Math 1432 - 13209

Jeff Morgan - 651 PGH - 11-noon MWF
http://www.math.uh.edu/~jmorgan/Math1432

Test 2 is will be given in CASA starting February 14. Start registering on January 31st at 12:01am.

Homework 2 is posted and due on Monday.

EMCF04 was due this morning at 9am. **EMCF05** is due Monday morning at 9am.

Online Quizzes 1 and 2 are Available on CourseWare, and **Quiz 1** expires tomorrow tonight.

Poppers start next Monday! Get your forms from the UC Book Store.

Access Codes are due on Sunday! Get yours from the UC Book Store.

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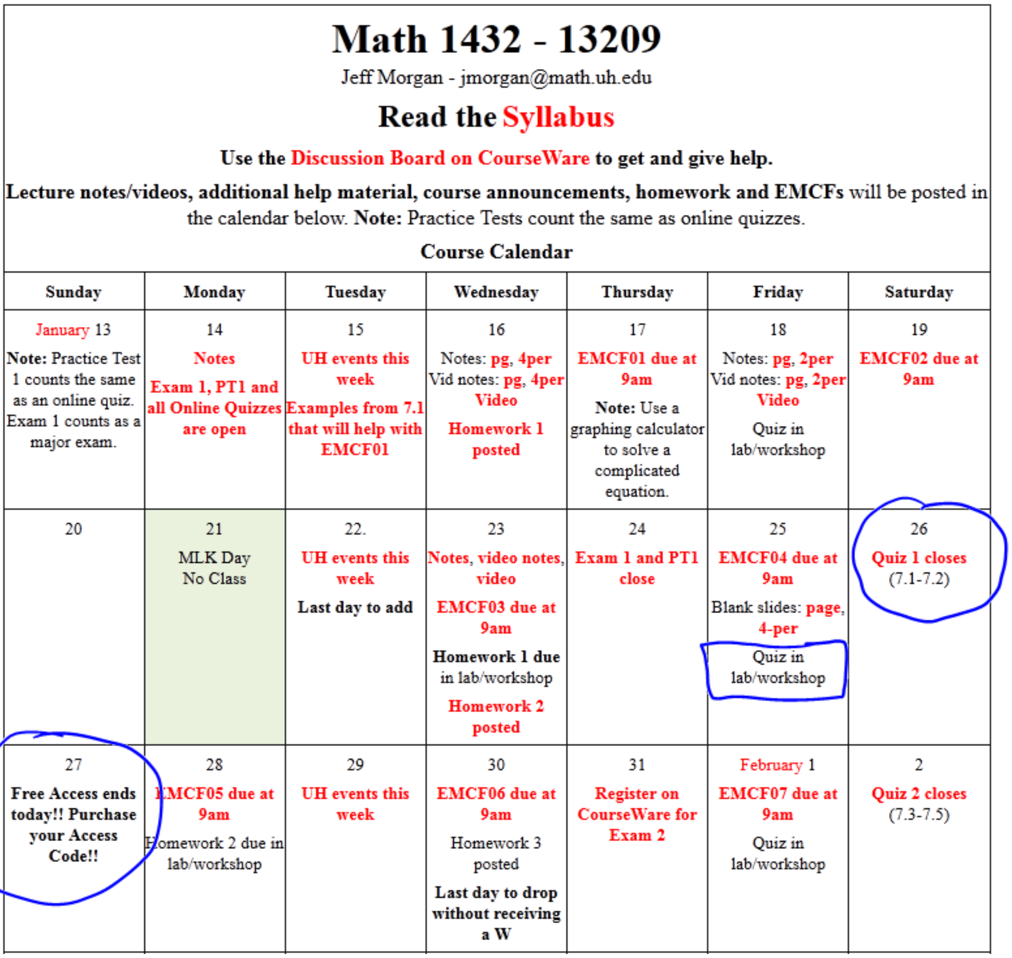

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Read the Syllabus

Use the Discussion Board on CourseWare to get and give help.

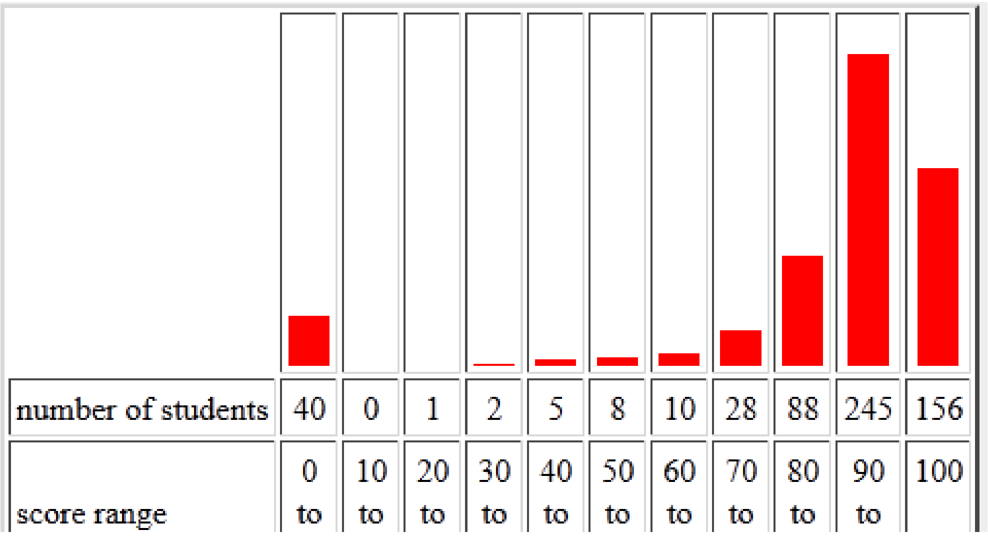
Lecture notes/videos, additional help material, course announcements, homework and EMCFs will be posted in the calendar below. **Note:** Practice Tests count the same as online quizzes.

Course Calendar

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
January 13 **Note:** Practice Test 1 counts the same as an online quiz. Exam 1 counts as a major exam.	14 **Notes** **Exam 1, PT1 and all Online Quizzes are open**	15 **UH events this week** **Examples from 7.1 that will help with EMCF01**	16 Notes: **pg**, **4per** Vid notes: **pg**, **4per** **Video** **Homework 1 posted**	17 **EMCF01 due at 9am** **Note:** Use a graphing calculator to solve a complicated equation.	18 Notes: **pg**, **2per** Vid notes: **pg**, **2per** **Video** Quiz in lab/workshop	19 **EMCF02 due at 9am**
20	21 MLK Day No Class	22. **UH events this week** **Last day to add**	23 **Notes**, **video notes**, **video** **EMCF03 due at 9am** **Homework 1 due** in lab/workshop **Homework 2 posted**	24 **Exam 1 and PT1 close**	25 **EMCF04 due at 9am** Blank slides: **page**, **4-per** Quiz in lab/workshop	26 **Quiz 1 closes** (7.1-7.2)
27 **Free Access ends today!! Purchase your Access Code!!**	28 **EMCF05 due at 9am** Homework 2 due in lab/workshop	29 **UH events this week**	30 **EMCF06 due at 9am** Homework 3 posted **Last day to drop without receiving a W**	31 **Register on CourseWare for Exam 2**	February 1 **EMCF07 due at 9am** Quiz in lab/workshop	2 **Quiz 2 closes** (7.3-7.5)

11:59 pm

Test 1 Scores



number of students	40	0	1	2	5	8	10	28	88	245	156
score range	0 to	10 to	20 to	30 to	40 to	50 to	60 to	70 to	80 to	90 to	100

89 99

Please tell you high school friends and former teachers about our

(*) **High School Mathematics Contest**

February 9th
University of Houston Free

http://mathcontest.uh.edu

$$y = (2+\sin(x))^{3x} \qquad \ln(y) = \ln\left((2+\sin(x))^{3x}\right)$$

$$\Rightarrow \ln(y) = \underline{3x\ln(2+\sin(x))}$$

diff wrt x

$$\frac{1}{y}y' = \boxed{3x\frac{\cos(x)}{2+\sin(x)} + \ln(2+\sin(x))\cdot 3}$$

$$\Rightarrow y' = y(\text{This stuff})$$

Review Examples:

$$\frac{d}{dx}\boxed{(2+\sin(x))^{3x}} = $$

logarithmic differentiation

$$= (2+\sin(x))^{3x}\left[\frac{3x\cos(x)}{2+\sin(x)} + 3\ln(2+\sin(x))\right]$$

$$\frac{d}{dx}\ln\left(\cos(2x)+3\right) = \frac{1}{\cos(2x)+3}\cdot(-2\sin(2x))$$

$$= \frac{-2\sin(2x)}{\cos(2x)+3}$$

$$\int \tan(3x)\,dx = -\frac{1}{3}\int \frac{-3\sin(3x)}{\boxed{\cos(3x)}}dx = -\frac{1}{3}\int\frac{1}{u}du$$

$u = \cos(3x)$

$du = -3\sin(3x)dx$

$$= -\frac{1}{3}\ln(|u|) + C$$

$$= -\frac{1}{3}\ln(|\cos(3x)|) + C$$

Other

Consequences of $\int \frac{1}{u}du = \ln\left(|u|\right)+C$.

$$\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = -\ln\left(|\cos(x)|\right)+C$$

$$= \ln\left(|\sec(x)|\right)+C$$

$$\int \cot(x)dx = \int \frac{\cos(x)}{\sin(x)}dx = \ln\left(|\sin(x)|\right)+C$$

$$= -\ln\left(|\csc(x)|\right)+C$$

$$\int \sec(x)dx =$$

$$= \int \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}dx$$

$$= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\boxed{\tan(x)}+\boxed{\sec(x)}}dx$$

$$= \ln\left(|\tan(x)+\sec(x)|\right)+C$$

$$\int \csc(x)dx =$$

similar

$$\int \csc(x)\frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}dx = \int \frac{\csc^2(x)+\csc(x)\cot(x)}{\boxed{\csc(x)}+\boxed{\cot(x)}}dx$$

$$= -\ln\left(|\csc(x)+\cot(x)|\right)+C$$

u substitution versions:

$$\int \tan(u)du = \ln(|\sec(u)|) + C$$

$$\int \cot(u)du = -\ln(|\csc(u)|) + C$$

$$\int \sec(u)du = \ln(|\sec(u) + \tan(u)|) + C$$

$$\int \csc(u)du = -\ln(|\csc(u) + \cot(u)|) + C$$

New

Exponential Growth and Decay

Introduction:

- Population Growth
- Radioactive Decay
- Investment
- Mixing Problems

Common Theme: There is a quantity that changes at a rate proportional to the amount present.

$u(t) \equiv$ quantity at time t.

$$\frac{d}{dt}u(t) = k\,u(t)$$

constant

Differential Equations

$y \equiv$ quantity at time t

$$y' = ky$$

Note: If we use x instead of t then

$$\frac{dy}{dx} = ky.$$

How do we solve $y' = \underset{\uparrow}{k}y \iff y = C_1e^{kt}$

constant

Assume the independent variable is $\underline{\underline{t}}$.

We want to find y.

One approach

$$y' = ky$$

$$y' + (-k)y = 0$$

$$\underbrace{e^{-kt}y' + e^{-kt}(-k)y} = 0$$

$$\frac{d}{dt}\left(e^{-kt}y\right) = 0$$

$$\Rightarrow \quad e^{-kt}y = C_1$$

$$\Rightarrow \quad \boxed{y = C_1e^{kt}}$$

Another approach

$$\frac{y'}{y} = k$$

Integrate wrt t.

$$\ln(|y|) = kt + \tilde{C}$$

$$\Rightarrow \quad |y| = e^{kt+\tilde{C}} = e^{kt}e^{\tilde{C}}$$

$$\Rightarrow \quad y = \pm e^{\tilde{C}}e^{kt}$$

$$\boxed{y = C_1e^{kt}}$$

$$y' = ky \iff y = Ce^{kt}$$

Example: Find a function that satisfies $\underbrace{y' = -2y}$ and $\overbrace{y(0) = 3}^{\text{initial data}}$.

$$y = Ce^{-2t} \Rightarrow 3 = Ce^{0} \Rightarrow C = 3$$

$$\Rightarrow \boxed{y = 3e^{-2t}}$$

Example: Find a function that satisfies $\underbrace{y'(x) - 3y(x) = 0}$ and $\underbrace{y(0) = 2.51}$.

$$y' = 3y \Rightarrow y = Ce^{3x} \longrightarrow 2.51 = Ce^{0} \Rightarrow C = 2.51$$

$x \equiv$ ind. variable name

$$\Rightarrow \boxed{y = 2.51e^{3x}}$$

Example: Give all functions that satisfy $u'(t) = 0.3u(t)$.

$$\boxed{u(t) = Ce^{0.3t}}$$

Examples: Suppose a culture of bacteria is growing in such a way that the change in the number of bacteria is proportional to the number present. The number of bacteria double every 200 minutes and there are currently 5,000 bacteria in the culture. How many bacteria were present 2 hours ago?

Radio-active substances change at a rate proportional to the amount present. What is the half-life of a radio-active substance if it takes 10 years for 28% of the substance to decay?

After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long would it take the sample to decay to 10% of its original amount?

Suppose a culture of bacteria is growing in such a way that the change in the number of bacteria is proportional to the number present. The number of bacteria double every 200 minutes and there are currently 5,000 bacteria in the culture. How many bacteria were present 2 hours ago?

$u(t) \equiv$ # of bacteria at time t.

in hours

currently $\Longleftrightarrow t=0$

$$\underline{u'(t) = k\,u(t)}$$

$$\Downarrow$$

$$u(t) = C e^{kt}$$

$$\boxed{u(0) = 5{,}000}$$

twice as many every $\underbrace{200 \text{ minutes.}}$ $\hookrightarrow \frac{10}{3}$ hours

$$5000 = Ce^{0}$$

$$C = 5000$$

$$\Rightarrow \boxed{u(t) = 5000e^{kt}}$$

Find $u(-2)$.

$$10{,}000 = 5{,}000\,e^{k\cdot\frac{10}{3}}$$

$$2 = e^{k\cdot\frac{10}{3}} \Rightarrow \underbrace{2^{3/10} = e^{k}}$$

$$u(t) = 5000\,(e^{k})^{t} = 5000\cdot 2^{\frac{3}{10}t}$$

$$\Rightarrow u(-2) = 5000\cdot 2^{-6/10}$$

$$= 3298.7697$$

i.e. roughly 3299 bacteria 2 hours ago.

Radio-active substances change at a rate proportional to the amount present. What is the half-life of a radio-active substance if it takes 10 years for 28% of the substance to decay?

$u(t) \equiv$ amount of the radio-active substance at time t (in years)

$u'(t) = k u(t)$

$\Downarrow$

$u(t) = Ce^{kt}$

- half-life $\equiv$ the amount of time it takes for $\frac{1}{2}$ of the substance to decay.

$\downarrow$

Find this.

We know $u(10) = .72\, u(0)$.

$\cancel{C} e^{10k} = .72 \cancel{C} e^{0}$

$\Rightarrow e^{10k} = .72$

$10k = \ln(.72)$

$\Rightarrow k = \frac{\ln(.72)}{10}$. $\Rightarrow u(t) = C e^{\frac{\ln(.72)}{10} t}$

Half Life? Call it T.

$\boxed{u(T) = \frac{1}{2} u(0)}$

$\cancel{C} e^{\frac{\ln(.72)}{10} T} = \frac{1}{2} \cancel{C} e^{0}$

Solve for T.

$\frac{\ln(.72)}{10} T = \ln(1/2)$

$\Rightarrow T = \frac{10 \ln(.5)}{\ln(.72)}$

$= 21.1001\cdots$ years

After 3 days a sample of radon-222 decayed to 58% of its original amount. What is the half-life of radon-222? How long would it take the sample to decay to 10% of its original amount?

$u(t) \equiv$ amount of radon-222 at time t (days).

$$u'(t) = k\,u(t)$$

- $u(3) = .58\,u(0)$
- Find the half-life.
- $\boxed{\text{Find } \tau \text{ so that } u(\tau) = \frac{1}{10}u(0).}$

$$\Downarrow$$

$$u(t) = Ce^{kt}$$

$$\cancel{C}e^{3k} = .58\,\cancel{C}$$

$$\Rightarrow e^{3k} = .58 \Rightarrow 3k = \ln(.58) \Rightarrow k = \tfrac{1}{3}\ln(.58).$$

$$\Rightarrow \boxed{u(t) = Ce^{\frac{1}{3}\ln(.58)\,t}}$$

Half-Life: Call it T.

$$u(T) = \tfrac{1}{2}u(0)$$

$$\cancel{C}e^{\frac{1}{3}\ln(.58)\,T} = \tfrac{1}{2}\cancel{C}$$

$$\Rightarrow \tfrac{1}{3}\ln(.58)\,T = \ln(.5)$$

$$\Rightarrow T = 3\frac{\ln(.5)}{\ln(.58)} = 3.817\cdots \text{ days}$$

$$\cancel{C}e^{\frac{1}{3}\ln(.58)\,\tau} = \tfrac{1}{10}\cancel{C}$$

Solve for τ.

$$\tfrac{1}{3}\ln(.58)\,\tau = \ln(0.1)$$

$$\Rightarrow \tau = 3\frac{\ln(0.1)}{\ln(.58)} \text{ days.}$$