Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF

http://www.math.uh.edu/~jmorgan/Math1432

Test 1 and **Practice Test 1** are available on CourseWare. **Test 1** counts the same as a major exam. **Practice Test 1** counts the same as an online quiz. Both are due next Thursday.

Homework 1 is posted on the course homepage and due next Wednesday. **Homework 2** will be posted next Wednesday.

EMCF02 is due tomorrow morning at 9am. **EMCF03** is posted, and it is due next Wednesday morning at 9am.

Online Quizzes are Available on CourseWare.

Poppers start in week 3! Get your forms from the Book Store.

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Read the Syllabus

Use the Discussion Board on CourseWare to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 **Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 **Notes** **Exam 1, PT1 and all Online Quizzes are open**	15 **UH events this week** **Examples from 7.1 that will help with EMCF01**	16 Notes: **pg, 4per** Vid notes: **pg, 4per** **Video** **Homework 1 posted**	17 **EMCF01 due at 9am** **Note:** Use a graphing calculator to solve a complicated equation.	18 **Blank Slides** Quiz in lab/workshop	19 **EMCF02 due at 9am**
20	21 MLK Day No Class	22. **UH events this week** **Last day to add**	23 **EMCF03 due at 9am** **Homework 1 due** in lab/workshop Homework 2 posted	24 **Exam 1 and PT1 close**	25 **EMCF04 due at 9am** Quiz in lab/workshop	26 **Quiz 1 closes** (7.1-7.2)

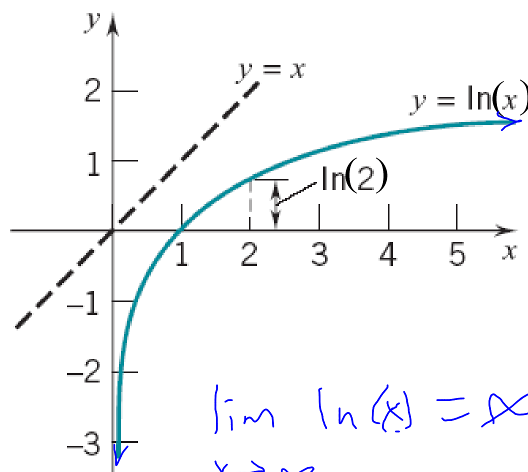
Recall the natural logarithm. $\underline{\underline{\ln(x)}} = \log_e(x)$

$$\ln(x) = \int_1^x \frac{1}{t}\,dt \,, \quad x > 0$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \,, \quad x > 0$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)}\,u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$



$$\lim_{x\to\infty} \ln(x) = \infty$$

very $\underline{\underline{\text{slow}}}$

$$\lim_{x\to 0^+} \ln(x) = -\infty$$

The base of the natural logarithm is called *e*.

$$\ln(x) = \log_e(x)$$

What is *e* ?

***e* is an irrational number.**

An excellent approximation is

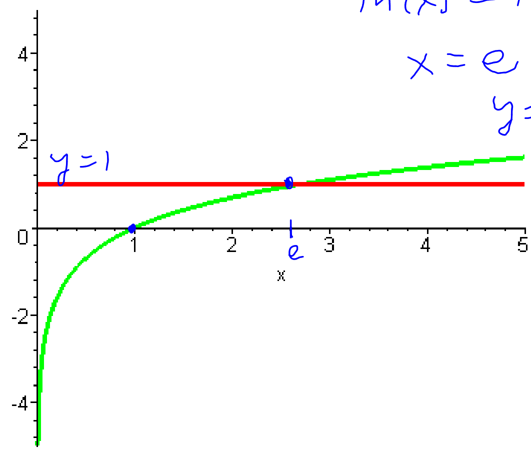
$$e = 2.71828182845904523536028747135266249775724709 3...$$

How do we find the value of *e*?

Here is a *very poor* method for approximating *e*.

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$



$\ln(x) = 1$

$x = e$

$\underline{\underline{y = \ln(x)}}$

Note that

$$\underline{\underline{1}} = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\ln(1+h) - \overbrace{\ln(1)}^{=0}}{h}$$

$$= \lim_{h \to 0} \frac{\ln(1+h)}{h}$$

$\boxed{\text{let } h = \frac{1}{n} \text{ with } n \to \infty}$

$$= \lim_{n \to \infty} \frac{\ln\left(1+\frac{1}{n}\right)}{1/n}$$

$$= \lim_{n \to \infty} n \ln\left(1+\frac{1}{n}\right)$$

$$1 = \lim_{n \to \infty} \ln\left(\underbrace{\left(1+\frac{1}{n}\right)^n}\right)$$

$$\therefore \quad \lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n = e$$

Never use $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ as an approximating tool!!

Why?

$$\begin{bmatrix} n & \left(1+\frac{1}{n}\right)^n \\ 1 & 2. \\ 10 & 2.593742460 \\ 100 & 2.704813829 \\ 1000 & 2.716923932 \\ 10000 & 2.718145927 \\ 100000 & 2.718268237 \end{bmatrix}$$

$e =$ **2.7182818284590...**

and it very slow bogs down

A better approximation...

$$e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots + \frac{1}{1\cdot 2\cdot 3\cdot\cdots\cdot n} + \cdots$$

Recall:

$$\boxed{\begin{array}{c}\textbf{Converting between bases:}\\ \text{If } a,b,x>0 \text{ and } a,b\neq 1, \text{ then}\\ \log_b(x)=\dfrac{\log_a(x)}{\log_a(b)}\end{array}}$$

Observation: There is a constant k so that

$$\underbrace{\frac{\ln(x)}{\ln(10)} = \log_{10}(x)}\, = k\int_1^x \frac{1}{t}\,dt$$

What is k ?

$$k = \frac{1}{\ln(10)}$$

$$\frac{d}{dx}\log_{10}(x) = \frac{d}{dx}\left(\frac{1}{\ln(10)}\ln(x)\right)$$

$$= \frac{1}{x\ln(10)}$$

$$\frac{d}{dx}\log_5(x) = \frac{1}{x\ln(5)}$$

$a>0$

$a\neq 1$

$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

Recall:

Converting between bases:

If $a, b, x > 0$ and $a, b \neq 1$, then

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Observation: Suppose $a > 0$ and $a \neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{1}{u(x)\ln(a)} \cdot u'(x)$$

Examples: $\frac{d}{dx}\log_{10}\left(x^2+1\right) = \frac{1}{(x^2+1)\ln(10)} \cdot 2x = \boxed{\frac{2x}{(x^2+1)\ln(10)}}$

4. Use a slope chart to determine the shape of the graph of $y = \log_{10}\left(x^2+1\right)$.

$y' = 0$ iff $x = 0$

y'	$- - -$	$- -$	0	$+ + +$	$+ + +$
x		-1	0	1	

local shape $\cup$

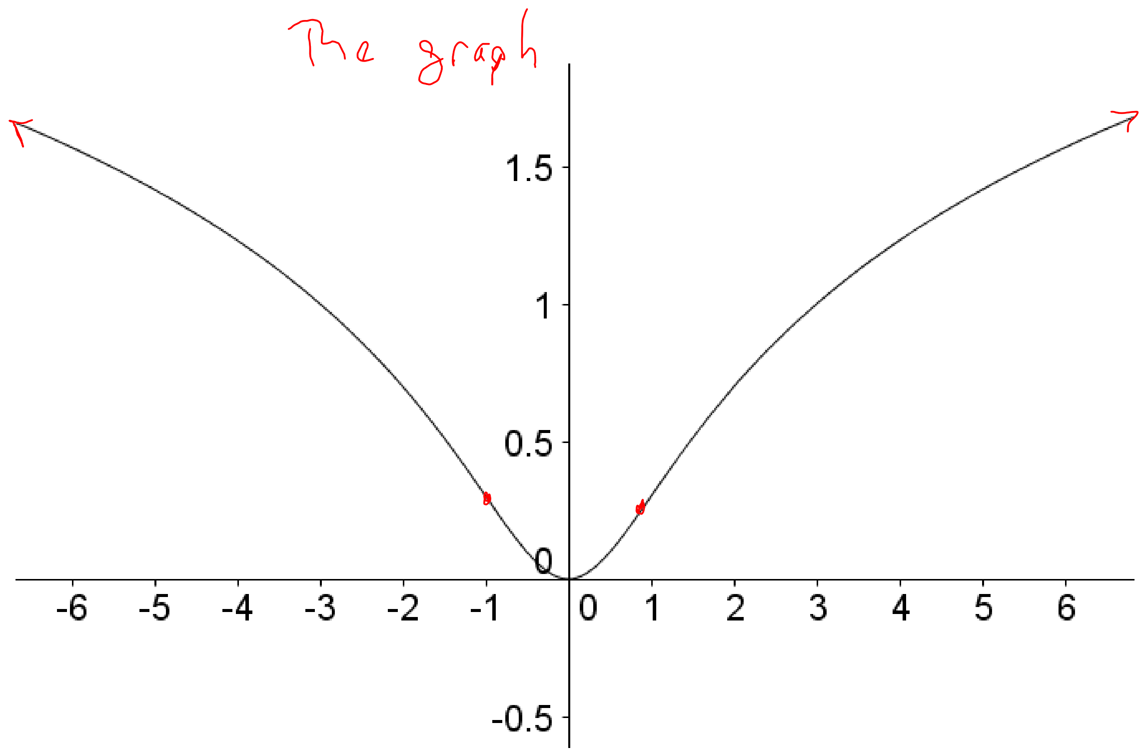
Refine with y''.

$$y'' = \frac{1}{\ln(10)} \cdot \frac{(x^2+1)\cdot 2 - 2x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{1}{\ln(10)} \cdot \frac{2 - 2x^2}{(x^2+1)^2}$$

y''	$- - -$		0	$+ + +$	0	$- - -$		
x	-2		-1	0	1		2	
	C.D.			C.U.		C.D.		

$y'' = 0 \iff x = \pm 1.$


The graph
1.5
1
0.5
0
-0.5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

Question: Suppose $f(x) = \ln(x)$. Is this function invertible, and if so, what is $f^{-1}(x)$?

$y = \ln(x) = \log_e(x)$

$x = e^y$

yes. It is increasing.

$\Rightarrow f^{-1}(x) = e^x$

Discuss domain, range and graphs for both f and f^{-1}.

	Domain	**Range**
$f(x) = \ln(x)$	$(0, \infty)$	$(-\infty, \infty)$
$f^{-1}(x) = e^x$	$(-\infty, \infty)$	$(0, \infty)$

Comment: $e^x = \exp(x)$

"the exponential function"

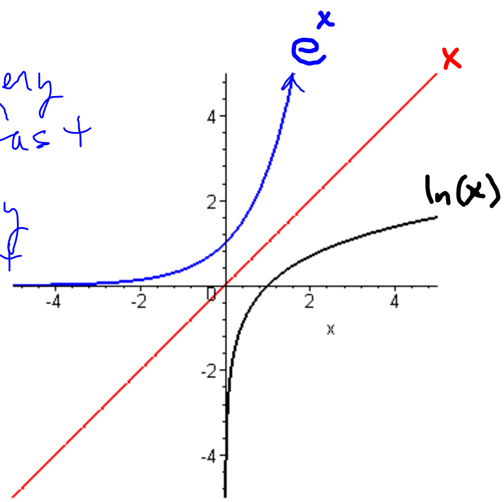
$e = 2.718281828459045235360287471352662497757247093...$

Notation: $\overset{= e^x}{\exp(x)}$ **is the inverse of** $\ln(x)$

$$\boxed{\exp(x) = e^x}$$

Properties:

1. $\lim_{x \to -\infty} \exp(x) = 0$ very fast
2. $\lim_{x \to \infty} \exp(x) = \infty$ very fast
3. $\exp(0) = e^0 = 1$
4. $\exp(1) = e^1 = e$
5. $\ln(\exp(x)) = x$
6. $\exp(\ln(x)) = x$



b/c $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

What about the derivative of exp(x)?

$$\frac{d}{dx}e^x = ?? \quad e^x$$

Recall : $(f^{-1})'(b) = \dfrac{1}{f'(a)} = a = e^b$

where $f(a) = b$.

$f(x) = \ln(x)$

$f'(x) = \dfrac{1}{x}$

$f'(a) = \dfrac{1}{a}$

$\ln(a) = b$

$a = e^b$

Consequences:

$$\boxed{\exp(x) = e^x}$$

$$\frac{d}{dx}\exp(x) = \exp(x) = e^x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\,\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$$

$$\int e^u\,du = e^u + C$$

Examples: $$\frac{d}{dx}e^{2x+1} = e^{2x+1} \cdot 2 = 2 \cdot e^{2x+1}$$

$$\frac{d}{dx}e^{x\sin(x)} = e^{x\sin(x)} \cdot \left(x\cos(x) + \sin(x)\right)$$

$$\frac{d}{dx}\exp\left(x^2 + x\right) = e^{x^2+x} \cdot (2x+1)$$
$$= (2x+1)\,e^{x^2+x}$$

$u = \sin(x)$

$du = \cos(x)dx$

$$\int \cos(x) e^{\boxed{\sin(x)}} dx = \int e^u du = e^u + C$$
$$= e^{\sin(x)} + C$$