

## Math 1432 - 13209

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Office Hours: 11:00 - Noon MWF  
**Course Homepage**

 <http://www.math.uh.edu/~jmorgan/Math1432>

**All of the important course information  
is posted at this site!**

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**Read the Syllabus**

Use the **Discussion Board on CourseWare** to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. Note: Practice Tests count the same as online quizzes.


### Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 Note: Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 Blank sheets Exam, PT, and all Online Quizzes are open	15 UH events this week	16 Homework 1 posted	17 EMCF01 due at 9am	18 Quiz in lab/workshop	19 EMCF02 due at 9am
20	21 MLK Day No Class	22 Last day to add	23 EMCF01 due at 9am Homework 1 due in lab/workshop Homework 2 posted	24 Exam 1 and PT1 close	25 EMCF04 due at 9am Quiz in lab/workshop	26 Quiz 1 closes (7.1-7.2)
27 Free Access ends today!! Purchase your Access Code!!	28 EMCF05 due at 9am Homework 2 due in lab/workshop	29	30 EMCF06 due at 9am Homework 3 posted Last day to drop without receiving a W	31 Register on CourseWare for Exam 2	February 1 EMCF07 due at 9am Quiz in lab/workshop	2 Quiz 2 closes (7.3-7.5)

2 attempts

Count me a quiz.

## Key Points

CourseWare Accounts  
 Access Codes  
Textbook  
Homework  
Daily Poppers  
EMCF  
Written Quizzes  
Online Quizzes  
Exams and Final Exam

<http://www.math.uh.edu/~jmorgan/Math1432>



## CourseWare Accounts

<http://www.casa.uh.edu>

Textbook, online quizzes, EMCF, discussion board, exam scheduler  
**and Test 1!!**

**Access Codes**

2 attempts

All students must purchase an Access Code from the University Book Store and enter it on CourseWare. You have unrestricted access for two weeks. An Access Code is required after that time. The Access Code gives you online access to the textbook, online quizzes, EMCFs and discussion board.

### Textbook

You do not need to purchase a physical copy of the text. You will have access to the text electronically on CourseWare for the first two weeks of the course, and also thereafter, provided you enter your **Course Access Code**.

If you want a physical copy of the text for the course, then purchase CALCULUS, 9th edition. Authors: Salas/Hille/Etgen. Publisher: John Wiley & Sons, Inc.

**Even if you purchase a physical copy of the text, you will still need the Course Access Code** to access the additional learning materials, including the online electronic quizzes and EMCF assignments.

### Written Homework

A new assignment will be given every week. Watch the course homepage for more information.

Note: If you do not submit your homework in the proper form, then your written quiz grade reverts to a ZERO.

<http://www.math.uh.edu/~jmorgan/Math1432>

### \* Daily Poppers \*

Daily Poppers will be given in lecture starting in week 3. You need a special "Popper" form. Go to the **University Center Bookstore** and ask for the packet for Math 1432, Section 13209.

<http://www.math.uh.edu/~jmorgan/Math1432>

### EMCF

"EMCF" = "Electronic Multiple Choice Form"

EMCF answer sheets are available through *CourseWare* at <http://www.casa.uh.edu>, and the questions will be posted on the course homepage.

In fact, EMCF01 is posted and due on Wednesday morning.

<http://www.math.uh.edu/~jmorgan/Math1432>

### Written Quizzes

Written quizzes will be given every Friday in recitation. You are responsible for all of the material covered through Wednesday each week.

<http://www.math.uh.edu/~jmorgan/Math1432>

### Online Quizzes

All Online Quizzes, Practice Test 1 and Test 1 are available NOW through CourseWare at

<http://www.casa.uh.edu>

2 attempts

### Four Tests and a Final Exam

Test 1 is available online NOW at <http://www.casa.uh.edu>. Tests 2, 3 and 4 will be proctored in CASA. The final exam is comprehensive. Dates will be announced in class at least 2 weeks in advance. The exam scheduler will be available on CourseWare at least 2 weeks prior to each exam. More information is available at

<http://www.math.uh.edu/~jmorgan/Math1432>

2 attempts  
the highest score is recorded

### Attendance and Classroom Behavior

- Come to class on time.
- Be prepared to start on time.
- Turn off your cell phone.
- Do not read the newspaper, cruise the web, or do anything that might disturb other students.
- Pay attention.
- Ask and answer questions.

<http://www.math.uh.edu/~jmorgan/Math1432>



### Grades:

- 400 points** - Tests 1, 2, 3, 4 (100 points each)
- 100 points** - weekly written quizzes and online quizzes
- 100 points** - poppers and EMCFs (equally weighted)
- 200 points** - final exam
- 800 points total**

### Notes:

1. The percentage grade on the final exam can be used to replace your lowest test score.
2. The practice tests count as online quizzes, and the practice final exam counts as 2 online quizzes.

**Question:** What determines whether a function is invertible?

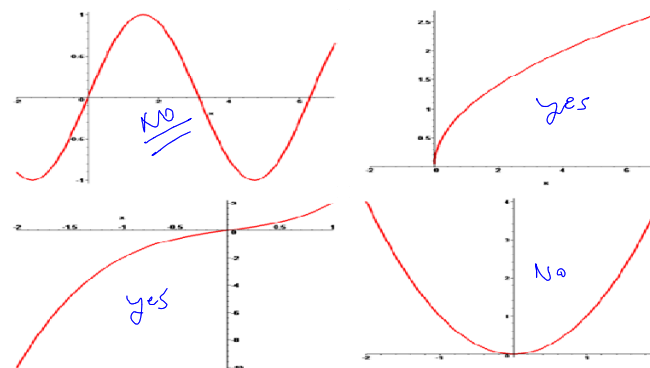
Geometrically? *Horizontal line test.*

Regardless of the choice of horizontal line, it will intersect the graph of the function at no more than one point.

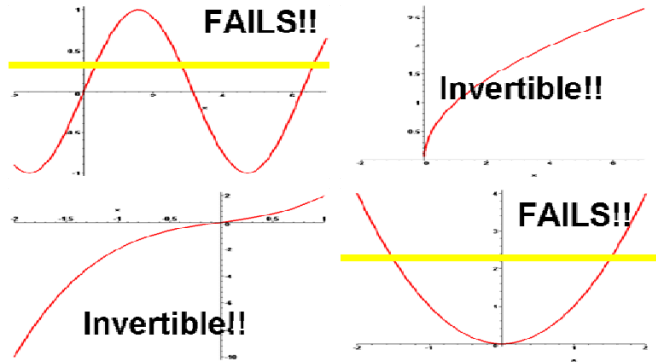
Algebraically? *f is 1-1.*

i.e. If  $x_1, x_2$  are in the domain of  $f$  so that  $f(x_1) = f(x_2)$ , then  $x_1 = x_2$ .

### Which Functions are Invertible?



### Which Functions are Invertible?



**Definition:** A function  $f$  is said to be *one-to-one* if there are no two distinct numbers in the domain of  $f$  at which  $f$  takes on the same value.

i.e. if and only if

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2$$

← invertible

**Theorem:** If  $f$  is a one-to-one function, then there is one and only one function  $g$  with domain equal to the range of  $f$  that satisfies the equation

$$f(g(x)) = x \text{ for all } x \text{ in the range of } f.$$

g is the inverse of f

Also  $g(f(x)) = x$  |  $g = f^{-1}$

$$f^{-1}(f(x)) = x \text{ and } f(f^{-1}(x)) = x$$

### Notation

We will use  $f^{-1}(x)$  to denote the inverse of  $f(x)$ .

**Note:**  $f(f^{-1}(x)) = x$

and

$$f^{-1}(f(x)) = x$$

### How can we find the formula for the inverse of a function?

1. Start with  $y = f(x)$ .
2. Solve this equation for  $x$  in terms of  $y$ .  
This gives an equation that looks like  $x = g(y)$ .
3. Switch!!! Write  $y = g(x)$ .
4. The function  $g$  is the inverse of  $f$ .

Truth  
↕  
Typically,  
you can't.

**Example:** Is  $f(x) = 2x - 3$  invertible? If so, find its inverse.

Yes  
The graph is a line with slope 2, so it satisfies the HLT.

$$\begin{aligned} \text{fun: } f(f^{-1}(x)) &= \\ &= 2f^{-1}(x) - 3 \\ &= (x+3) - 3 = x \end{aligned}$$

Solve for  $x$ .

$$y = 2x - 3 \quad \left\{ \begin{aligned} f^{-1}(f(x)) &= \frac{1}{2}(f(x) + 3) \\ &= \frac{1}{2}(2x) = x \end{aligned} \right.$$

$$y + 3 = 2x \Rightarrow x = \frac{1}{2}(y + 3)$$

Switch variables  $y = \frac{1}{2}(x + 3)$

$$f^{-1}(x) = \frac{1}{2}(x + 3)$$

**Theorem:** If  $f$  is either an increasing function or a decreasing function on an interval, then  $f$  is an invertible function on that interval.

**Question:** What tool do we have to determine whether a function is increasing or decreasing on an interval?

Derivative

If  $f'(x) > 0$  at all but finitely many points on an interval, then  $f$  is increasing on the interval.... and consequently invertible!!

If  $f'(x) < 0$  at all but finitely many points on an interval, then  $f$  is decreasing on the interval.... and consequently invertible!!

**Example:** Show that  $f(x) = x^3 + 3x$  is invertible on the interval  $[0, 10]$ .

$$f'(x) = 3x^2 + 3 > 0$$

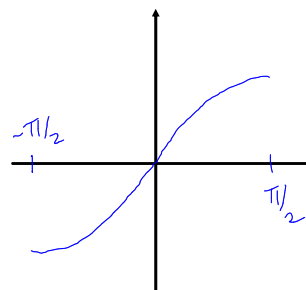
for all  $x$ ,

So  $f'(x)$  is certainly positive on  $[0, 10]$ .

$\therefore f$  is increasing  $\Rightarrow f$  is invertible.

Note: It is possible, but very difficult to "write  $f^{-1}(x)$ ".

**Example:** Show that  $f(x) = \sin(x)$  is invertible on the interval  $[-\pi/2, \pi/2]$ .



$$f'(x) = \cos(x)$$

is positive except at  $x = \pm \frac{\pi}{2}$

i.e.  $f'(x)$  is mostly positive on  $[-\pi/2, \pi/2]$   
 $\Rightarrow f$  is increasing.

$\therefore f^{-1}(x)$  exists.

### Relationships Between a Function and Its Inverse

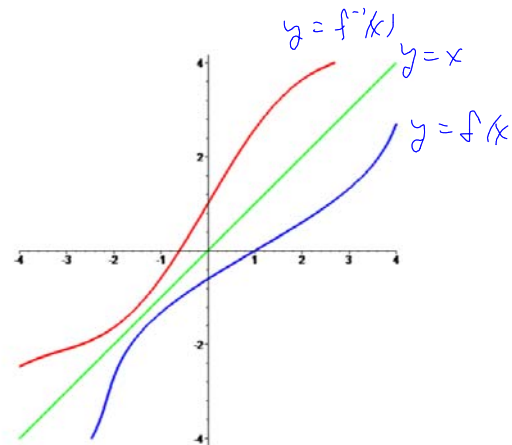
Graphs

$f$  and  $f^{-1}$  have graphs that are mirror images across  $y=x$ .

Continuity  
Derivatives

) →

### The Graphs of $f$ and $f^{-1}$ are Symmetric about the Line $y=x$



**Theorem:** If  $f(x)$  is continuous and invertible on an interval, then  $f^{-1}(x)$  is continuous.

Why??

No holes or breaks in  $y=f(x)$   
 $\Rightarrow$  no holes or breaks  
 in the mirror image.

### The Inverse Function Theorem

If  $f(x)$  is differentiable and invertible on an interval, and  $f'(x)$  is nonzero, then  $f^{-1}(x)$  is differentiable. Also, if  $a$  is in the interval,  $f(a)=b$ , and  $f'(a) \neq 0$ , then

$$a = f^{-1}(b) \quad (f^{-1})'(b) = \frac{1}{f'(a)}$$

Why?

$$f(f^{-1}(x)) = x$$

Diff wrt  $x$ .

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$$

$$f'(f^{-1}(b)) \cdot (f^{-1})'(b) = 1$$

$$f'(a) \cdot (f^{-1})'(b) = 1 \Rightarrow$$

Amazing.  
 You can differentiate

$f^{-1}(x)$   
 w/o knowing  
 $f^{-1}(x)$ .

**Example:** We saw earlier that  $f(x) = x^3 + 3x$   $f'(x) = 3x^2 + 3$  is invertible on the interval  $[0, 10]$ .

Find  $(f^{-1})'(4)$ .

$$\begin{aligned}
 (f^{-1})'(4) &= \frac{1}{f'(a)} \quad \text{where} \\
 &= \frac{1}{f'(1)} \quad \leftarrow \begin{array}{l} a^3 + 3a = 4 \\ a = 1 \\ f(a) = 4. \end{array} \\
 &= \frac{1}{3+3} = \frac{1}{6}.
 \end{aligned}$$

**Example:** We saw earlier that  $f(x) = \sin(x)$  is invertible on the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .  
Find  $(f^{-1})'(1/2)$ .

Then give the equation of the tangent line to the graph of  $f^{-1}(x)$  at  $x = 1/2$ .

$$\begin{aligned}
 (f^{-1})'(1/2) &= \frac{1}{f'(a)} \quad \text{where } f(a) = 1/2 \\
 \sin(a) &= 1/2 \quad \text{for } -\frac{\pi}{2} \leq a \leq \frac{\pi}{2} \\
 \Rightarrow a &= \pi/6. \quad f'(x) = \cos(x) \\
 \therefore (f^{-1})'(1/2) &= \frac{1}{f'(\pi/6)} = \frac{1}{\cos(\pi/6)} = \frac{2}{\sqrt{3}} \\
 \text{Tangent Line at } 1/2: \quad \text{Point} &= (1/2, f^{-1}(1/2)) \\
 &= (1/2, \pi/6) \\
 \text{slope} &= (f^{-1})'(1/2) = \frac{2}{\sqrt{3}} \\
 \text{Equation:} \quad y - \frac{\pi}{6} &= \frac{2}{\sqrt{3}}(x - 1/2)
 \end{aligned}$$