Math 1432 - 13209
Jeff Morgan - 651 PGH - 11-noon MWF
http://www.math.uh.edu/~jmorgan/Math1432

Read the Syllabus!
Test 1 is available online at CourseWare.
Practice Test 1 is also available and counts as an online quiz.
Homework 1 is posted on the course homepage and due next Wednesday.
EMCF01 is due tomorrow morning at 9am. **A help video was posted.**
EMCF02 is due on Saturday morning at 9am.
Your first written quiz will be given in lab/workshop on Friday.
Online Quiz 1 is Available on CourseWare.
Poppers start week 3. Get your packet at the UC Book Store.
Access Codes are required to continue to access the online text, EMCF, quizzes, etc... See the deadline on the course homepage.

CourseWare = http://www.casa.uh.edu

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Read the Syllabus

Use the Discussion Board on CourseWare to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 **Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 **Notes** **Exam 1, PT1 and all Online Quizzes are open**	15 **UH events this week** **Examples from 7.1 that will help with EMCF01**	16 **Blank Slides** **Homework 1 posted**	17 **EMCF01 due at 9am**	18 Quiz in lab/workshop	19 **EMCF02 due at 9am**
20	21 MLK Day No Class	22. **Last day to add**	23 EMCF03 due at 9am **Homework 1 due** in lab/workshop Homework 2 posted	24 **Exam 1 and PT1 close**	25 EMCF04 due at 9am Quiz in lab/workshop	26 **Quiz 1 closes** (7.1-7.2)

We started discussing inverse functions on Monday. The notes are posted. In addition, I posted a video that might help with EMCF01.

Today.... More Inverse Functions and Logarithms

Make sure you read sections 7.2 and 7.3.

"Review" What were you taught about Logarithms in highschool algebra? (base 10 answers...)

$\log_{10}(1) = 0$

$\log_{10}(10) = 1$

$\log_{10}\left(\frac{1}{10}\right) = -1$

$\log_{10}(x) = y$ iff $x = 10^y$

$\log_{10}(100) = 2$

$\log_{10}(10^x) = x$

"Review" **Properties:** If A and B are positive real numbers, and r is a real number, then

$\log_{10}(AB) = \log_{10}(A) + \log_{10}(B)$

$\log_{10}\left(\frac{A}{B}\right) = \log_{10}(A) - \log_{10}(B)$

$\log_{10}(A^r) = r\log_{10}(A)$

$\log_{10}\left(\frac{1}{A}\right) = -\log_{10}(A)$

"Review"
For arbitrary bases...

Properties: If A and B are positive real numbers, r is a real number, and n is a positive real number which IS NOT 1, then

$\log_n(1) = 0$

$\log_n(n^x) = x$

$\log_n(AB) = \log_n(A) + \log_n(B)$

$\log_n\left(\frac{A}{B}\right) = \log_n(A) - \log_n(B)$

$\log_n(A^r) = r\log_n(A)$

$\log_n\left(\frac{1}{A}\right) = -\log_n(A)$

Also, for $x > 0$

$\log_n(x) = y$

iff

$x = n^y$

"Review" Change of base:

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

$a, b, c > 0$ and $a, c \neq 1$

The Truth!

The defining property of logarithmic functions:

If $n > 0$ with $n \neq 1$ and $A, B > 0$ then
$\log_n(AB) = \log_n(A) + \log_n(B)$

All other properties follow from this property.

So...

We say that a real valued function L defined on the positive real numbers is a logarithmic function if and only if L is not the zero function and

$$L(AB) = L(A) + L(B)$$

(all other properties follow from this one)

(*) **Question: Does $L'(x)$ exist for $x > 0$? If so, what is $L'(x)$?**

A: For $x, y > 0$

$$L(xy) = L(x) + L(y)$$

$$\frac{d}{dy} L(xy) = \frac{d}{dy}\left[L(x) + L(y)\right]$$

$$L'(xy)\cdot x = L'(y)$$

Evaluate at $y = 1$. $\Rightarrow$ $L'(x)x = L'(1)$

$$\Rightarrow L'(x) = \frac{\boxed{L'(1)}}{x}$$

Depends on the base of L.

In fact, we could prove...

If L is a logarithmic function then L is differentiable and there is a constant C so that

$$L'(x) = C\cdot\frac{1}{x}$$

As a result, all logarithmic functions are invertible. Why?

The constant depends upon the base of the logarithm.

Questions:

For $x > 0$ define

1. Is there a function whose derivative is $\frac{1}{x}$? $L(x) = \int_1^x \frac{1}{t}\,dt$

2. Is this a logarithmic function?

yes

3. Is there any other logarithmic function with this derivative?

NO

Question : How can we show that $L(x) = \int_1^x \frac{1}{t}\,dt$ is a logarithmic function?

Note: $L(1) = \int_1^1 \frac{1}{t}\,dt = 0$

Answer : We need to show $L(AB) = L(A) + L(B)$ for $A, B > 0$.

$$\frac{d}{dx}L(Ax) = L'(Ax)\cdot A = \frac{1}{Ax}\cdot A = \frac{1}{x}$$

equal

$$\frac{d}{dx}\left[L(A) + L(x)\right] = 0 + L'(x) = \frac{1}{x}$$

$\therefore$ There is a constant C so that

$$\boxed{L(Ax) = L(A) + L(x)} + C$$

for every $A, x > 0$.

Put in $A = x = 1$.

$$L(1) = L(1) + L(1) + C$$

$$0 = 0 + 0 + C \Rightarrow C = 0.$$

We refer to this special logarithmic function as the *natural logarithm*, and we denote it as $\ln(x)$.

$$\ln(x) = \int_1^x \frac{1}{t}\,dt, \quad x > 0$$

$\ln(x)$ satisfies the logarithmic properties and...

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)} \cdot u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$

$$\ln(|u|) = \begin{cases} \ln(-u), & u < 0 \\ \ln(u), & u > 0 \end{cases}$$

$$\frac{d}{du}\ln(u) = \begin{cases} \frac{d}{du}\ln(-u), & u < 0 \\ \frac{d}{du}\ln(u), & u > 0 \end{cases}$$

$$= \begin{cases} \frac{1}{-u}\cdot(-1), & u < 0 \\ \frac{1}{u}, & u > 0 \end{cases}$$

$$= \begin{cases} \frac{1}{u}, & u < 0 \\ \frac{1}{u}, & u > 0 \end{cases}$$

Example: Solve the equation $\ln((3x+3)(x+5)) = 2\ln(x+5)$

$$\ln((3x+3)(x+5)) = \ln((x+5)^2)$$

Note: ln is invertible.

$\Rightarrow (3x+3)(x+5) = (x+5)^2$

Note: $x=-5$ solves this equation, BUT $x=-5$ is not allowed above since $\ln(0)$ is undefined!

So $x=-5$ is NOT a solution.

$(3x+3) = (x+5)$

$2x = 2 \Rightarrow \boxed{x=1}$

Check the original equation.

$\ln((3x+3)(x+5)) = 2\ln(x+5)$

$\ln(6\cdot 6) \stackrel{?}{=} 2\ln(6)$

$\ln(6^2) \stackrel{?}{=} \ln(6^2)$ Yes ✓

$\therefore$ $x=1$ is the solution.

Examples: $\dfrac{d}{dx}\ln(x^2+1) = \dfrac{1}{x^2+1}\cdot 2x = \dfrac{2x}{x^2+1}$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)}\,u'(x)$$

$u = 2x+3$

$du = 2dx$

$$\int \frac{1}{2x+3}\,dx = \frac{1}{2}\int \frac{2\,dx}{2x+3} = \frac{1}{2}\ln(|2x+3|) + C$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$

$$\frac{1}{6}\int \frac{6x}{3x^2+5}\,dx = \frac{1}{6}\ln(|3x^2+5|) + C$$

$$= \frac{1}{6}\ln(3x^2+5) + C$$

Example: Give the y-intercept of the tangent line to the graph of $f(x) = \ln(3x^2-2)$ at $x=1$.

1. Get the tangent line.

point: $(1, f(1)) = (1, 0)$

slope: $f'(1) = 6$

$f'(x) = \dfrac{1}{3x^2-2}\cdot 6x \Rightarrow f'(1) = 6$

$\Rightarrow \quad y = 6(x-1)$

2. Get the y-intercept.

$y = 6x - 6$

$\therefore$ the y-intercept is -6.

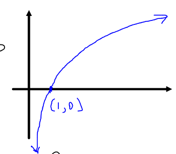
i.e. the line passes through $(0,-6)$.

Can we graph $y = \ln(x)$?

Domain: $x > 0$

Behavior near ∞: $\lim_{x\to\infty} \ln(x) = \infty$ ← very slowly

Behavior near 0: $\lim_{x\to 0^+} \ln(x) = -\infty$



Range: $(-\infty, \infty)$

Slope and Concavity: $\frac{d}{dx}\ln(x) = \frac{1}{x} > 0$

$\Rightarrow \ln(x)$ is increasing.

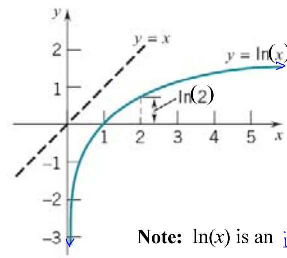
$\frac{d^2}{dx^2}\ln(x) = \frac{-1}{x^2} < 0$

$\Rightarrow$ concave down.

x-intercept:

$\ln(1) = 0 \Rightarrow (1,0)$ is the x-intercept.

The Graph...



Note: $\ln(x)$ is an invertible function.

We will study the inverse of this function in a few days.

What is the "base" of $\ln(x)$?

$$\ln(x) = \log_e(x)$$

e = 2.71828182845904523536028747135266249775724709369995957496696762772407....

(irrational)

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