Math 1432 - 13209

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Test 1 and **Practice Test 1** are available on CourseWare. **Test 1** counts the same as a major exam. **Practice Test 1** counts the same as an online quiz. Both are due next Thursday.

Homework 1 is posted on the course homepage and due next Wednesday. **Homework 2** will be posted next Wednesday.

EMCF02 is due tomorrow morning at 9am. **EMCF03** is posted, and it is due next Wednesday morning at 9am.

Online Quizzes are Available on CourseWare.

Poppers start in week 3! Get your forms from the Book Store.

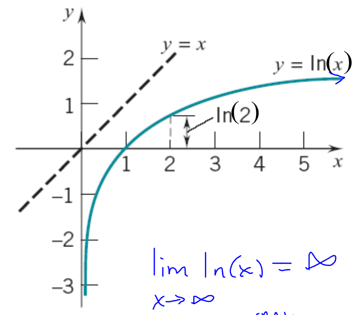
Recall the natural logarithm.

$$\ln(x) = \int_1^x \frac{1}{t}\,dt, \quad x>0$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}, \quad x>0$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)}u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$



$\lim_{x\to\infty} \ln(x) = \infty$

grows very slowly

$\lim_{x\to 0^+} \ln(x) = -\infty$

The base of the natural logarithm is called *e*.

$$\underline{\ln}(x) = \log_e(x)$$

What is *e* ?

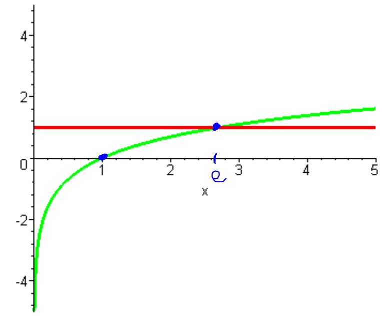
***e* is an irrational number.**

An excellent approximation is

$e =$ 2.71828182845904523536028747135266249775724709...

How do we find the value of *e*?

Here is a *very poor* method for approximating *e*.



$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

Note that

$$1 = f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\ln(1+h) - \overbrace{\ln(1)}^{=0}}{h}$$

$$= \lim_{h\to 0} \frac{\ln(1+h)}{h}$$

set $h = \frac{1}{n}$

let $n \to \infty$

$$= \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n})}{1/n}$$

$$= \lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right)$$

Hm...

$\ln(x) = 1$

$x = e$

$$1 = \lim_{n\to\infty} \ln\left(\left(1+\frac{1}{n}\right)^n\right)$$

$$\therefore \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e.$$

Never use $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ as an approximating tool!!

Why?

$e = 2.7182818284590...$

n	$\left(1+\frac{1}{n}\right)^n$
1	2.
10	2.593742460
100	2.704813829
1000	2.716923932
10000	2.718145927
100000	2.718268237

The approx is slow and it stalls at 10^{16}. $\longrightarrow$

A better approximation...

$$e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots + \frac{1}{1\cdot 2\cdot 3\cdot \cdots \cdot n} + \cdots$$

Recall:

Converting between bases:
If $a,b,x>0$ and $a,b\neq 1$, then
$$\log_b(x)=\frac{\log_a(x)}{\log_a(b)}$$

Observation: There is a constant k so that

$$\frac{\log_e(x)}{\log_e(10)} = \log_{10}(x) = k\int_1^x \frac{1}{t}\,dt = k\ln(x)$$

$$= k\log_e(x)$$

$$= \frac{\ln(x)}{\ln(10)}$$

What is k ? $\quad k = \frac{1}{\ln(10)}$

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)} \Rightarrow \frac{d}{dx}\log_{10}(x) = \frac{1}{x\ln(10)}$$

$$\log_5(x) = \frac{\ln(x)}{\ln(5)} \Rightarrow \frac{d}{dx}\log_5(x) = \frac{1}{x\ln(5)}$$

Recall:

Converting between bases:
If $a,b,x>0$ and $a,b\neq 1$, then
$$\log_b(x)=\frac{\log_a(x)}{\log_a(b)}$$

Observation: Suppose $a>0$ and $a\neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{d}{dx}\frac{\ln(u(x))}{\ln(a)}$$

$$= \frac{1}{u(x)\ln(a)}\cdot u'(x)$$

Examples: $\frac{d}{dx}\log_{10}(x^2+1) = \frac{1}{(x^2+1)\ln(10)} \cdot 2x = \boxed{\frac{2x}{(x^2+1)\ln(10)}}$

Use a slope chart to determine the shape of the graph of $\underline{y = \log_{10}(x^2+1)}$.

Domain: x^2+1 is always > 0 so the domain is $(-\infty, \infty)$.

Note: $y' = 0$ when $x = 0$.

y': $- - - - - -$ 0 $+ + + + + +$

x: -1, 0, 1

local shape: $\cup$

$$y' = \frac{2x}{(x^2+1)\ln(10)}$$

$$y'' = \frac{1}{\ln(10)} \cdot \frac{(x^2+1)\cdot 2 - 2x\cdot 2x}{(x^2+1)^2}$$

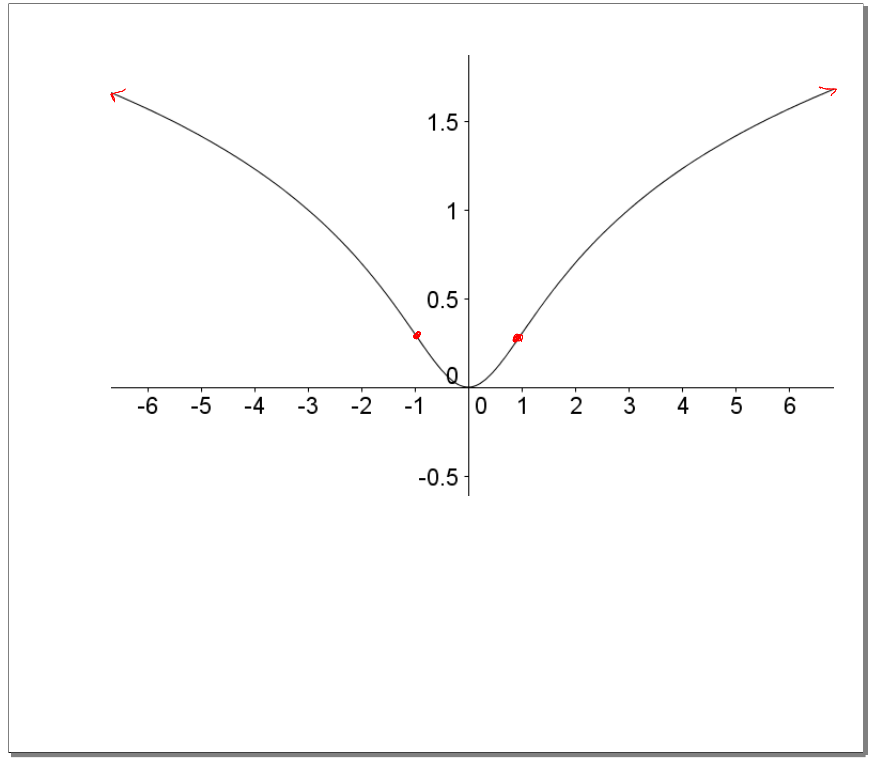
$$= \frac{1}{\ln(10)} \cdot \frac{2-2x^2}{(x^2+1)^2}$$

Note: $y'' = 0 \iff 2-2x^2 = 0 \iff x = \pm 1$

y'': $- - - - -$ 0 $+ + + +$ 0 $- - - -$

x: -2, -1, 0, 1, 2

C.D. C.U. C.D.



Question: Suppose $f(x)=\ln(x)$. Is this function invertible, and if so, what is $f^{-1}(x)$?

yes

$y=\ln(x)=\log_e(x)$ $\quad x=e^y$

$\ln(x)$ is increasing. $\quad \Rightarrow f^{-1}(x)=e^x$

Discuss domain, range and graphs for both f and f^{-1}.

	Domain	Range
$f(x)=\ln(x)$	$(0,\infty)$	$(-\infty,\infty)$
$f^{-1}(x)=e^x$	$(-\infty,\infty)$	$(0,\infty)$

$\parallel$

$\exp(x) \equiv$ the exponential function.

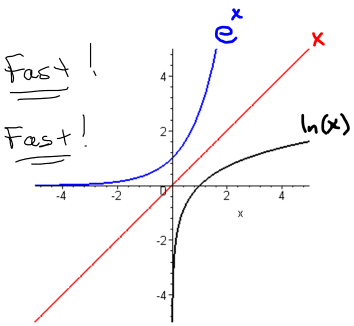
$e=$ 2.718281828459045235360287471352662497757247093...

Notation: exp(x) is the inverse of ln(x)

$$\boxed{\exp(x)=e^x}$$

Properties:

1. $\lim\limits_{x\to-\infty}\exp(x)=0$ Fast!
2. $\lim\limits_{x\to\infty}\exp(x)=\infty$ Fast!
3. $\exp(0)=e^0=1$
4. $\exp(1)=e^1=e$
5. $\ln(\exp(x))=x$
6. $\exp(\ln(x))=x$



What about the derivative of exp(x)?

$$\frac{d}{dx}e^x = ?? \qquad e^x$$

Recall : $\left(f^{-1}\right)'(b) = \frac{1}{f'(a)} = a = e^b$

where $f(a) = b$.

set $f(x) = \ln(x)$. $f^{-1}(x) = e^x$

$f'(x) = \frac{1}{x}$

$f'(a) = \frac{1}{a}$ $\quad \ln(a) = b$

$a = e^b$

$$\frac{d}{dx}e^x = e^x$$

Consequences:

$$\boxed{\exp(x) = e^x}$$

$$\frac{d}{dx}\exp(x) = \exp(x)$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\cdot u'$$

$$\frac{d}{dx}e^u = e^u u'$$

$$\int e^u\,du = e^u + C$$

Examples: $\frac{d}{dx}e^{2x+1} = e^{2x+1} \cdot 2 = 2 \cdot e^{2x+1}$

$\frac{d}{dx}e^{x\sin(x)} = e^{x \cdot \sin(x)} \cdot \left(x\cos(x) + \sin(x)\right)$

$\frac{d}{dx}\exp(x^2+x) = \exp(x^2+x) \cdot (2x+1)$

$= (2x+1)e^{x^2+x}$

$u = \sin(x)$

$du = \cos(x)dx$

$\int \cos(x)e^{\boxed{\sin(x)}}dx = \int e^u du = e^u + C$

$= e^{\sin(x)} + C.$