Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF

http://www.math.uh.edu/~jmorgan/Math1432

Test 1 and **Practice Test 1** are available on CourseWare. **Test 1** counts the same as a major exam. **Practice Test 1** counts the same as an online quiz. Both are due next Thursday.

Homework 1 is posted on the course homepage and due next Wednesday. **Homework 2** will be posted next Wednesday.

EMCF02 is due tomorrow morning at 9am. **EMCF03** is posted, and it is due next Wednesday morning at 9am.

Online Quizzes are Available on CourseWare.

Poppers start in week 3! Get your forms from the Book Store.

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Read the Syllabus

Use the Discussion Board on CourseWare to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 **Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 **Notes** **Exam 1, PT1 and all Online Quizzes are open**	15 **UH events this week** **Examples from 7.1 that will help with EMCF01**	16 Notes: **pg, 4per** Vid notes: **pg, 4per** **Video** **Homework 1 posted**	17 **EMCF01 due at 9am** **Note:** Use a graphing calculator to solve a complicated equation.	18 **Blank Slides** Quiz in lab/workshop	19 **EMCF02 due at 9am**
20	21 MLK Day No Class	22. **UH events this week** **Last day to add**	23 **EMCF03 due at 9am** **Homework 1 due** in lab/workshop Homework 2 posted	24 **Exam 1 and PT1 close**	25 **EMCF04 due at 9am** Quiz in lab/workshop	26 **Quiz 1 closes** (7.1-7.2)

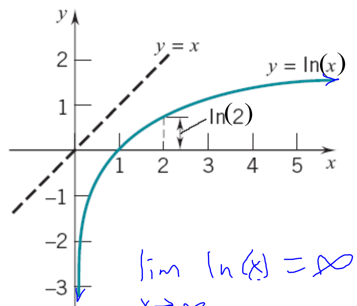
Recall the natural logarithm. $\ln(x) = \log_e(x)$

$\ln(x) = \int_1^x \frac{1}{t}\,dt\,, \quad x > 0$

$\frac{d}{dx}\ln(x) = \frac{1}{x}\,, \quad x > 0$

$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)}\,u'(x)$

$\int \frac{1}{u}\,du = \ln(|u|) + C$



$\lim_{x\to\infty} \ln(x) = \infty$ very slow

$\lim_{x\to 0^+} \ln(x) = -\infty$

The base of the natural logarithm is called *e*.

$$\ln(x) = \log_e(x)$$

What is *e* ?

***e* is an irrational number.**

An excellent approximation is

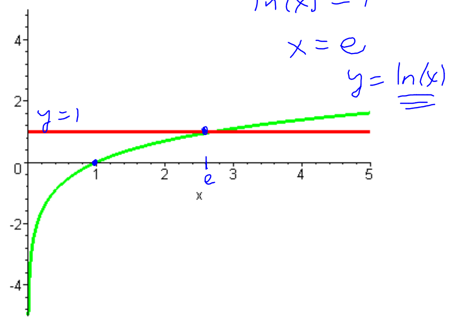
$e =$ 2.71828182845904523536028747135266249775724709 3...

How do we find the value of *e*?

$\ln(x) = 1$

$x = e$

Here is a *very poor* method for approximating *e*.



$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

Note that

$$1 = f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\ln(1+h) - \underbrace{\ln(1)}_{=0}}{h}$$

$$= \lim_{h\to 0} \frac{\ln(1+h)}{h}$$

$\boxed{\text{let } h = \frac{1}{n} \text{ with } n \to \infty}$

$$= \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{1/n}$$

$$= \lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right)$$

$$1 = \lim_{n\to\infty} \ln\left(\left(1+\frac{1}{n}\right)^n\right)$$

$$\therefore \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

Never use $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ as an approximating tool!!

Why?

$e = 2.7182818284590...$

and it very slow bogs down

n	$\left(1+\frac{1}{n}\right)^n$
1	2.
10	2.593742460
100	2.704813829
1000	2.716923932
10000	2.718145927
100000	2.718268237

A better approximation...

$$e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots + \frac{1}{1\cdot 2\cdot 3\cdot \cdots \cdot n} + \cdots$$

Recall:

Converting between bases:
If $a,b,x>0$ and $a,b\neq 1$, then

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Observation: There is a constant k so that

$$\underbrace{\frac{\ln(x)}{\ln(10)} = \log_{10}(x)} = k\int_1^x \frac{1}{t}\,dt$$

What is k? $k = \frac{1}{\ln(10)}$

$$\frac{d}{dx}\log_{10}(x) = \frac{d}{dx}\left(\frac{1}{\ln(10)}\ln(x)\right)$$

$$= \frac{1}{x\ln(10)}$$

$$\frac{d}{dx}\log_5(x) = \frac{1}{x\ln(5)}$$

$a>0$
$a\neq 1$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

Recall:

Converting between bases:

If $a, b, x > 0$ and $a, b \neq 1$, then

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Observation: Suppose $a > 0$ and $a \neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{1}{u(x)\ln(a)} \cdot u'(x)$$

Examples: $\frac{d}{dx}\log_{10}(x^2+1) = \frac{1}{(x^2+1)\ln(10)} \cdot 2x = \boxed{\frac{2x}{(x^2+1)\ln(10)}}$

4) Use a slope chart to determine the shape of the graph of $y = \log_{10}(x^2+1)$.

$y' = 0$ iff $x = 0$

y': – – – – – – 0 + + + + + +

x: –1 0 1

local shape

Refine with y''.

$$y'' = \frac{1}{\ln(10)} \cdot \frac{(x^2+1)\cdot 2 - 2x \cdot 2x}{(x^2+1)^2}$$

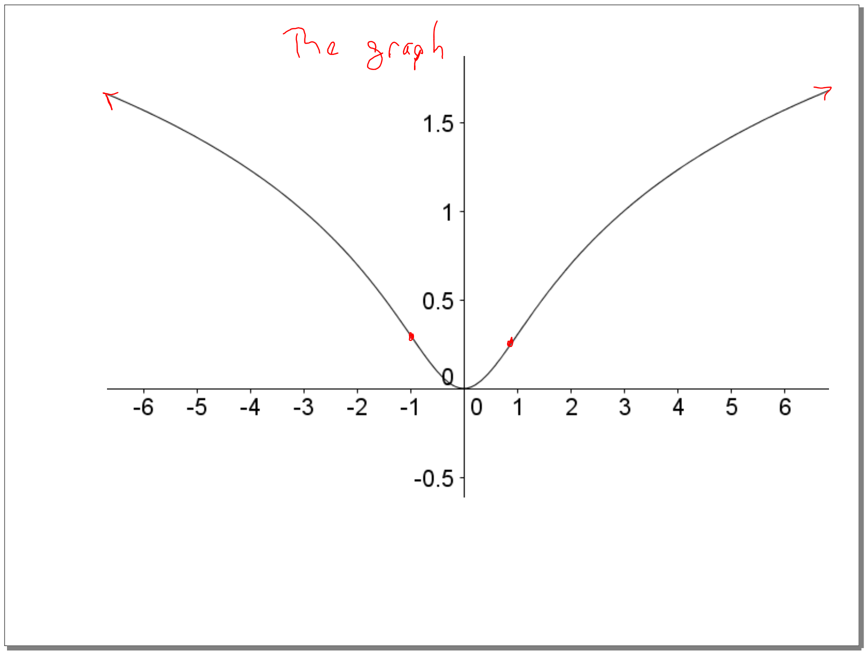
$$= \frac{1}{\ln(10)} \cdot \frac{2 - 2x^2}{(x^2+1)^2}$$

$y'' = 0 \iff x = \pm 1.$

y'': – – – 0 + + + 0 – – –

x: –2 –1 0 1 2

C.D. C.U. C.D.



Question: Suppose $f(x)=\ln(x)$. Is this function invertible, and if so, what is $f^{-1}(x)$?

yes. It is increasing.

$y=\ln(x)=\log_e(x)$

$x=e^y$

$\Rightarrow f^{-1}(x)=e^x$

Discuss domain, range and graphs for both f and f^{-1}.

	Domain	Range
$f(x)=\ln(x)$	$(0,\infty)$	$(-\infty,\infty)$
$f^{-1}(x)=e^x$	$(-\infty,\infty)$	$(0,\infty)$

Comment: $e^x=\exp(x)$

"the exponential function"

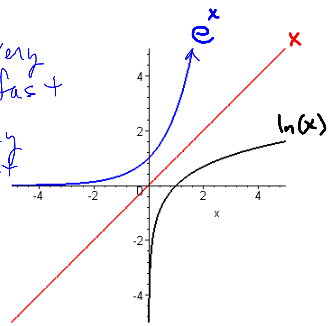
$e = 2.718281828459045235360287471352662497757247093...$

Notation: $\mathbf{exp}(x)$ **is the inverse of** $\mathbf{ln}(x)$ $\quad // \, e^x$

$$\boxed{\exp(x) = e^x}$$

Properties:

1. $\lim\limits_{x\to-\infty} \exp(x) = 0$ very fast
2. $\lim\limits_{x\to\infty} \exp(x) = \infty$ very fast
3. $\exp(0) = e^0 = 1$
4. $\exp(1) = e^1 = e$
5. $\ln(\exp(x)) = x$
6. $\exp(\ln(x)) = x$



b/c $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

What about the derivative of exp(x)?

$$\frac{d}{dx} e^x = ?? \quad e^x$$

Recall : $(f^{-1})'(b) = \dfrac{1}{f'(a)} = a = e^b$

where $f(a) = b$.

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f'(a) = \frac{1}{a}$

$\ln(a) = b$

$a = e^b$

Consequences:

$$\exp(x) = e^x$$

$$\frac{d}{dx}\exp(x) = \exp(x) = e^x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

$$\int e^u\,du = e^u + C$$

Examples:

$$\frac{d}{dx}e^{2x+1} = e^{2x+1}\cdot 2 = 2\cdot e^{2x+1}$$

$$\frac{d}{dx}e^{x\sin(x)} = e^{x\sin(x)}\cdot\left(x\cos(x) + \sin(x)\right)$$

$$\frac{d}{dx}\exp\left(x^2 + x\right) = e^{x^2+x}\cdot(2x+1) = (2x+1)\,e^{x^2+x}$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$\int \cos(x)e^{\boxed{\sin(x)}}dx = \int e^u\,du = e^u + C = e^{\sin(x)} + C$$