$

you.

14. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$ ← centered at 0.

b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$

15. For each of the problems in number 14, give the antiderivate F of the power series so that F(0)=0.

also give the interval and radius of convergence for the original series, the derivate and the anti-deriv.

Radius of Convergence: Determined by ABS Conv.

Check $\sum \frac{(n+1)|x|^n}{n^2+2}$

ratio test: $\lim_{n\to\infty} \left[\frac{\frac{(n+2)|x|^{n+1}}{(n+1)^2+2}}{\frac{(n+1)|x|^n}{n^2+2}} \right]$

$$= \lim_{n\to\infty} \frac{(n^2+2)(n+2)|x|^{n+1}}{(n+1)((n+1)^2+2)|x|^n}$$

$$= \lim_{n\to\infty} \frac{(n+2)}{(n+1)} \frac{(n^2+2)}{(n^2+2n+3)} \frac{|x|^{n+1}}{|x|^n}$$

$\to 1 \qquad \to 1 \qquad \to |x|$

$= |x|$

∴ Conv. for $|x|<1$ and diverges for $|x|>1$.

Radius of conv. is $R=1$.

Interval? One of $[-1,1]$, $[-1,1)$, $(-1,1]$, $(-1,1)$.

Plug the values $x=-1,1$ into original series

$$\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2} = f(x)$$

Radius of conv. is 1.

$\underline{x=-1}$: $\sum \frac{(n+1)(-1)^n}{n^2+2}$

Alt. Series.

Use A.S.T. to show it converges.

$\underline{x=1}$: $\sum \frac{(n+1)}{n^2+2}$ diverges (LCT with $\sum \frac{1}{n}$)

$\therefore$ Interval of convergence is $[-1,1)$.

$$f'(x) = \sum_{n=1}^{\infty} \frac{(n+1)n\, x^{n-1}}{n^2+2}$$

Radius of conv is 1 b/c it is the same as the radius of conv. for $f(x)$.

$\underline{\text{Interval}}$: Check $x=-1,1$

$\underline{x=-1}$: $\sum_{n=1}^{\infty} \frac{(n+1)n(-1)^n}{n^2+2}$ diverges b/c terms do not go to 0.

$\underline{x=1}$: $\sum_{n=1}^{\infty} \frac{(n+1)n}{n^2+2}$ $\underline{\text{Same deal}}$.

$\therefore$ the interval of conv. for $f'(x)$ is $(-1,1)$.

Now look at the anti-deriv. $F(x)$ where $F(0)=0$.

Next page.

$$\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2} = f(x)$$

$$F(x) = \int f(x)\,dx = \int \sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}\,dx$$

$$= \sum_{n=0}^{\infty} \int \frac{(n+1)}{n^2+2} x^n\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(n+1)x^{n+1}}{(n^2+2)(n+1)} + C$$

<u>AND</u> $F(0) = 0. \Rightarrow C = 0.$

$$\Rightarrow F(x) = \sum_{n=0}^{\infty} \frac{(n+1)x^{n+1}}{(n^2+2)(n+1)} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n^2+2}$$

Radius of convergence is $R = 1$ b/c it is the same as the radius of conv. for $f(x)$.

<u>Interval?</u> Check $x=-1$, $x=1$.

<u>$x=-1$:</u> $\sum \frac{(-1)^{n+1}}{n^2+2}$. Converges. (even absolutely)

<u>$x=1$:</u> $\sum \frac{1}{n^2+2}$ converges

$\therefore$ The interval of conv is $[-1,1]$.

21. Determine the convergence or divergence for each series with the given general term:

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n^3}}$	Div.	p-series
$\sum_{n=1}^{\infty}\frac{2^n}{n^3}$	Div.	terms do not go to 0
$\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n}\right) = \sum\frac{-1}{(n+1)n}$	Conv. ABS	LCT p-series
$\sum_{n=1}^{\infty}\frac{3^{2n}}{n!}$	Conv.	ratio
$\sum_{n=1}^{\infty}\cos(\pi n) = \sum(-1)^n$	Div	Terms do not go to 0.
$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n} = \sum\frac{1}{\sqrt{n}}$	Div.	p-series
$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n^2}{3n^3+1}$	Conv.	A.S.T.
$\sum_{n=0}^{\infty}3\left(-\frac{1}{2}\right)^n$	Conv.	Geom. Series test
$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$	Conv.	Integral Test
$\sum_{n=1}^{\infty}ne^{-n^3} = \sum\frac{n}{e^{n^3}}$	Conv.	ratio or root
$\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^n$	Div.	Terms do not go to 0
$\sum_{n=1}^{\infty}\frac{1}{n^3+1}$	Conv.	LCT with $\sum\frac{1}{n^3}$
$\sum_{n=0}^{\infty}\left(\frac{2}{9}\right)^n$	Conv.	Geom Series test
$\sum_{n=1}^{\infty}\frac{n^2}{2^n}$	Conv.	ratio or root

$\sum_{n=1}^{\infty}(0.34)^n$	Conv.	Geom Series Test
$\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$	Conv.	P-series
$\sum_{n=1}^{\infty}\frac{1}{2n+1}$	Div.	LCT with $\sum \frac{1}{n}$

$f(x)=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}x^{2n}$. Give $f^{(8)}(0)$, $f^{(21)}(0)$ and $f^{(22)}(0)$.

$$= \sum_{m=0} \frac{f^{(m)}(0)}{m!}x^m$$

in front of x^8 term. $\rightarrow n=4$

$$\frac{f^{(8)}(0)}{8!} = \frac{(-1)^4}{4^2+1} \Rightarrow f^{(8)}(0) = \frac{8!}{17}$$

$f^{(21)}(0)=0$ since there is no x^{21} term.

$$\frac{f^{(22)}(0)}{22!} = \frac{(-1)^{11}}{11^2+1}$$

Solve.

Give a power series expansion for $f(x)=\frac{1}{(x+1)^2}$ centered at 0.

Note: $\frac{d}{dx}\frac{1}{1+x} = \frac{-1}{(1+x)^2} \Rightarrow -\frac{d}{dx}\frac{1}{1+x} = \frac{1}{(1+x)^2}$

Give a value of n so that the Taylor polynomial of degree n centered at 0 can be used to approximate $\cos(x/2)$ on the interval $[-1,1]$ with error no greater than 0.01.

$$-\frac{d}{dx}\left[\frac{1}{1-(-x)}\right] = -\frac{d}{dx}\sum_{n=0}^{\infty}(-x)^n$$

$\frac{1}{1-r}$

rad. of conv. 1.

$$= -\frac{d}{dx}\sum_{n=0}^{\infty}(-1)^n x^n$$

$$= \sum_{n=1}^{\infty}(-1)^{n+1}\, n\, x^{n-1}$$

rad. of conv. is 1.