Math 1432 - 13209
Jeff Morgan - 651 PGH
jmorgan@math.uh.edu
Office Hours: 11:00 - Noon MWF
Course Homepage

[Link: http://www.math.uh.edu/~jmorgan/Math1432]

All of the important course information is posted at this site!

---

Key Points

CourseWare Accounts
  Access Codes
  Textbook
  Homework
  Daily Poppers
  EMCF
  Written Quizzes
  Online Quizzes
  Exams and Final Exam

[Link: http://www.math.uh.edu/~jmorgan/Math1432]
Textbook

You do not need to purchase a physical copy of the text. You will have access to the text electronically on CourseWare for the first two weeks of the course, and also thereafter, provided you enter your Course Access Code.

If you want a physical copy of the text for the course, then purchase CALCULUS, 9th edition. Authors: Salas/Hille/Eisen. Publisher: John Wiley & Sons, Inc.

Even if you purchase a physical copy of the text, you will still need the Course Access Code to access the additional learning materials, including the online electronic quizzes and EMCF assignments.

Written Homework

A new assignment will be given every week. Watch the course homepage for more information.

Note: If you do not submit your homework in the proper form, then your written quiz grade reverts to a ZERO.

http://www.math.uh.edu/~jmorgan/Math1432

Daily Poppers

Daily Poppers will be given in lecture starting in week 3. You need a special "Popper" form. Go to the University Center Bookstore and ask for the packet for Math 1432, Section 13209.

http://www.math.uh.edu/~jmorgan/Math1432

EMCF

"EMCF" = "Electronic Multiple Choice Form"

EMCF answer sheets are available through CourseWare at http://www.casa.uh.edu and the questions will be posted on the course homepage.

In fact, EMCF01 is posted and due on Wednesday morning.

http://www.math.uh.edu/~jmorgan/Math1432

Written Quizzes

Written quizzes will be given every Friday in recitation. You are responsible for all of the material covered through Wednesday each week.

http://www.math.uh.edu/~jmorgan/Math1432

Online Quizzes

All Online Quizzes, Practice Test 1 and Test 1 are available NOW through CourseWare at http://www.casa.uh.edu

2 attempts

Test 1 is available online NOW at http://www.casa.uh.edu Tests 2, 3 and 4 will be proctored in CASA. The final exam is comprehensive. Dates will be announced in class at least 2 weeks in advance. The exam schedule will be available on CourseWare at least 2 weeks prior to each exam. More information is available at

http://www.math.uh.edu/~jmorgan/Math1432

Four Tests and a Final Exam

2 attempts

the highest score is recorded
Attendance and Classroom Behavior

- Come to class on time.
- Be prepared to start on time.
- Turn off your cell phone.
- Do not read the newspaper, cruise the web, or do anything that might disturb other students.
- Pay attention.
- Ask and answer questions.

http://www.math.uh.edu/~jmorgan/Math1432

Grades:

400 points - Tests 1, 2, 3, 4 (100 points each)
100 points - weekly written quizzes and online quizzes
100 points - poppers and EMCFs (equally weighted)
200 points - final exam
800 points total

Notes:

1. The percentage grade on the final exam can be used to replace your lowest test score.
2. The practice tests count as online quizzes, and the practice final exam counts as 2 online quizzes.

Question: What determines whether a function is invertible?

Geometrically? **Horizontal line test:**

Regardless of the choice of horizontal line, it will intersect the graph of the function at no more than one point.

Algebraically? \( f \) is 1-1, 

i.e. if \( x_1, x_2 \) are in the domain of \( f \) so that \( f(x_1) = f(x_2) \), then \( x_1 = x_2 \).

Which Functions are Invertible?
Which Functions are Invertible?

Fail!!

Invertible!!

Fail!!

Invertible!!

Definition: A function $f$ is said to be one-to-one if there are no two distinct numbers in the domain of $f$ at which $f$ takes on the same value.

i.e. if and only if

$f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Theorem: If $f$ is a one-to-one function, then there is one and only one function $g$ with domain equal to the range of $f$ that satisfies the equation

$f(g(x)) = x$ for all $x$ in the range of $f$.

$g$ is the inverse of $f$.

Note: $f(f^{-1}(x)) = x$

and

$f^{-1}(f(x)) = x$.

How can we find the formula for the inverse of a function?

1. Start with $y = f(x)$.

2. Solve this equation for $x$ in terms of $y$. This gives an equation that looks like $x = g(y)$.

3. Switch!!! Write $y = g(x)$.

4. The function $g$ is the inverse of $f$.

Note: You can’t.
Example: Is \( f(x) = 2x - 3 \) invertible? If so, find its inverse.

\[
\text{The graph is a line with slope } 2, \text{ so it satisfies the HLT.}\n\]

\[
\begin{align*}
\text{Solve for } x. \\
2f^{-1}(x) - 3 &= (x + 3) - 3 = x \\
f^{-1}(x) &= \frac{1}{2}(x + 3) \\
\end{align*}
\]

Theorem: If \( f \) is either an increasing function or a decreasing function on an interval, then \( f \) is an invertible function on that interval.

Question: What tool do we have to determine whether a function is increasing or decreasing on an interval?

\[\text{Derivative}\]

If \( f'(x) > 0 \) at all but finitely many points on an interval, then \( f \) is increasing on the interval... and consequently invertible!!

If \( f'(x) < 0 \) at all but finitely many points on an interval, then \( f \) is decreasing on the interval... and consequently invertible!!

Example: Show that \( f(x) = x^2 + 3x \) is invertible on the interval \([0, 10]\).

\[
f'(x) = 3x^2 + 3 > 0 \quad \text{for all } x.
\]

Note: It is possible, but very difficult to write \( f^{-1}(x) \).

So \( f'(x) \) is certainly positive on \([0, 10]\).

\[\Rightarrow f \text{ is increasing} \quad \Rightarrow f \text{ is invertible}.
\]

Example: Show that \( f(x) = \sin(x) \) is invertible on the interval \([-\pi/2, \pi/2]\).

\[
f'(x) = \cos(x)\]

is positive except at \( x = \pm \frac{\pi}{2} \).

\[\Rightarrow f \text{ is invertible}.
\]

\[
f^{-1}(x) \text{ exists}.
\]
Relationships Between a Function and Its Inverse

Graphs

\( f \) and \( f^{-1} \) have graphs that are mirror images across \( y = x \).

Continuity

Derivatives

The Graphs of \( f \) and \( f^{-1} \) are Symmetric about the Line \( y = x \)

Theorem: If \( f(x) \) is continuous and invertible on an interval, then \( f^{-1}(x) \) is continuous.

Why? No holes or breaks in \( y = f(x) \) \( \Rightarrow \) no holes or breaks in the mirror image.

The Inverse Function Theorem

If \( f(x) \) is differentiable and invertible on an interval, and \( f'(x) \) is nonzero, then \( f^{-1}(x) \) is differentiable. Also, if \( a \) is in the interval, \( f(a) = b \), and \( f'(a) \neq 0 \), then

\[
\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}
\]

Why?

\[ f(f^{-1}(x)) = x \]

Differentiate with respect to \( x \).

Thus \( f^{-1} \) is differentiable.

Why?

\[ f'(f^{-1}(x)) f^{-1}'(k) = 1 \]

\[ f'(f^{-1}(b)) f^{-1}'(b) = 1 \Rightarrow \]
Example: We saw earlier that $f(x) = x^3 + 3x$ is invertible on the interval $[0, 10]$.

Find $(f^{-1})'(4)$.

\[ (f^{-1})'(4) = \frac{1}{f'(a)} \]

where \( f(a) = 4 \).

\[ a^3 + 3a = 4 \]
\[ a = 1 \]

\[ f'(a) = 3a^2 + 3 \]
\[ f'(1) = 6 \]

Then give the equation of the tangent line to the graph of $f''(y)$ at $y = 1/2$.

\[ (f^{-1})''(4) = \frac{1}{f''(a)} \]

where $f(a) = 4$.

\[ \sin(a) = \frac{1}{2} \]
\[ \Rightarrow -\frac{\pi}{2} < a < \frac{\pi}{2} \]
\[ \Rightarrow a = \frac{\pi}{6} \]

\[ f'(a) = \cos(\pi/6) \]
\[ \Rightarrow (f^{-1})'(4) = \frac{1}{f''(\pi/6)} = \frac{1}{6\sqrt{3}} = \frac{\sqrt{3}}{18} \]

Tangent line at $4$: \[ y = \frac{\sqrt{3}}{18} + \frac{\pi}{6} \]

Equation: \[ y - \frac{\pi}{6} = \frac{\sqrt{3}}{18}(x - \frac{1}{2}) \]