Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF

http://www.math.uh.edu/~jmorgan/Math1432

Test 1 and **Practice Test 1** are available on online CourseWare. Test 1 counts the same as a major exam. Practice Test 1 counts the same as an online quiz. Both are due on Thursday (tomorrow).

Homework 2 is posted and due next Monday.

EMCF03 was due this morning at 9am. **EMCF04** is due on Friday morning at 9am.

Online Quizzes are Available on CourseWare.

Poppers start next Monday! Get your forms from the book store in the University Center.

Access Codes are due by Sunday. Purchase yours at the book store in the University Center.

http://www.math.uh.edu/~jmorgan/Math1432

Math 1432 - 13209

Jeff Morgan - jmorgan@math.uh.edu

Read the Syllabus

Use the Discussion Board on CourseWare to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 **Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 **Notes** **Exam 1, PT1 and all Online Quizzes are open**	15 **UH events this week** **Examples from 7.1 that will help with EMCF01**	16 Notes: **pg, 4per** Vid notes: **pg, 4per** **Video** **Homework 1 posted**	17 **EMCF01 due at 9am** **Note:** Use a graphing calculator to solve a complicated equation.	18 Notes: **pg, 2per** Vid notes: **pg, 2per** **Video** Quiz in lab/workshop	19 **EMCF02 due at 9am**
20	21 MLK Day No Class	22. **UH events this week** **Last day to add**	23 **Blank Slides** **EMCF03 due at 9am** **Homework 1 due** in lab/workshop **Homework 2 posted**	24 **Exam 1 and PT1 close**	25 **EMCF04 due at 9am** Quiz in lab/workshop	26 **Quiz 1 closes** (7.1-7.2)
27 **Free Access ends today!! Purchase your Access Code!!**	28 EMCF05 due at 9am Homework 2 due in lab/workshop	29	30 EMCF06 due at 9am Homework 3 posted **Last day to drop without receiving a W**	31 **Register on CourseWare for Exam 2**	February 1 EMCF07 due at 9am Quiz in lab/workshop	2 **Quiz 2 closes** (7.3-7.5)

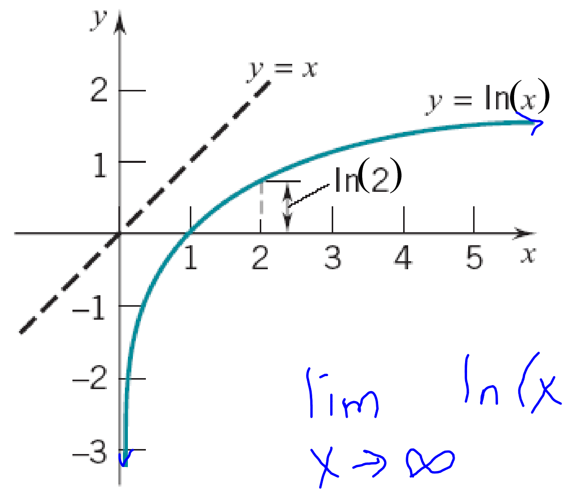
Recall the natural logarithm. $\ln(x) = \log_e(x)$

$$\ln(x) = \int_1^x \frac{1}{t}\,dt \quad , \quad x > 0$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \quad , \quad x > 0$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)}\,u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$



$$\lim_{x\to\infty} \ln(x) = \infty$$

very slow

$$\lim_{x\to 0^+} \ln(x) = -\infty$$

e = 2.71828182845904523536028747135266249775724709 3...

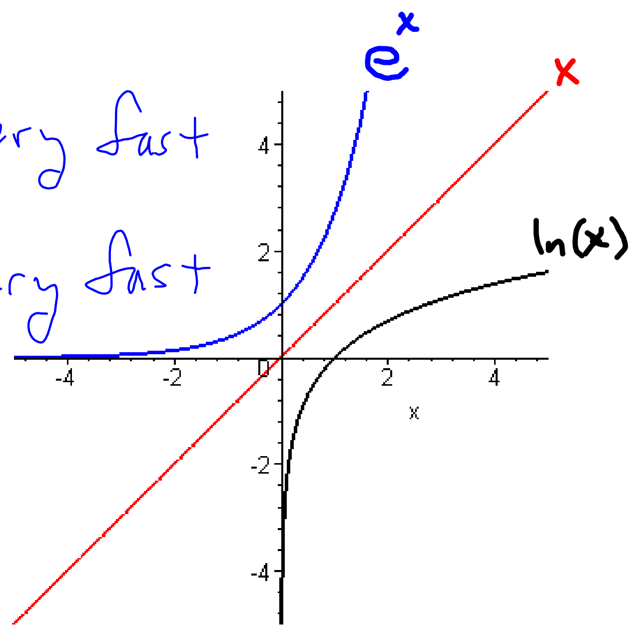
...and it's inverse...

Notation: exp(x) is the inverse of ln(x)

$$\boxed{\exp(x) = e^x}$$

Properties:

1. $\lim\limits_{x \to -\infty} \exp(x) = 0$ very fast
2. $\lim\limits_{x \to \infty} \exp(x) = \infty$ very fast
3. $\exp(0) = 1$
4. $\exp(1) = e$
5. $\ln(\exp(x)) = x$
6. $\exp(\ln(x)) = x$



$$\boxed{\ln(x) = \log_e(x)}$$

Also, recall

$$\boxed{\exp(x) = e^x}$$

$$\frac{d}{dx}\exp(x) = \exp(x)$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\int e^u\,du = e^u + C$$

and...

Suppose $a > 0$ and $a \neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{1}{u(x)\ln(a)} \cdot u'(x)$$

Example: $$\frac{d}{dx}\log_5\left(x^2+\cos(x)+2\right) = \frac{1}{\left(x^2+\cos(x)+2\right)\ln(5)} \cdot \left(2x-\sin(x)\right)$$

Examples:

Give the domain of $f(x) = \sqrt{1 - \ln(2x)}$

We need $\quad 2x > 0 \quad$ and $\quad 1 - \ln(2x) \geq 0$

$x > 0 \quad$ and $\quad \ln(2x) \leq 1$

$x > 0 \quad$ and $\quad 2x \leq e$

Domain: $\quad 0 < x \leq \frac{e}{2}$.

Give an equation for the tangent line to the graph of

$f(x) = \underbrace{x \ln(x)}_{\text{product}}$ at $x = 1$.

Point: $\quad (1, f(1)) = (1, 0)$

Slope: $\quad f'(1) = 1$

$f'(x) = x \cdot \frac{1}{x} + \ln(x) = 1 + \ln(x)$

Tangent Line: $\quad y = x - 1$.

Examples:

Show that $f(x) = x + e^{2x}$ is an invertible function, and give the equation of the tangent line to the graph of $f^{-1}(x)$ at $x = 1$.

$f'(x) = 1 + 2\underbrace{e^{2x}}_{\text{positive}} > 0 \Rightarrow f$ is increasing

$\Rightarrow f$ is invertible.

Point: $(1, f^{-1}(1)) = (1, 0)$

Solve $f(x) = 1$

$x + e^{2x} = 1$

$x = 0$.

slope: $(f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{3}$

Tangent line: $y = \frac{1}{3}(x-1) \Leftrightarrow \boxed{y = \frac{1}{3}x - \frac{1}{3}}$ T.L. for f^{-1}

Remark: let's find the T.L. to $f(x)$ at $x = 0$.

Point: $(0, f(0)) = (0, 1)$

slope: $f'(0) = 3$

Tangent line: $\boxed{y = 3x + 1}$ T.L. for f

These are inverses.

Why?

$3\left(\frac{1}{3}x - \frac{1}{3}\right) + 1 = x$

$\frac{1}{3}(3x+1) - \frac{1}{3} = x$

$$\int \frac{\sin(\ln(x))}{x}\,dx =$$

you

Question: What is the derivative of 2^x?

Note: $2 = e^{\ln(2)} \Rightarrow 2^x = \left(e^{\ln(2)}\right)^x = e^{x\ln(2)}$

So, $\frac{d}{dx} 2^x = \frac{d}{dx} e^{x\ln(2)} = e^{x\ln(2)} \ln(2)$

$= 2^x \ln(2)$

What is the derivative of a^x, if $a > 0$ and a is not 1?

$$\frac{d}{dx} a^x = a^x \ln(a).$$

$$\frac{d}{dx} a^{u(x)} = a^{u(x)} \ln(a)\, u'(x)$$

In summary, **if $a > 0$ with $a \neq 1$ then**

$$\frac{d}{dx}a^{u(x)} = a^{u(x)} \ln(a)\, u'(x)$$

$$\frac{d}{dx}\log_a(u(x)) = \frac{1}{u(x)\ln(a)} u'(x)$$

$$\int a^u du = \frac{1}{\ln(a)} a^u + C$$

Examples:

$$\frac{d}{dx}2^{\cos(3x)} = 2^{\cos(3x)} \ln(2)\cdot(-3\sin(3x))$$

$$= -3\ln(2)\sin(3x)\,2^{\cos(3x)}$$

$$\int 3^{\cos(x)}\sin(x)dx = -\int \underbrace{3^{\cos(x)}}_{3^u}\underbrace{(-\sin(x))dx}_{du}$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$= -\int 3^u\,du$$

$$= -\frac{1}{\ln(3)}3^u + C$$

$$= -\frac{1}{\ln(3)}3^{\cos(x)} + C.$$

$f(x)$	Domain	Range	$f^{-1}(x)$	$f'(x)$
e^x	$(-\infty, \infty)$	$(0, \infty)$	$\ln(x)$	e^x
$\ln(x)$	$(0, \infty)$	$(-\infty, \infty)$	e^x	$\frac{1}{x}$
10^x	$(-\infty, \infty)$	$(0, \infty)$	$\log_{10}(x)$	$10^x \ln(10)$
$\log_{10}(x)$	$(0, \infty)$	$(-\infty, \infty)$	10^x	$\frac{1}{x \ln(10)}$
a^x	$(-\infty, \infty)$	$(0, \infty)$	$\log_a(x)$	$a^x \ln(a)$
$\log_a(x)$	$(0, \infty)$	$(-\infty, \infty)$	a^x	$\frac{1}{x \ln(a)}$

$a > 0, \ a \neq 1.$

(logarithmic differentiation)

How can we used logarithms to differentiate complicated exponential functions?

Suppose $a(x)$ and $b(x)$ are differentiable and $a(x)>0$.

$$\frac{d}{dx}a(x)^{b(x)} = \quad y'$$

$$y = \underline{a(x)}^{b(x)}$$

$$\ln(y) = \ln\left(a(x)^{b(x)}\right)$$

$$\ln(y) = b(x)\ln(a(x))$$

Diff wrt x

$$\frac{1}{y}y' = b(x)\frac{a'(x)}{a(x)} + \ln(a(x))\,b'(x)$$

$$\Rightarrow \quad y' = y\left(\text{This}\right)$$

Example: $$\frac{d}{dx}\left(x^2+1\right)^{\sin(x)} = y'$$

$$y = (x^2+1)^{\sin(x)}$$

$$\ln(y) = \ln\left((x^2+1)^{\sin(x)}\right)$$

$$\ln(y) = \sin(x)\ln(x^2+1)$$

Diff wrt x.

$$\frac{1}{y}y' = \sin(x)\frac{2x}{x^2+1} + \ln(x^2+1)\cos(x)$$

$$\Rightarrow y' = (x^2+1)^{\sin(x)}\left[\frac{2x\sin(x)}{x^2+1} + \ln(x^2+1)\cos(x)\right]$$

How can we use logarithms to differentiate complicated products?

(logarithmic differentiation)

Suppose a, b, c and d are differentiable and positive.

$$\frac{d}{dx}\left[a(x)b(x)c(x)d(x)\right] =$$

let $y = a(x)b(x)c(x)d(x)$

we want $\frac{dy}{dx}$.

$$\ln(y) = \ln\left(a(x)b(x)c(x)d(x)\right)$$

$$\Rightarrow \ln(y) = \ln(a(x)) + \ln(b(x)) + \ln(c(x)) + \ln(d(x))$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{a'(x)}{a(x)} + \frac{b'(x)}{b(x)} + \frac{c'(x)}{c(x)} + \frac{d'(x)}{d(x)}$$

$$\Rightarrow \frac{dy}{dx} = a(x)b(x)c(x)d(x)\left(\frac{a'(x)}{a(x)} + \frac{b'(x)}{b(x)} + \frac{c'(x)}{c(x)} + \frac{d'(x)}{d(x)}\right)$$

$$= \underbrace{a'bcd + ab'cd + abc'd + abcd'}_{\text{generalized product rule}}$$

Example : (logarithmic differentiation)

$$\frac{d}{dx}\Big[\underbrace{(2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}}_{\substack{\| \\ y}}\Big]=$$

$$\ln(y) = \ln\left((2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}\right)$$

$$\ln(y) = 12\ln|2x+1| + 7\ln|\cos(x)+1| + 8\ln|2-3x|$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{24}{2x+1} + \frac{-7\sin(x)}{\cos(x)+1} + \frac{-24}{2-3x}$$

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}$$

$$\therefore \quad \frac{dy}{dx} = (2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}\left[\frac{24}{2x+1} + \frac{-7\sin(x)}{\cos(x)+1} + \frac{-24}{2-3x}\right]$$