Homework is due TODAY in recitation.

Poppers start TODAY!!
You must have the proper form.

Quiz 1 closed on Saturday.

Test 2 starts on February 14th.
Registration for Test 2 starts on January 31st at 12:01am.
Please tell you high school friends and former teachers about our
High School Mathematics Contest

February 9th
University of Houston

http://mathcontest.uh.edu
Use a No. 2 Pencil. Do Not Write Outside of This Box.

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Number
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1. $1 + 2 = 3$

2. The answer is $-17$.

3. The answer is $-2.1356$. 

Do not write 1 or 1 or 2 or 4. Write 1 2 4 5 6 7 8 9 0
Popper P01

4. The answer is $-\frac{23}{421}$

5. The answer is 0.5.

Note: Decimals should be accurate to 4 places after the decimal.

$5.12678214 \rightarrow 5.1267$
Popper P01

6. Give the slope of the tangent line to the graph of \( f(x) = e^{2x} - 3x \) at \( x = 0 \).

\[
\begin{align*}
  f'(x) &= 2e^{2x} - 3 \\
  f'(0) &= 2 - 3 = -1
\end{align*}
\]
Popper P01

7. Give the domain of the function $g(x) = \ln(1 - 3x)$.

1. $x > 0$
2. $x > 1/3$
3. $x < 1/3$
4. $x < -1/3$
5. None of the above.

Write the number associated with your answer choice.
Today...

Inverse Trigonometric Functions

Section 7.7
Question: Is this an invertible function?

\[ f(x) = \sin(x) \]

\[ \text{NO} \]
**Question:** Is this an invertible function?

\[ f(x) = \tan(x) \]

\[ \mathbb{N} \not\supset \mathbb{O} \]
Restricted Versions of these Functions

\[ f(x) = \sin(x) \] restricted \[ \sin(x) \]
on \([-\pi/2, \pi/2]\)

\[ f(x) = \tan(x) \] restricted \[ \tan(x) \]
on \((-\pi/2, \pi/2)\)

These are invertible functions!!
Let \( f(x) = \sin(x) \) for \( x \in \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \), which restricts \( \sin(x) \).

This function is invertible, and we denote its inverse by \( \sin^{-1}(x) \) or \( \arcsin(x) \).

\( \sin(x) \) on \( \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right] \)

\( \arcsin(x) \) on \( [-1, 1] \)
Let \( f(x) = \tan(x) \) for \( x \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \), \( \exists \text{ restricted } \tan(x) \).

This function is invertible, and we denote its inverse by \( \tan^{-1}(x) \) or \( \arctan(x) \).

\( x = -\frac{\pi}{2} \) \( \Rightarrow \ y = \frac{\pi}{2} \)

\( x = \frac{\pi}{2} \) \( \Rightarrow \ y = -\frac{\pi}{2} \)

\( \arctan(x) \) on \( (-\infty, \infty) \)

\( \lim_{x \to \infty} \arctan(x) = \frac{\pi}{2} \)

\( \lim_{x \to -\infty} \arctan(x) = -\frac{\pi}{2} \)
<table>
<thead>
<tr>
<th>Function</th>
<th>Domain</th>
<th>Range</th>
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<tbody>
<tr>
<td>sin(x) *</td>
<td>$[-\frac{\pi}{2}, \frac{\pi}{2}]$</td>
<td>[-1, 1]</td>
</tr>
<tr>
<td>arcsin(x)</td>
<td>[-1, 1]</td>
<td>$[-\frac{\pi}{2}, \frac{\pi}{2}]$</td>
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<tr>
<td>tan(x) *</td>
<td>$(-\frac{\pi}{2}, \frac{\pi}{2})$</td>
<td>$(-\infty, \infty)$</td>
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<tr>
<td>arctan(x)</td>
<td>$(-\infty, \infty)$</td>
<td>$(-\frac{\pi}{2}, \frac{\pi}{2})$</td>
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* - Restricted version
**Question:** What is the derivative of \( \arcsin(x) \)?

\[
f(x) = \sin(x) \quad \text{on} \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]
\]

\[
f^{-1}(x) = \arcsin(x)
\]

\[
f'(x) = \cos(x)
\]

\[
\frac{d}{dx} \arcsin(x) = (f^{-1})'(x) = \frac{1}{f'(a)} = \frac{1}{\cos(a)}
\]

where \( f(a) = x \)

i.e., \( \sin(a) = x \)

\[
\Rightarrow \cos(a) = \sqrt{1-x^2}
\]

\[
\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}
\]
**Question:** What is the derivative of \( \arctan(x) \)?

\[
\begin{align*}
f(x) &= \tan(x) \quad \text{on} \quad (-\frac{\pi}{2}, \frac{\pi}{2}) \\
f^{-1}(x) &= \arctan(x) \quad \Rightarrow \quad f'(x) = \sec^2(x) \\
\frac{d}{dx} \arctan(x) &= (f^{-1})'(x) = \frac{1}{f'(a)} = \frac{1}{\sec^2(a)} \\
\text{where} \quad f(a) &= x \\
\text{i.e.} \quad \tan(a) &= x \\
\end{align*}
\]

\[
\begin{align*}
\sec^2(a) &= \left(\sqrt{1+x^2}\right)^2 \\
&= 1+x^2 \\
\frac{d}{dx} \arctan(x) &= \frac{1}{1+x^2}
\end{align*}
\]
Chain Rule Derivative Formulas

\[ \frac{d}{dx} \arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \]

\[ \frac{d}{dx} \arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx} \]

Consequences

\[ \int \frac{1}{\sqrt{1-u^2}} \, du = \arcsin(u) + C \]

\[ \int \frac{1}{1+u^2} \, du = \arctan(u) + C \]
**Example:** Give the domain of \( f(x) = \arctan(\ln(x)) \) and find its derivative.

\[
\text{Domain: } (0, \infty).
\]

\[
f'(x) = \frac{1}{1+(\ln(x))^2} \cdot \frac{1}{x} = \frac{1}{x(1+(\ln(x))^2)}
\]
Popper P01

8. Give the slope of the tangent line to the graph of \( f(x) = \arctan(4 - 2x) \) at \( x = \frac{3}{2} \).

\[
\begin{align*}
\frac{df}{dx} &= \frac{1}{1 + (4 - 2x)^2} \cdot (-2) \\
\Rightarrow \quad f' \left( \frac{3}{2} \right) &= \frac{1}{1 + 1} (-2) = -1
\end{align*}
\]
Example: Give the domain of \( g(x) = \arcsin\left(\frac{e^x}{2}\right) \), and find an equation for the tangent line to the graph of this function at \( x = 0 \).

\[ -1 \leq \frac{e^x}{2} \leq 1 \]
\[ e^x \leq 2 \]
\[ x \leq \ln(2) \]
\[ \text{Domain: } (-\infty, \ln(2)] \]

Point = \( (0, \arcsin(\frac{1}{2})) = (0, \frac{\pi}{6}) \)

Slope = \( g'(0) = \frac{1}{\sqrt{1 - \frac{1}{4} e^{2x}}} \cdot \frac{1}{2} e^x \)

\[ \left(\frac{1}{2} e^x\right)^2 = \frac{1}{4} e^{2x} \]
\[ g'(0) = \frac{1}{\sqrt{\frac{3}{4}}} \cdot \frac{1}{2} = \frac{1}{\sqrt{3}} \]
\[ = \frac{\sqrt{3}}{3} \]

Tangent Line: \( y - \frac{\pi}{6} = \frac{\sqrt{3}}{3} x \)
Example: Compute $\int \frac{x}{\sqrt{1-x^4}} \, dx = \frac{1}{2} \int \frac{2x}{\sqrt{1-(x^2)^2}} \, dx$

$u = x^2$
$du = 2x \, dx$

$= \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2} \arcsin(u) + C$

$= \frac{1}{2} \arcsin(x^2) + C$
Example: Compute \( \int \frac{e^x}{e^{2x} + 1} \, dx = \int \frac{e^x}{1 + (e^x)^2} \, dx \)

\( u = e^x \)
\( du = e^x \, dx \)

\( = \int \frac{du}{1 + u^2} \)

\( = \arctan(u) + C \)

\( = \arctan(e^x) + C \)