Homework is due TODAY in recitation.

Poppers start TODAY!!
You must have the proper form.

Quiz 1 closed on Saturday.

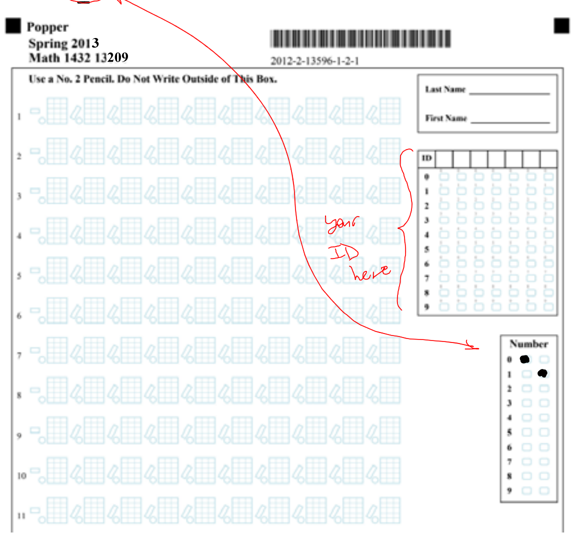
Test 2 starts on February 14th.
Registration for Test 2 starts on January 31st at 12:01am.

Please tell you high school friends and former teachers about our
High School Mathematics Contest

February 9th
University of Houston

http://mathcontest.uh.edu

Popper P01



Popper P01

Do not write 1 or 1 or 2 or 4

Write 1 2 4 5 6 7 8 9 0

1. 1 + 2 = 3

3

2. The answer is –17.

-17

3. The answer is –2.1356.

-2.1356

Popper P01

4. The answer is $-23/421$

-23/421

5. The answer is 0.5.

0.5

or

.5

or

1/2

Note: Decimals should be accurate to 4 places after the decimal.

$\underbrace{5.1267}8214$

$\hookrightarrow$ 5.1267

Popper P01

6. Give the slope of the tangent line to the graph of $f(x) = e^{2x} - 3x$ at $x = 0$.

-1

$$f'(x) = 2e^{2x} - 3$$

$$f'(0) = 2 - 3 = -1$$

Popper P01

7. Give the domain of the function $g(x) = \ln(1 - 3x)$.

1. $x > 0$
2. $x > 1/3$
3. $x < 1/3$
4. $x < -1/3$
5. None of the above.

Write 1, 2, 3, 4 or 5

$1 - 3x > 0$

$\frac{1}{3} > x$

Write the number associated with your answer choice.

3

Today...

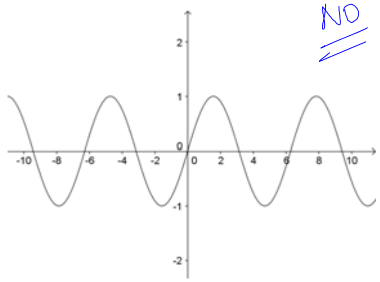
Inverse Trigonometric Functions

Section 7.7

Question: Is this an invertible function?

$f(x) = \sin(x)$

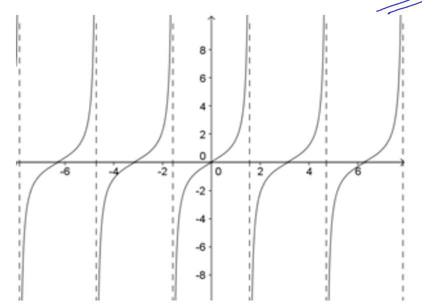
NO



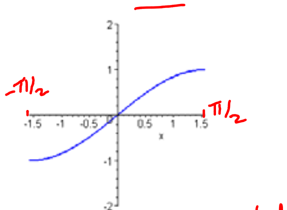
Question: Is this an invertible function?

$f(x) = \tan(x)$

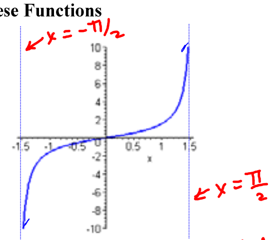
NO



Restricted Versions of these Functions



$f(x) = \sin(x)$ on $[-\pi/2, \pi/2]$ } restricted $\sin(x)$



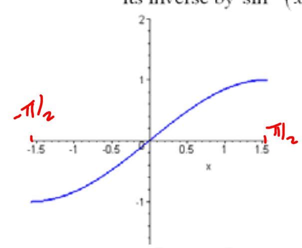
$f(x) = \tan(x)$ on $(-\pi/2, \pi/2)$ } restricted $\tan(x)$

These are invertible functions!!

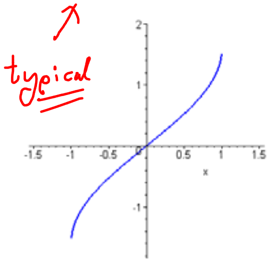
Let $f(x) = \sin(x)$ for $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. } restricted $\sin(x)$

This function is invertible, and we denote its inverse by $\sin^{-1}(x)$ or $\arcsin(x)$.

typical



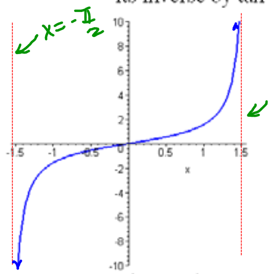
$\sin(x)$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



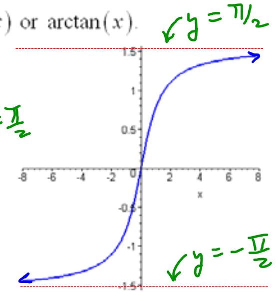
$\arcsin(x)$ on $[-1,1]$

Let $f(x) = \tan(x)$ for $\underline{\underline{x \in}} \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ } restricted $\tan(x)$

This function is invertible, and we denote its inverse by $\tan^{-1}(x)$ or $\arctan(x)$.



$\tan(x)$ on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



$\arctan(x)$ on $(-\infty, \infty)$

$$\lim_{x \to \infty} \arctan(x) = \pi/2$$

$$\lim_{x \to -\infty} \arctan(x) = -\pi/2$$

	Domain	Range
$\sin(x)$ *	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1, 1]$
$\arcsin(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\tan(x)$ *	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$(-\infty, \infty)$
$\arctan(x)$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

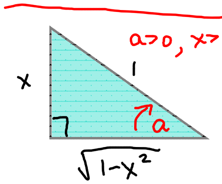
*** - Restricted version**

Question: What is the derivative of $\arcsin(x)$?

$f(x) = \sin(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\longrightarrow$ $f'(x) = \cos(x)$

$f^{-1}(x) = \arcsin(x)$

$$\frac{d}{dx}\arcsin(x) = (f^{-1})'(x) = \frac{1}{f'(a)} = \frac{1}{\cos(a)}$$



where $f(a) = x$

i.e. $\sin(a) = x$

$\Rightarrow \cos(a) = \sqrt{1-x^2}$

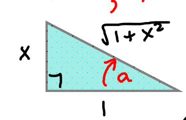
$$\boxed{\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}}$$

Question: What is the derivative of $\arctan(x)$?

$f(x) = \tan(x)$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$ $\longrightarrow$ $f'(x) = \sec^2(x)$

$f^{-1}(x) = \arctan(x)$

$$\frac{d}{dx}\arctan(x) = (f^{-1})'(x) = \frac{1}{f'(a)} = \frac{1}{\sec^2(a)}$$



where $f(a) = x$

i.e. $\tan(a) = x$

$\sec^2(a) = \left(\sqrt{1+x^2}\right)^2 = 1 + x^2$

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

Chain Rule Derivative Formulas

$$\frac{d}{dx}\arcsin(u) = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\arctan(u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

Consequences

$$\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C$$

$$\int \frac{1}{1+u^2}\,du = \arctan(u) + C$$

Example: Give the domain of $f(x) = \arctan(\ln(x))$ and find its derivative.

does not restrict anything.

$x > 0$

Domain : $(0, \infty)$.

$$f'(x) = \frac{1}{1+(\ln(x))^2}\cdot\frac{1}{x} = \frac{1}{x\left(1+(\ln(x))^2\right)}$$

Popper P01

8\. Give the slope of the tangent line to the graph of $f(x) = \arctan(4-2x)$ at $x = \frac{3}{2}$.

$$f'(x) = \frac{1}{1+(4-2x)^2}\cdot(-2)$$

$$\Rightarrow \quad f'(3/2) = \frac{1}{1+1}(-2) = -1$$

Example: Give the domain of $g(x) = \arcsin\left(\frac{e^x}{2}\right)$, and find an equation for the tangent line to the graph of this function at $x = 0$.

$-1 \le \boxed{\frac{e^x}{2}} \le 1$

automatic

always > 0

$e^x \le 2$

$x \le \ln(2)$

Domain : $(-\infty, \ln(2)]$.

Point $= (0, \arcsin(\tfrac{1}{2})) = (0, \pi/6)$

Slope $= g'(0) = \frac{\sqrt{3}}{3}$

$$g(x) = \arcsin\left(\tfrac{1}{2}e^x\right) \Rightarrow g'(x) = \frac{1}{\sqrt{1-\frac{1}{4}e^{2x}}}\cdot\tfrac{1}{2}e^x$$

$$\left(\tfrac{1}{2}e^x\right)^2 = \tfrac{1}{4}e^{2x}$$

$$g'(0) = \frac{1}{\sqrt{3/4}}\cdot\frac{1}{2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Tangent Line :

$$y - \frac{\pi}{6} = \frac{\sqrt{3}}{3}x$$

Example: Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx. = \frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx$

$u = x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{1}{2}\arcsin(u) + C$$

$$= \frac{1}{2}\arcsin(x^2) + C$$

Example: Compute $\int \frac{e^x}{e^{2x}+1}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx$

$u = e^x$

$du = e^x\,dx$

$$= \int \frac{du}{1+u^2}$$

$$= \arctan(u) + C$$

$$= \arctan(e^x) + C$$