Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF

http://www.math.uh.edu/~jmorgan/Math1432

Test 1 and **Practice Test 1** are available on CourseWare. **Test 1** counts the same as a major exam. **Practice Test 1** counts the same as an online quiz. Both are due next Thursday.

Homework 1 is posted on the course homepage and due next Wednesday. **Homework 2** will be posted next Wednesday.

EMCF02 is due tomorrow morning at 9am. **EMCF03** is posted, and it is due next Wednesday morning at 9am.

Online Quizzes are Available on CourseWare.

Poppers start in week 3! Get your forms from the Book Store.

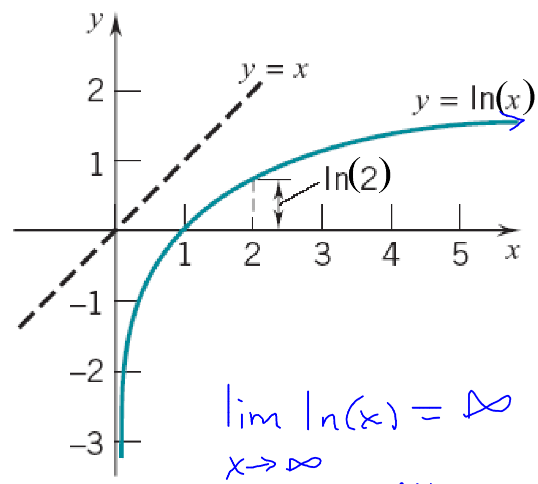
Recall the natural logarithm.

$$\ln(x) = \int_1^x \frac{1}{t}\,dt, \quad x>0$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}, \quad x>0$$

$$\frac{d}{dx}\ln\left(u(x)\right) = \frac{1}{u(x)}u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$



$\lim_{x\to\infty} \ln(x) = \infty$

grows very slowly

$\lim_{x\to 0^+} \ln(x) = -\infty$

The base of the natural logarithm is called *e*.

$$\ln(x) = \log_e(x)$$

What is *e* ?

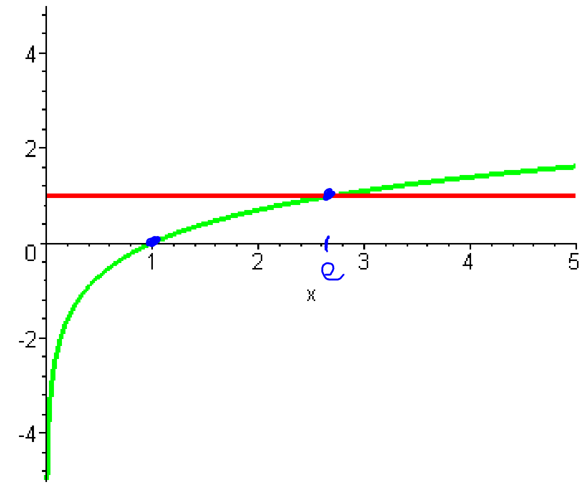
***e* is an irrational number.**

An excellent approximation is

$$e = 2.718281828459045235360287471352662497757247 09...$$

How do we find the value of *e*?

Here is a *very poor* method for approximating *e*.



$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

Note that

$$1 = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\ln(1+h) - \overbrace{\ln(1)}^{=0}}{h}$$

$$= \lim_{h \to 0} \frac{\ln(1+h)}{h}$$

set $h = \frac{1}{n}$

let $n \to \infty$

$$= \lim_{n \to \infty} \frac{\ln\left(1 + \frac{1}{n}\right)}{1/n}$$

$$= \lim_{n \to \infty} n \ln\left(1 + \frac{1}{n}\right)$$

Hm...

$$1 = \lim_{n \to \infty} \ln\left(\left(1 + \frac{1}{n}\right)^n\right)$$

$$\ln(x) = 1$$

$$x = e$$

$$\therefore \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

Never use $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$ as an approximating tool!!

Why?

$e =$ **2.7182818284590...**

$$\begin{bmatrix} n & \left(1+\frac{1}{n}\right)^n \\ 1 & 2. \\ 10 & 2.593742460 \\ 100 & 2.704813829 \\ 1000 & 2.716923932 \\ 10000 & 2.718145927 \\ 100000 & 2.718268237 \end{bmatrix}$$

The approx is slow and it stalls at 10^{16}.

A better approximation...

$$e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots + \frac{1}{1\cdot 2\cdot 3\cdot \cdots \cdot n} + \cdots$$

Recall:

Converting between bases:

If $a, b, x > 0$ and $a, b \neq 1$, then

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Observation: There is a constant k so that

$$\frac{\log_e(x)}{\log_e(10)} = \log_{10}(x) = k\int_1^x \frac{1}{t}\,dt = k\ln(x)$$

$$= k \log_e(x)$$

$$= \frac{\ln(x)}{\ln(10)}$$

What is k ?

$$k = \frac{1}{\ln(10)}$$

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)} \Rightarrow \frac{d}{dx}\log_{10}(x) = \frac{1}{x\ln(10)}$$

$$\log_5(x) = \frac{\ln(x)}{\ln(5)} \Rightarrow \frac{d}{dx}\log_5(x) = \frac{1}{x\ln(5)}$$

Recall:

Converting between bases:

If $a, b, x > 0$ and $a, b \neq 1$, then

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Observation: Suppose $a > 0$ and $a \neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{d}{dx} \frac{\ln(u(x))}{\ln(a)}$$

$$= \frac{1}{u(x)\ln(a)} \cdot u'(x)$$

Examples: $\frac{d}{dx}\log_{10}\left(x^2+1\right) = \frac{1}{(x^2+1)\ln(10)} \cdot 2x = \boxed{\frac{2x}{(x^2+1)\ln(10)}}$

Use a slope chart to determine the shape of the graph of $\underline{y=\log_{10}\left(x^2+1\right)}$

<u>Domain</u>: x^2+1 is always >0 so the domain is $(-\infty, \infty)$.

<u>Note</u>: $y' = 0$ when $x = 0$.

y': $- - - - - -$ 0 $+ + + + + +$

x: -1, 0, 1

local shape: $\cup$ (at $x = 0$)

$$y' = \frac{2x}{(x^2+1)\ln(10)}$$

$$y'' = \frac{1}{\ln(10)} \cdot \frac{(x^2+1)\cdot 2 - 2x \cdot 2x}{(x^2+1)^2}$$

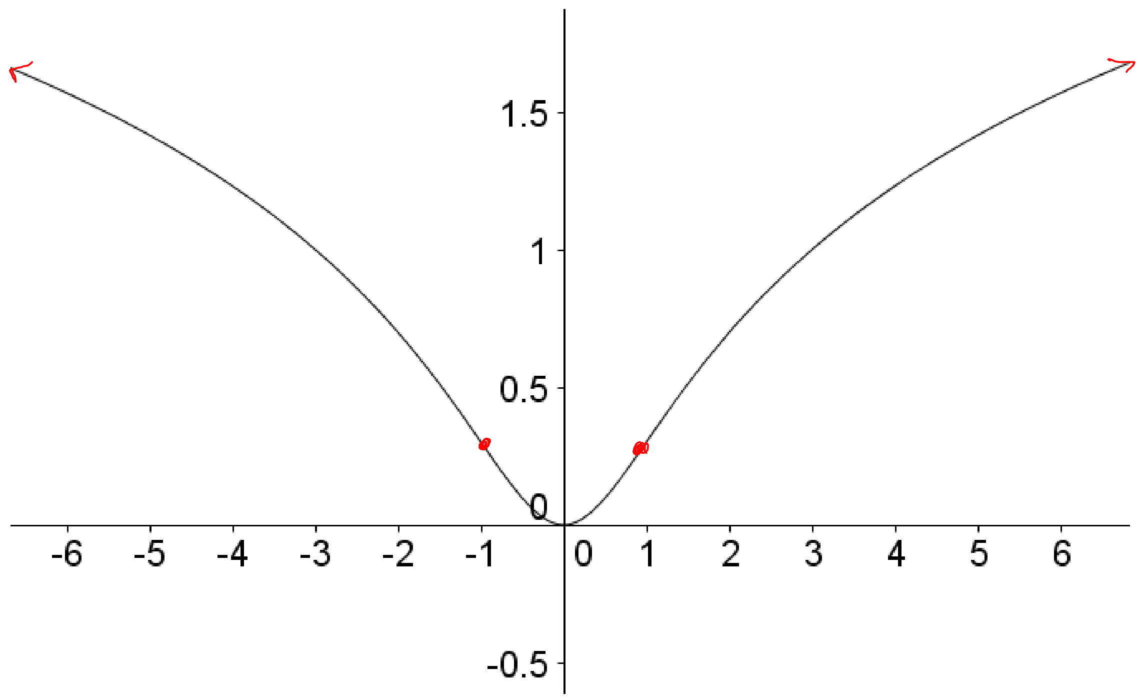
$$= \frac{1}{\ln(10)} \cdot \frac{2-2x^2}{(x^2+1)^2}$$

<u>Note</u>: $y'' = 0 \iff 2-2x^2 = 0 \iff x = \pm 1$

y'': $- - - - -$ 0 $+ + + +$ 0 $- - - -$

x: -2, -1, 0, 1, 2

C.D. (for $x<-1$) C.U. (for $-1<x<1$) C.D. (for $x>1$)



1.5
1
0.5
0
-0.5
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6

Question: Suppose $f(x) = \ln(x)$. Is this function invertible, and if so, what is $f^{-1}(x)$?

yes $\quad$ $\ln(x)$ is increasing.

$y = \ln(x) = \log_e(x)$ $\quad$ $x = e^y$

$\Rightarrow f^{-1}(x) = e^x$

Discuss domain, range and graphs for both f and f^{-1}.

	Domain	**Range**
$f(x) = \ln(x)$	$(0, \infty)$	$(-\infty, \infty)$
$f^{-1}(x) = e^x$	$(-\infty, \infty)$	$(0, \infty)$

$e^x = \exp(x) \equiv$ the exponential function.

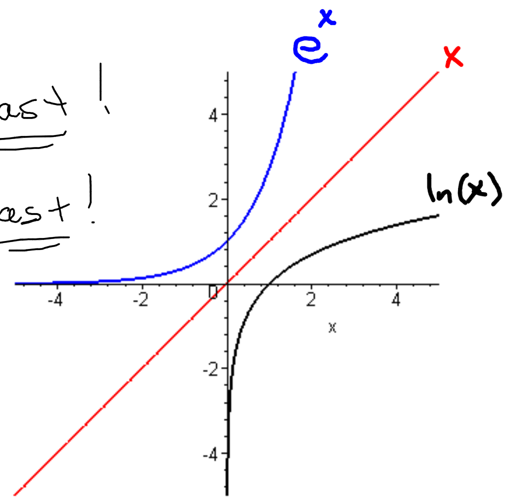
$e = 2.7182818284590452353602874713526624977572470\,93...$

Notation: **exp(x) is the inverse of ln(x)**

$$\boxed{\exp(x) = e^x}$$

Properties:

1. $\lim_{x \to -\infty} \exp(x) = 0$ Fast!
2. $\lim_{x \to \infty} \exp(x) = \infty$ Fast!
3. $\exp(0) = e^0 = 1$
4. $\exp(1) = e^1 = e$
5. $\ln(\exp(x)) = x$
6. $\exp(\ln(x)) = x$



What about the derivative of exp(x)?

$$\frac{d}{dx}e^x = ?? \quad e^x$$

Recall : $(f^{-1})'(b) = \frac{1}{f'(a)} = a = e^b$

where $f(a) = b$.

set $f(x) = \ln(x)$. $f^{-1}(x) = e^x$

$f'(x) = \frac{1}{x}$

$f'(a) = \frac{1}{a}$ $\quad \ln(a) = b$

$a = e^b$

$$\frac{d}{dx}e^x = e^x$$

Consequences:

$$\boxed{\exp(x) = e^x}$$

$$\frac{d}{dx}\exp(x) = \exp(x)$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\cdot u'$$

$$\frac{d}{dx}e^u = e^u u'$$

$$\int e^u\,du = e^u + C$$

Examples: $\dfrac{d}{dx}e^{2x+1} = e^{2x+1} \cdot 2 = 2 \cdot e^{2x+1}$

$$\frac{d}{dx}e^{x\sin(x)} = e^{x \cdot \sin(x)} \cdot \left(x\cos(x) + \sin(x)\right)$$

$$\frac{d}{dx}\exp\left(x^2 + x\right) = \exp\left(x^2+x\right) \cdot (2x+1)$$
$$= (2x+1)e^{x^2+x}$$

$u = \sin(x)$

$du = \cos(x)dx$

$$\int \cos(x)e^{\boxed{\sin(x)}}dx = \int e^u du = e^u + C$$
$$= e^{\sin(x)} + C.$$