# Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF

http://www.math.uh.edu/~jmorgan/Math1432

**Test 1** and **Practice Test 1** are available on online CourseWare. Test 1 counts the same as a major exam. Practice Test 1 counts the same as an online quiz. Both are due on Thursday (tomorrow).

**Homework 2** is posted and due next Monday.

**EMCF03** was due this morning at 9am. **EMCF04** is due on Friday morning at 9am.

**Online Quizzes** are Available on CourseWare.

**Poppers** start next Monday! Get your forms from the book store in the University Center.

**Access Codes** are due by Sunday. Purchase yours at the book store in the university center.

**http://www.math.uh.edu/~jmorgan/Math1432**

# Math 1432 - 13209

Jeff Morgan - jmorgan@math.uh.edu

## Read the Syllabus

**Use the Discussion Board on CourseWare to get and give help.**

**Lecture notes/videos, additional help material, course announcements, homework and EMCFs** will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

**Course Calendar**

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| January 13<br>**Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam. | 14<br>**Notes**<br>**Exam 1, PT1 and all Online Quizzes are open** | 15<br>**UH events this week**<br>**Examples from 7.1 that will help with EMCF01** | 16<br>Notes: **pg, 4per**<br>Vid notes: **pg, 4per**<br>**Video**<br>**Homework 1 posted** | 17<br>**EMCF01 due at 9am**<br>**Note:** Use a graphing calculator to solve a complicated equation. | 18<br>Notes: **pg, 2per**<br>Vid notes: **pg, 2per**<br>**Video**<br>Quiz in lab/workshop | 19<br>**EMCF02 due at 9am** |
| 20 | 21<br>MLK Day<br>No Class | 22.<br>**UH events this week**<br>**Last day to add** | 23<br>**Blank Slides**<br>**EMCF03 due at 9am**<br>**Homework 1 due** in lab/workshop<br>**Homework 2 posted** | 24<br>**Exam 1 and PT1 close** | 25<br>**EMCF04 due at 9am**<br>Quiz in lab/workshop | 26<br>**Quiz 1 closes**<br>(7.1-7.2) |
| 27<br>**Free Access ends today!! Purchase your Access Code!!** | 28<br>EMCF05 due at 9am<br>Homework 2 due in lab/workshop | 29 | 30<br>EMCF06 due at 9am<br>Homework 3 posted<br>**Last day to drop without receiving a W** | 31<br>**Register on CourseWare for Exam 2** | February 1<br>EMCF07 due at 9am<br>Quiz in lab/workshop | 2<br>**Quiz 2 closes**<br>(7.3-7.5) |

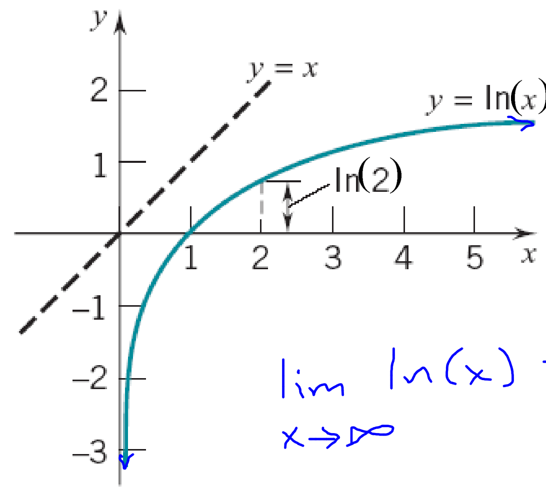
**Recall the natural logarithm.** $\ln(x) = \log_e(x)$

$$\ln(x) = \int_1^x \frac{1}{t}\,dt, \quad x > 0$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}\ln(u(x)) = \frac{1}{u(x)} \cdot u'(x)$$

$$\int \frac{1}{u}\,du = \ln(|u|) + C$$




$$\lim_{x \to \infty} \ln(x) = \infty$$

$$\lim_{x \to 0^+} \ln(x) = -\infty$$

e = 2.71828182845904523536028747135266249775724709 3...

**...and it's inverse...**

**Notation: exp($x$) is the inverse of ln($x$)**

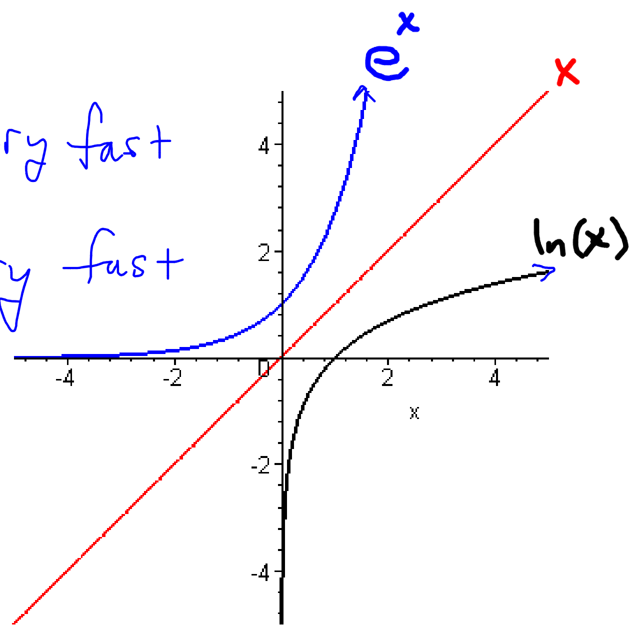
$$\boxed{\exp(x) = e^x}$$

Properties:

1. $\lim\limits_{x \to -\infty} \exp(x) = 0$
2. $\lim\limits_{x \to \infty} \exp(x) = \infty$
3. $\exp(0) = 1$
4. $\exp(1) = e$
5. $\ln(\exp(x)) = x$
6. $\exp(\ln(x)) = x$

very fast

very fast




$$\boxed{\ln(x) = \log_e(x)}$$

**Also, recall**

$$\boxed{\exp(x) = e^x}$$

$$\frac{d}{dx}\exp(x) = \exp(x)$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\exp(u) = \exp(u)\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\int e^u du = e^u + C$$

***and...***

Suppose $a > 0$ and $a \neq 1$.

$$\frac{d}{dx}\log_a(u(x)) = \frac{1}{u(x)\ln(a)}\,u'(x)$$

**Example:** $$\frac{d}{dx}\log_5\left(x^2+\cos(x)+2\right) = \frac{1}{(x^2+\cos(x)+2)\ln(5)}\cdot\left(2x-\sin(x)\right)$$

**Examples:**

Give the domain of $f(x) = \sqrt{1 - \ln(2x)}$

Needs: $2x > 0$ and $1 - \ln(2x) \geq 0$

$\boxed{x > 0}$ and $\ln(2x) \leq 1$

$\boxed{2x \leq e}$

Domain: $\boxed{0 < x \leq \frac{1}{2}e}$

Give an equation for the tangent line to the graph of $f(x) = x\ln(x)$ at $x = 1$.

product

Point: $(1, f(1)) = (1, 0)$

slope: $f'(1) = 1$

$f'(x) = x \cdot \frac{1}{x} + \ln(x) = 1 + \ln(x)$

Tangent Line: $y = (x-1)$ i.e. $\boxed{y = x - 1.}$

**Examples:**

Show that $f(x) = x + e^{2x}$ is an invertible function, and give the equation of the tangent line to the graph of $f^{-1}(x)$ at $x = 1$.

$f'(x) = 1 + 2\underbrace{e^{2x}}_{} > 0 \Rightarrow f$ is increasing

positive

$\therefore f$ is invertible.

Solve $x + e^{2x} = 1$

$\boxed{x = 0}$

point: $(1, f^{-1}(1)) = (1, 0)$

slope: $(f^{-1})'(1) = \dfrac{1}{f'(0)} = \dfrac{1}{3}$

Tangent Line:

$y = \frac{1}{3}(x - 1)$.

$$\int \frac{\sin(\ln(x))}{x}dx = \int \sin(\ln(x)) \cdot \frac{1}{x}\,dx = \int \sin(u)\,du$$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$= -\cos(u) + C$

$= -\cos(\ln(x)) + C$.

**Question:** What is the derivative of $2^x$?

<u>Note</u>: $2 = e^{\ln(2)} \Rightarrow 2^x = \left(e^{\ln(2)}\right)^x$

$\therefore 2^x = e^{x\ln(2)} \Rightarrow \dfrac{d}{dx} 2^x = e^{x\ln(2)} \cdot \ln(2)$

$= 2^x \cdot \ln(2)$

What is the derivative of $a^x$, if $a > 0$ and $a$ is not 1?

$$\frac{d}{dx} a^x = a^x \cdot \ln(a)$$

In summary, if $a > 0$ with $a \neq 1$ then

$$\frac{d}{dx} a^{u(x)} = a^{u(x)} \ln(a) \; u'(x)$$

$$\frac{d}{dx} \log_a\left(u(x)\right) = \frac{1}{u(x) \ln(a)} \cdot u'(x)$$

$$\int a^u \, du = \frac{1}{\ln(a)} a^u + C$$

**Examples:**

$$\frac{d}{dx}2^{\cos(3x)} = 2^{\cos(3x)} \ln(2) \cdot (-\sin(3x) \cdot 3)$$

$$= -3\ln(2)\sin(3x)\,2^{\cos(3x)}$$

$$\int 3^{\cos(x)}\sin(x)dx = -\int 3^{\cos(x)}(-\sin(x))\,dx$$

$$\int a^u du = \frac{1}{\ln(a)}a^u + C$$

$$= -\int 3^u du$$

$$u = \cos(x)$$

$$du = -\sin(x)\,dx$$

$$= -\frac{1}{\ln(3)}3^u + C$$

$$= -\frac{1}{\ln(3)}3^{\cos(x)} + C$$

| $f(x)$ | Domain | Range | $f^{-1}(x)$ | $f'(x)$ |
|---|---|---|---|---|
| $e^x$ | $(-\infty, \infty)$ | $(0, \infty)$ | $\ln(x)$ | $e^x$ |
| $\ln(x)$ | $(0, \infty)$ | $(-\infty, \infty)$ | $e^x$ | $\frac{1}{x}$ |
| $10^x$ | $(-\infty, \infty)$ | $(0, \infty)$ | $\log_{10}(x)$ | $10^x \ln(10)$ |
| $\log_{10}(x)$ | $(0, \infty)$ | $(-\infty, \infty)$ | $10^x$ | $\frac{1}{x \ln(10)}$ |
| $a^x$ | $(-\infty, \infty)$ | $(0, \infty)$ | $\log_a(x)$ | $a^x \ln(a)$ |
| $\log_a(x)$ | $(0, \infty)$ | $(-\infty, \infty)$ | $a^x$ | $\frac{1}{x \ln(a)}$ |

$a > 0$, $a \neq 1$.

$*$ (logarithmic differentiation)

How can we used logarithms to differentiate complicated exponential functions?

Suppose $a(x)$ and $b(x)$ are differentiable and $\underline{a(x) > 0}$.

$$\frac{d}{dx}\boxed{a(x)^{b(x)}} = ? \qquad a(x)^{b(x)}\left(b(x)\frac{1}{a(x)}a'(x) + \ln(a(x))\cdot b'(x)\right)$$

$$y = \underline{a(x)^{b(x)}}$$

$$\ln(y) = \ln\left(a(x)^{b(x)}\right)$$

$$\Rightarrow \quad \ln(y) = b(x)\ln(a(x))$$

Diff wrt $x$

$$\frac{1}{y}y' = b(x)\frac{1}{a(x)}a'(x) + \ln(a(x))\cdot b'(x)$$

$$\Rightarrow \quad y' = \underline{y}\left(b(x)\frac{1}{a(x)}a'(x) + \ln(a(x))\cdot b'(x)\right)$$

**Example:**

$$\frac{d}{dx}\left(x^2+1\right)^{\sin(x)} = y'$$

$$= \left(x^2+1\right)^{\sin(x)}\left(\frac{2x\sin(x)}{x^2+1} + \ln\left(x^2+1\right)\cos(x)\right)$$

$$y = \left(x^2+1\right)^{\sin(x)}$$

$$\ln(y) = \ln\left(\left(x^2+1\right)^{\sin(x)}\right)$$

$$\Rightarrow \quad \ln(y) = \sin(x)\ln\left(x^2+1\right)$$

Diff wrt $x$

$$\frac{1}{y}y' = \sin(x)\frac{2x}{x^2+1} + \ln(x^2+1)\cos(x)$$

$$\Rightarrow \quad y' = y\left(\frac{2x\sin(x)}{x^2+1} + \ln(x^2+1)\cos(x)\right)$$

How can we use logarithms to differentiate complicated products?

(logarithmic differentiation)

**Suppose $a, b, c$ and $d$ are differentiable and positive.**

$$\frac{d}{dx}\left[\underbrace{a(x)b(x)c(x)d(x)}\right] = y' =$$

$$= a(x)b(x)c(x)d(x)\left(\frac{a'(x)}{a(x)} + \frac{b'(x)}{b(x)} + \frac{c'(x)}{c(x)} + \frac{d'(x)}{d(x)}\right)$$

$$= a'(x)b(x)c(x)d(x) + a(x)b'(x)c(x)d(x) + a(x)b(x)c'(x)d(x) + a(x)b(x)c(x)d'(x)$$

$$y = a(x)b(x)c(x)d(x)$$

$$\ln(y) = \ln\left(a(x)b(x)c(x)d(x)\right)$$

$$\Rightarrow \ln(y) = \ln(a(x)) + \ln(b(x)) + \ln(c(x)) + \ln(d(x))$$

Diff wrt $x$.

$$\frac{1}{y}y' = \frac{a'(x)}{a(x)} + \frac{b'(x)}{b(x)} + \frac{c'(x)}{c(x)} + \frac{d'(x)}{d(x)}$$

$$\Rightarrow y' = y\left(\frac{a'(x)}{a(x)} + \frac{b'(x)}{b(x)} + \frac{c'(x)}{c(x)} + \frac{d'(x)}{d(x)}\right)$$

Example : (logarithmic differentiation)

$$\frac{d}{dx}\left[\underbrace{(2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}}\right] = y'$$

$$y = (2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}$$

$$\ln(y) = \ln\left[(2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}\right]$$

$$\ln(y) = \ln\left((2x+1)^{12}\right) + \ln\left((\cos(x)+1)^{7}\right) + \ln\left((2-3x)^{8}\right)$$

$$\ln(y) = 12\ln(|2x+1|) + 7\ln(|\cos(x)+1|) + 8\ln(|2-3x|)$$

$$\Rightarrow \frac{1}{y}y' = \frac{24}{2x+1} + \frac{-7\sin(x)}{\cos(x)+1} + \frac{-24}{2-3x}$$

$$\Rightarrow y' = y\left(\frac{24}{2x+1} - \frac{7\sin(x)}{\cos(x)+1} - \frac{24}{2-3x}\right)$$

$$\Rightarrow y' = (2x+1)^{12}(\cos(x)+1)^{7}(2-3x)^{8}\left(\frac{24}{2x+1} - \frac{7\sin(x)}{\cos(x)+1} - \frac{24}{2-3x}\right)$$