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Homework is due TODAY in recitation.

Poppers start TODAY!!
You must have the proper form.

Quiz 1 closed on Saturday.

Test 2 starts on February 14th.
Registration for Test 2 starts on January 31st at 12:01am.

Please tell you high school friends and former teachers about our

High School Mathematics Contest

February 9th
University of Houston

http://mathcontest.uh.edu

Popper P01

Popper
Spring 2013
Math 1432 13209

2012-2-13596-1-2-1

Use a No. 2 Pencil. Do Not Write Outside of This Box.

1
2
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10
11

Last Name ____________

First Name ____________

ID							
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your ID here

Number		
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Do not write 1 or 1 or 2 or 4

Write 1 2 4

5 6

7 8

9 0

1. 1 + 2 = 3

1 3

2. The answer is –17.

2 -17

3. The answer is –2.1356.

3 -2.1356

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4. The answer is –23/421

4 -23/421

5. The answer is 0.5.

5 0.5

or

5 .5

or

5 1/2

Note: Decimals should be accurate to 4 places after the decimal.

5.12678214

→ 5.1267

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6. Give the slope of the tangent line to the graph of $f(x) = e^{2x} - 3x$ at $x = 0$.

6 -1

$$f'(x) = 2e^{2x} - 3$$

$$f'(0) = 2 - 3 = -1$$

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7. Give the domain of the function $g(x) = \ln(1 - 3x)$.

1. $x > 0$
2. $x > 1/3$
3. $x < 1/3$
4. $x < -1/3$
5. None of the above.

Write 1, 2, 3, 4 or 5

$1 - 3x > 0$

$\frac{1}{3} > x$

Write the number associated with your answer choice.

7 3

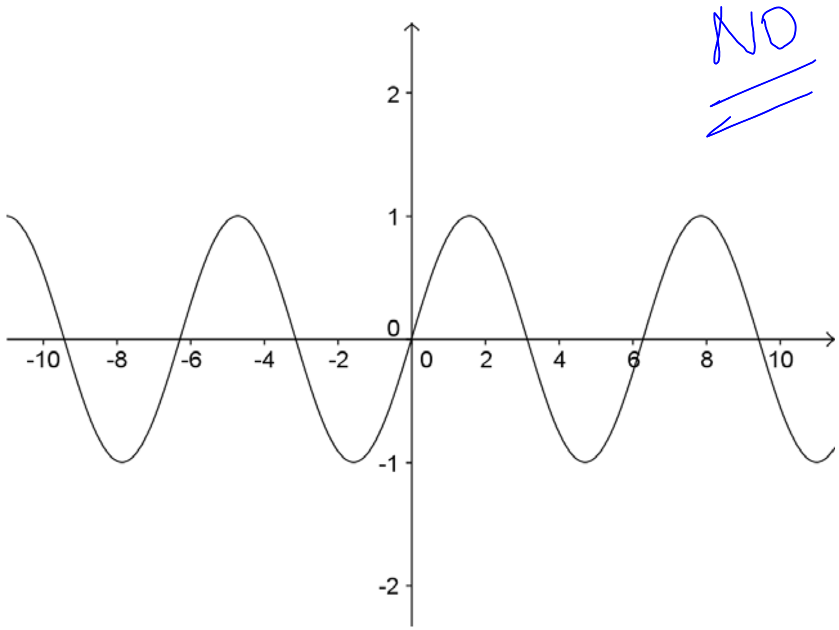
Today...

Inverse Trigonometric Functions

Section 7.7

Question: Is this an invertible function?

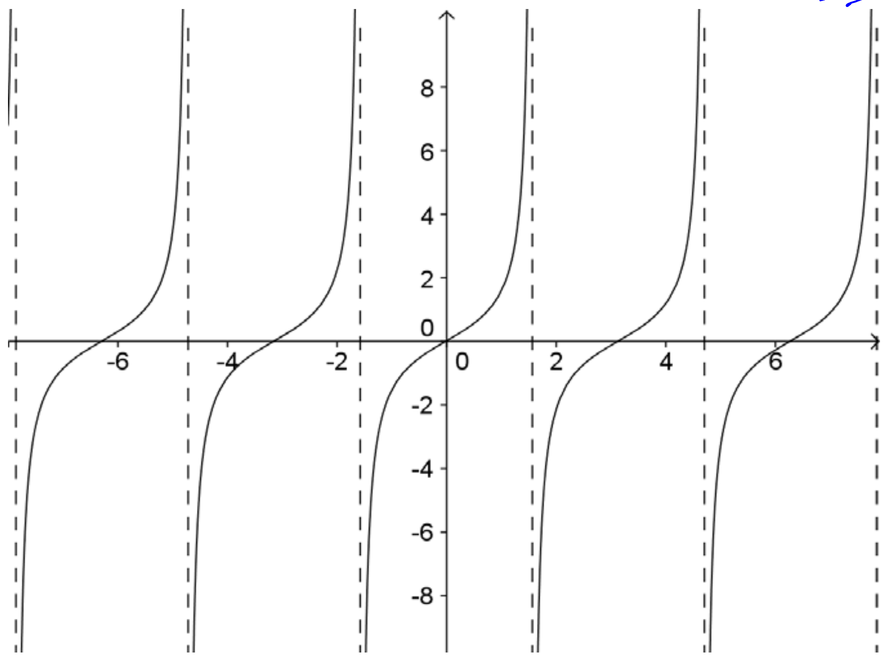
$$f(x) = \sin(x)$$



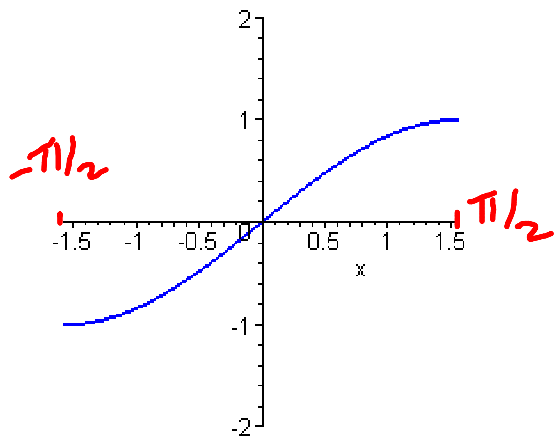
Question: Is this an invertible function?

$$f(x) = \tan(x)$$

NO

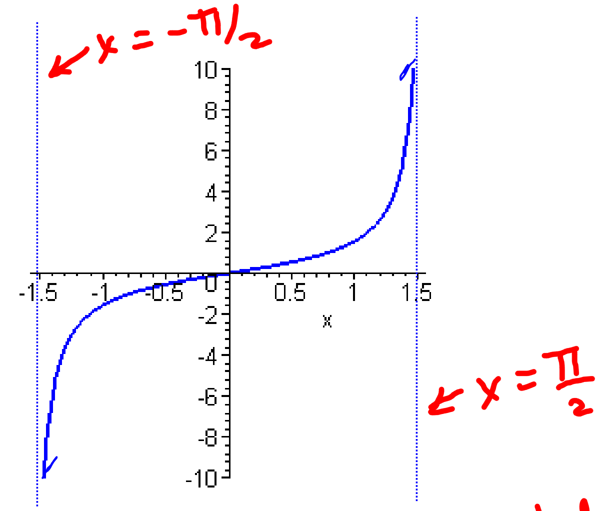


Restricted Versions of these Functions



$f(x) = \sin(x)$
on [-π/2,π/2]

restricted sin(x)



$f(x) = \tan(x)$
on (-π/2,π/2)

restricted tan(x)

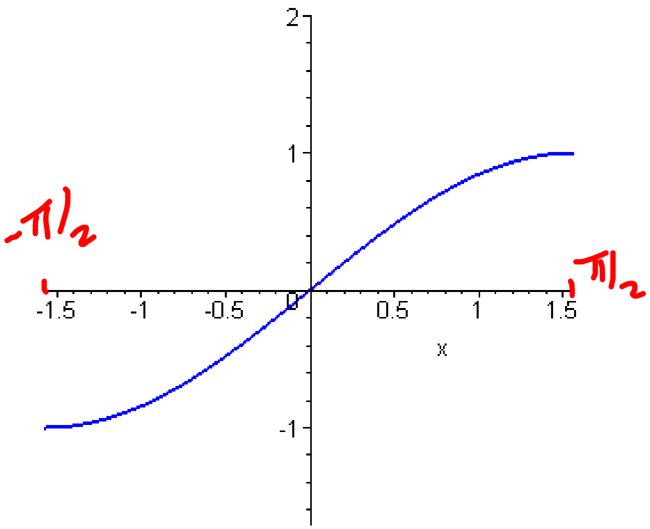
These are invertible functions!!

Let $f(x) = \sin(x)$ for $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. } restricted sin(x)

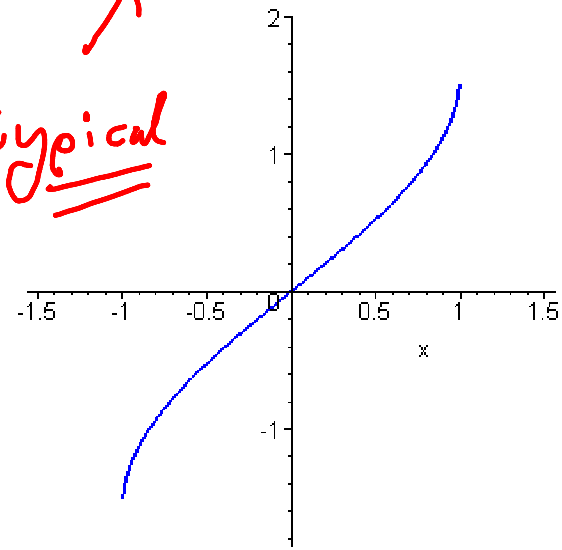
This function is invertible, and we denote

its inverse by $\sin^{-1}(x)$ or $\arcsin(x)$.

↗ typical



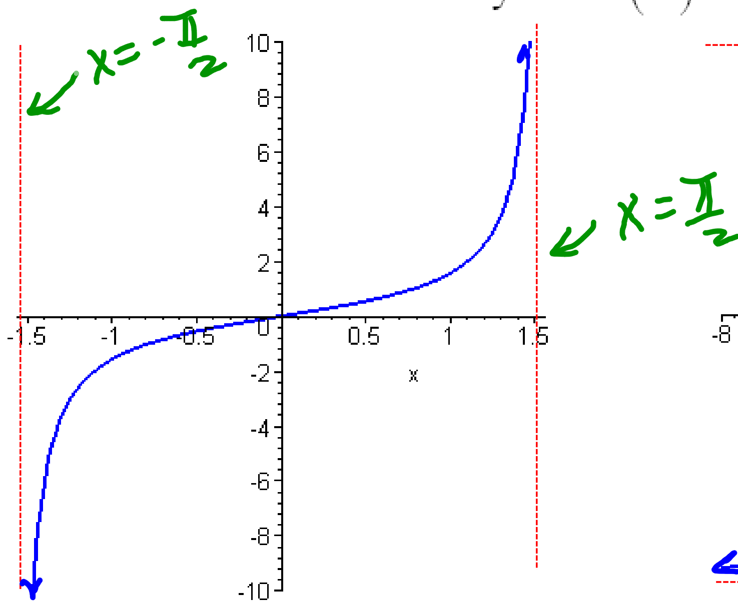
$\sin(x)$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



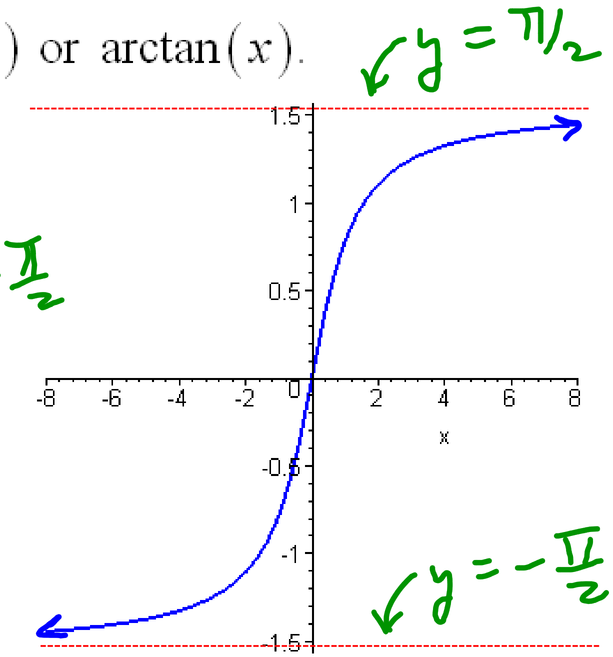
$\arcsin(x)$ on $[-1,1]$

Let $f(x) = \tan(x)$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$. } restricted tan(x)

This function is invertible, and we denote its inverse by $\tan^{-1}(x)$ or $\arctan(x)$.



$\tan(x)$ on $\left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$



$\arctan(x)$ on $(-\infty, \infty)$

$$\lim_{x \to \infty} \arctan(x) = \pi/2$$

$$\lim_{x \to -\infty} \arctan(x) = -\pi/2$$

	Domain	**Range**
sin(x) *	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1, 1]$
arcsin(x)	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
tan(x) *	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$(-\infty, \infty)$
arctan(x)	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

*** - Restricted version**

Question: What is the derivative of $\arcsin(x)$?

$f(x) = \sin(x)$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\rightarrow$ $f'(x) = \cos(x)$

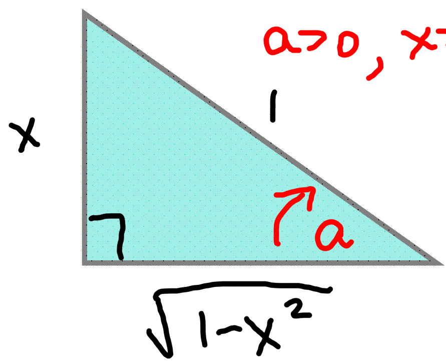
$f^{-1}(x) = \arcsin(x)$

$$\frac{d}{dx}\arcsin(x) = \left(f^{-1}\right)'(x) = \frac{1}{f'(a)} = \frac{1}{\cos(a)}$$

where $f(a) = x$

i.e. $\sin(a) = x$

$a > 0,\ x > 0$



$\Rightarrow \cos(a) = \sqrt{1-x^2}$

$$\boxed{\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}}$$

Question: What is the derivative of arctan(x) ?

$$f(x) = \tan(x) \quad \text{on} \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$f'(x) = \sec^2(x)$$

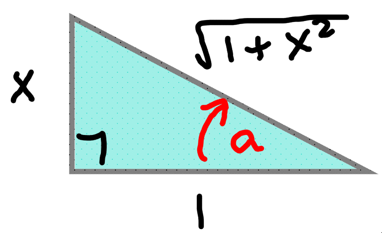
$$f^{-1}(x) = \arctan(x)$$

$$\frac{d}{dx}\arctan(x) = (f^{-1})'(x) = \frac{1}{f'(a)} = \frac{1}{\sec^2(a)}$$

where $f(a) = x$

i.e. $\tan(a) = x$

$a > 0, \; x > 0$



$$\sec^2(a) = \left(\sqrt{1+x^2}\right)^2 = 1 + x^2$$

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

Chain Rule Derivative Formulas	Consequences
$\frac{d}{dx}\arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$	$\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C$
$\frac{d}{dx}\arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$	$\int \frac{1}{1+u^2}\,du = \arctan(u) + C$

Example: Give the domain of $f(x) = \arctan(\ln(x))$ and find its derivative.

arctan: does not restrict anything.

ln: $x > 0$

Domain: $(0, \infty)$.

$$f'(x) = \frac{1}{1+(\ln(x))^2} \cdot \frac{1}{x} = \frac{1}{x(1+(\ln(x))^2)}$$

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8. Give the slope of the tangent line to the graph of $f(x) = \arctan(4 - 2x)$ at $x = \frac{3}{2}$.

8 -1

$$f'(x) = \frac{1}{1 + (4-2x)^2} \cdot (-2)$$

$$\Rightarrow \quad f'(3/2) = \frac{1}{1+1}(-2) = -1$$

Example: Give the domain of $g(x) = \arcsin\left(\frac{e^x}{2}\right)$, and find an equation for the tangent line to the graph of this function at $x = 0$.

$-1 \le \frac{e^x}{2} \le 1$

automatic

always > 0

$e^x \le 2$

$x \le \ln(2)$

Domain: $(-\infty, \ln(2)]$.

Point = $(0, \arcsin(\frac{1}{2})) = (0, \pi/6)$

Slope = $g'(0) = \frac{\sqrt{3}}{3}$

$g(x) = \arcsin(\frac{1}{2}e^x) \Rightarrow g'(x) = \frac{1}{\sqrt{1 - \frac{1}{4}e^{2x}}} \cdot \frac{1}{2}e^x$

$(\frac{1}{2}e^x)^2 = \frac{1}{4}e^{2x}$

$g'(0) = \frac{1}{\sqrt{3/4}} \cdot \frac{1}{2} = \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

Tangent Line:

$y - \frac{\pi}{6} = \frac{\sqrt{3}}{3}x$

Example: Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx$. $= \frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx$

$u = x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{1}{2}\arcsin(u) + C$$

$$= \frac{1}{2}\arcsin(x^2) + C$$

Example: Compute $\int \frac{e^x}{e^{2x}+1}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx$

$u = e^x$

$du = e^x\,dx$

$$= \int \frac{du}{1+u^2}$$

$$= \arctan(u) + C$$

$$= \arctan(e^x) + C$$