Homework is due TODAY in lab/workshop.

Poppers start TODAY!!
You must have the proper form.

Quiz 1 closed on Saturday.

Test 2 starts on February 14th.
Registration for Test 2 starts on January 31st at 12:01am.

Please tell you high school friends and former teachers about our

High School Mathematics Contest

February 9th

University of Houston

* **http://mathcontest.uh.edu**

Popper P01

Popper
Spring 2013
Math 1432 13209

2012-2-13596-1-2-1

Use a No. 2 Pencil. Do Not Write Outside of This Box.

Last Name Morgan

First Name Jeff

ID	0	0	8	8	4	3	2
0							
1							
2							
3							
4							
5							
6							
7							
8							
9							

Number		
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

1

2

3

4

5

6

7

8

9

10

11

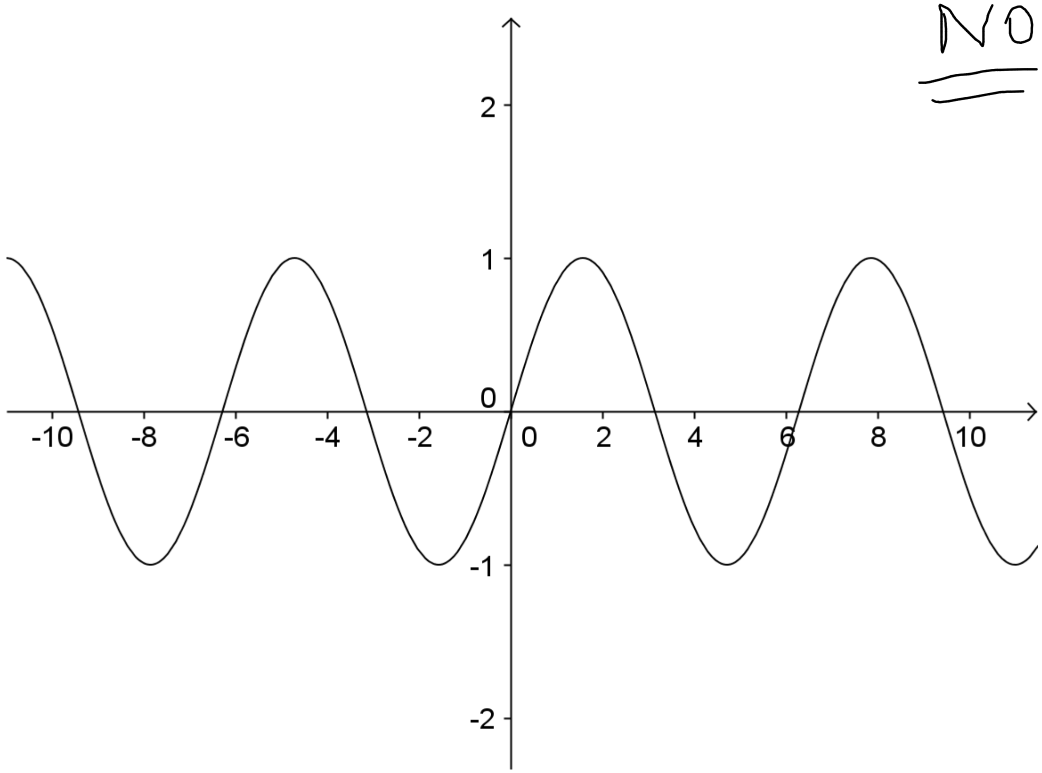
Today...

Inverse Trigonometric Functions

Section 7.7

Question: Is this an invertible function?

$$f(x) = \sin(x)$$

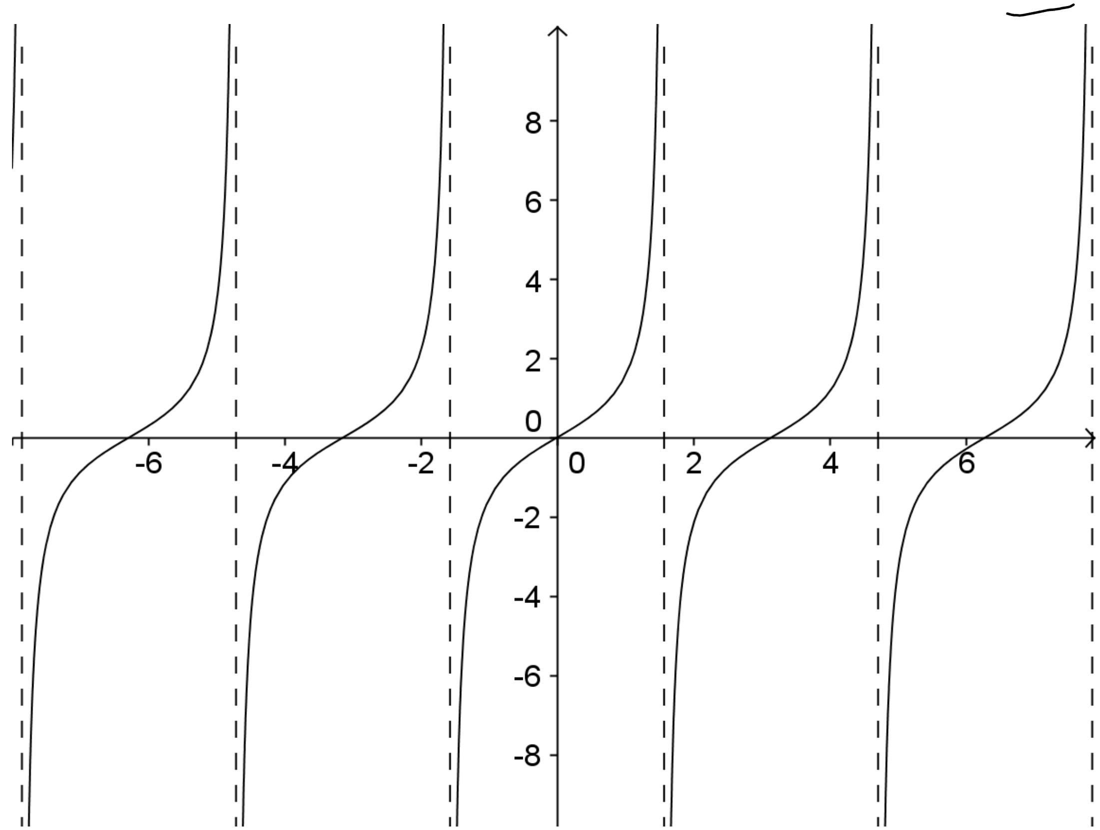


NO

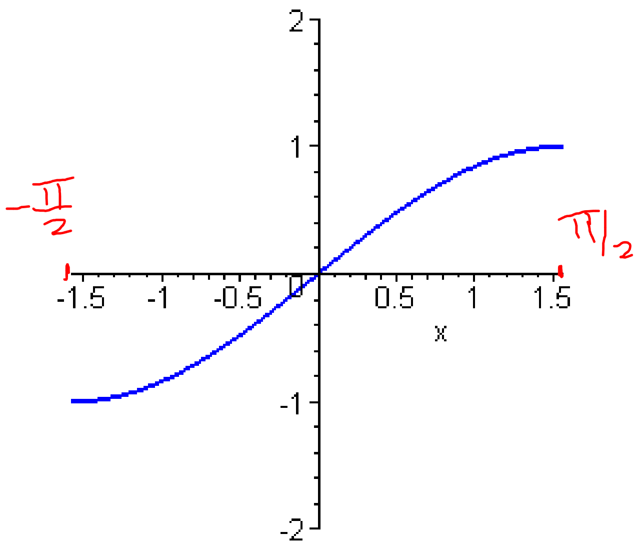
Question: Is this an invertible function?

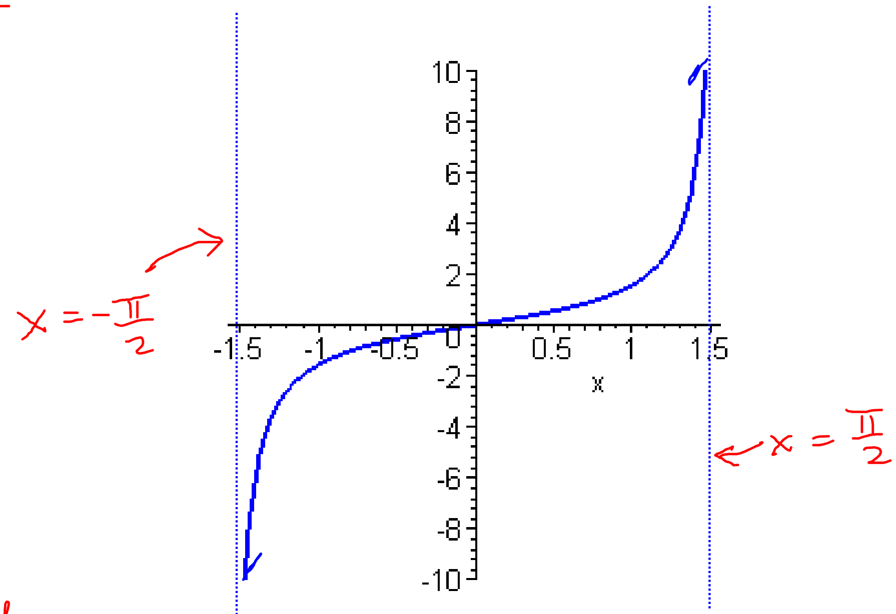
$$f(x) = \tan(x)$$

NO



Restricted Versions of these Functions





$f(x) = \sin(x)$
on $[-\pi/2, \pi/2]$ } restricted sin(x)

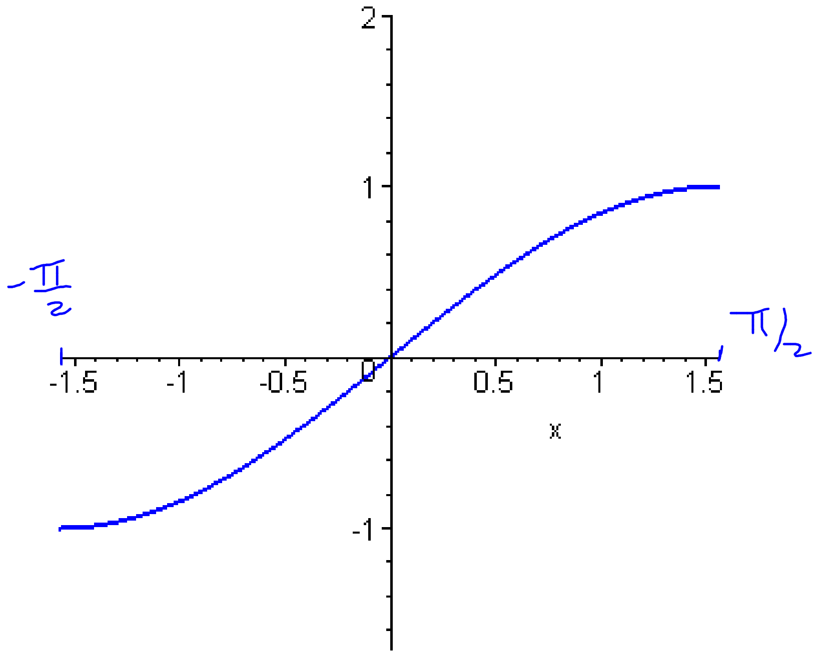
$f(x) = \tan(x)$
on $(-\pi/2, \pi/2)$ } restricted tan(x)

These are invertible functions!!

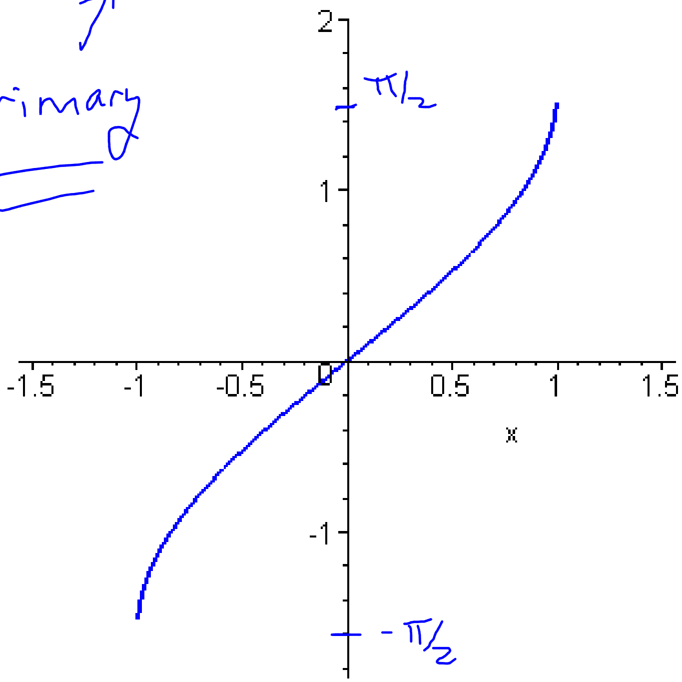
Let $f(x) = \sin(x)$ for $x \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$.

This function is invertible, and we denote its inverse by $\sin^{-1}(x)$ or arcsin(x).

primary



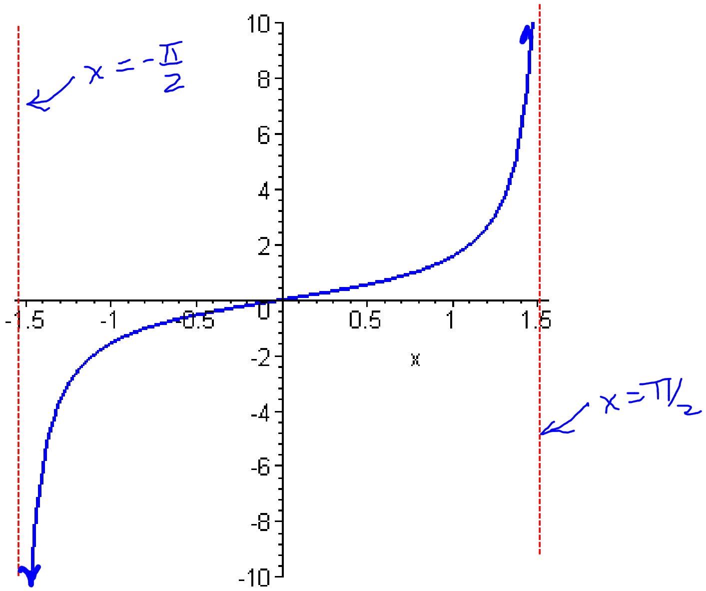
$\sin(x)$ on $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$



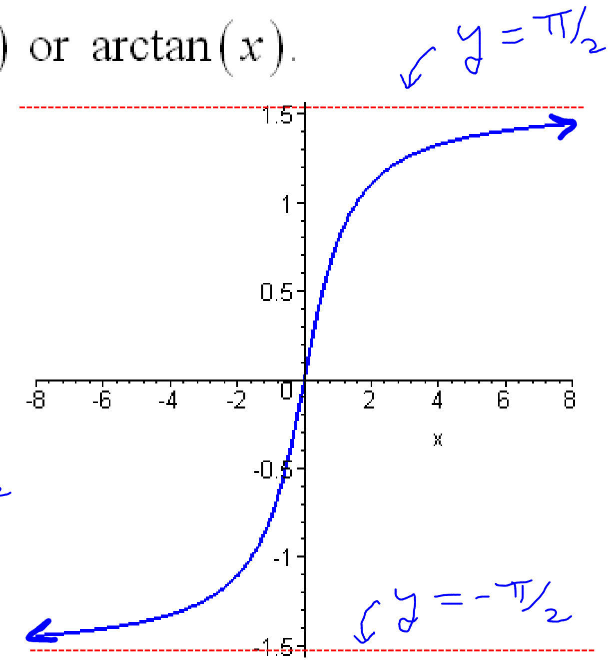
$\arcsin(x)$ on $[-1,1]$

Let $f(x) = \tan(x)$ for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$.

This function is invertible, and we denote its inverse by $\tan^{-1}(x)$ or $\arctan(x)$.



$\tan(x)$ on $\left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$



$\arctan(x)$ on $(-\infty, \infty)$

$\lim_{x \to \infty} \arctan(x) = \pi/2$

$\lim_{x \to -\infty} \arctan(x) = -\pi/2$

	Domain	Range
$\sin(x)$ *	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1, 1]$
$\arcsin(x)$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\tan(x)$ *	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$(-\infty, \infty)$
$\arctan(x)$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\boxed{\textbf{* - Restricted version}}$

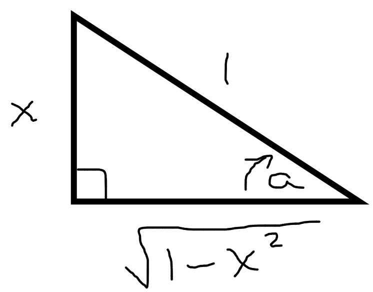
Question: What is the derivative of arcsin(x) ?

$f(x) = \sin(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $\longrightarrow$ $f'(x) = \cos(x)$

$f^{-1}(x) = \arcsin(x)$

$$\frac{d}{dx}\arcsin(x) = (f^{-1})'(x) = \frac{1}{f'(a)}$$

Case: $a > 0$, $x > 0$



$\cos(a) = \sqrt{1-x^2}$

where $f(a) = x$

i.e. $\sin(a) = x$

angle

$$= \frac{1}{\cos(a)}$$

$$= \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$

Question: What is the derivative of $\arctan(x)$?

$f(x) = \tan(x)$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f^{-1}(x) = \arctan(x)$ $\rightarrow$ $f'(x) = \sec^2(x)$

$$\frac{d}{dx}\arctan(x) = (f^{-1})'(x) = \frac{1}{f'(a)}$$

where $f(a) = x$

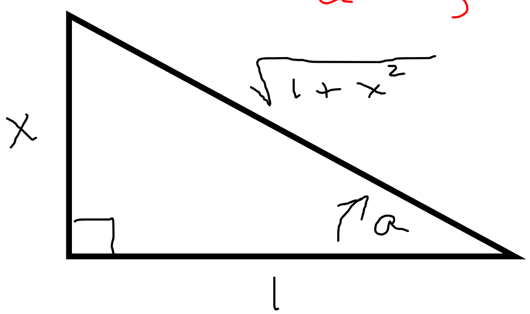
i.e. $\tan(a) = x$

$$= \frac{1}{\sec^2(a)}$$

$$= \cos^2(a)$$

$$= \frac{1}{1+x^2}$$

$a > 0$, $x > 0$



$$\cos^2(a) = \frac{1}{1+x^2}$$

Chain Rule Derivative Formulas	Consequences
$\frac{d}{dx}\arcsin(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$	$\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C$
$\frac{d}{dx}\arctan(u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$	$\int \frac{1}{1+u^2}\,du = \arctan(u) + C$

Example: Give the domain of $f(x) = \underline{\arctan} (\ln(x))$ and find its derivative.

$x > 0$

Domain: $x > 0$.

$$f'(x) = \frac{1}{1+(\ln(x))^2} \cdot \frac{1}{x} = \frac{1}{x\left(1+(\ln(x))^2\right)}$$

Example: Give the domain of $g(x) = \arcsin\left(\frac{e^x}{2}\right)$, and find an equation for the tangent line to the graph of this function at $x = 0$.

$$\underbrace{-1 \le}_{} \boxed{\frac{e^x}{2} \le 1} \iff e^x \le 2$$

$$x \le \ln(2)$$

positive (automatically)

Domain: $(-\infty, \ln(2)]$.

Tangent Line at $x=0$: Point $= (0, \arcsin(\tfrac{1}{2}))$

$= (0, \pi/6)$

$$\text{slope} = g'(0) = \frac{1}{\sqrt{3/4}} \cdot \frac{1}{2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$g(x) = \arcsin\left(\tfrac{1}{2}e^x\right) \Rightarrow g'(x) = \frac{1}{\sqrt{1 - \frac{1}{4}e^{2x}}} \cdot \tfrac{1}{2}e^x$$

Equation:

$$y - \frac{\pi}{6} = \frac{\sqrt{3}}{3}x$$

or $$y = \frac{\sqrt{3}}{3}x + \frac{\pi}{6}.$$

Example: Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx.$ $= \frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx$

$u = x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{1}{2}\arcsin(u) + C$$

$$= \frac{1}{2}\arcsin(x^2) + C$$

Example: Compute $\int \frac{e^x}{e^{2x}+1}\,dx$ $= \int \frac{e^x}{(e^x)^2+1}\,dx$

$u = e^x$

$du = e^x\,dx$

$$= \int \frac{du}{u^2+1}$$

$$= \arctan(u) + C$$

$$= \arctan(e^x) + C$$