Homework is posted.

Poppers started Monday!!
You must have the correct popper form.

Finish Quiz 2 and Start Quiz 3 asap.

Test 2 registration starts at 12:01am Thursday.

Please tell you high school friends and former teachers about our High School Mathematics Contest

February 9th
University of Houston

http://mathcontest.uh.edu

Math 1432 - 13209
Jeff Shurman - jshurman@math.uh.edu

Read the Syllabus

Use the Discussion Board on CourseWeb to get and give help.

Lecture notes/videos, additional help material, course announcements, homework and MCES will be posted on the calendar below. Note: Practice Tests count the same to online quizzes.

Course Calendar

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Homework 1 is posted.

Review: What is the solution to $y'' = ky$?

$y = Ce^{\frac{kt}{2}}$

$y(0) = C$ constant

$u' = -3u, u(0) = -3t$

$u(t) = Ce^{-3t}$

$u(4) = Ce^{-12} = 2$

$u(t) = 2e^{-3t}$
Aside on inverse trig integration formulas...

Assume $a > 0.$

\[ \int \frac{1}{u^2 + a^2} du = \frac{1}{a} \arctan \left( \frac{u}{a} \right) + C \]

\[ \int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \left( \frac{u}{a} \right) + C \]
Examples:

\[ \int \frac{1}{4 + x^2} \, dx = \int \frac{1}{2^2 + u^2} \, du = \frac{1}{2} \arctan \left( \frac{u}{2} \right) + C \]

\[ \int \frac{\cos(x)}{\sqrt{3 - \sin^2(x)}} \, dx = \int \frac{1}{\sqrt{3 - u^2}} \, du \]

Let \( u = \sin(x) \), then \( du = \cos(x) \, dx \)

\[ = \arcsin \left( \frac{u}{\sqrt{3}} \right) + C \]

\[ = \arcsin \left( \frac{\sin(x)}{\sqrt{3}} \right) + C \]

Hyperbolic Functions

\[ \cosh(x) = \frac{e^x + e^{-x}}{2} \]

\[ \sinh(x) = \frac{e^x - e^{-x}}{2} \]

\[ \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \]

\[ \coth(x) = \frac{\cosh(x)}{\sinh(x)} \]

\[ \text{sech}(x) = \frac{1}{\cosh(x)} \]

\[ \text{csch}(x) = \frac{1}{\sinh(x)} \]

Where do these names come from?

First, recall that sine and cosine are called circular functions. Do you recall why?

\[ x^2 + y^2 = 1 \]

\[ \cos^2(\theta) + \sin^2(\theta) = 1 \]
Now let's do some computations with cosh and sinh.

\[
\cosh^2(t) = \left[ \frac{1}{2} (e^t + e^{-t}) \right]^2 = \frac{1}{4} (e^{2t} + 2 + e^{-2t})
\]

\[
(a+b)^2 = a^2 + 2ab + b^2
\]

\[
\sinh^2(t) = \left[ \frac{1}{2} (e^t - e^{-t}) \right]^2 = \frac{1}{4} (e^{2t} - 2 + e^{-2t})
\]

\[
(a-b)^2 = a^2 - 2ab + b^2
\]

\[
\cosh^2(t) - \sinh^2(t) = \frac{2}{4} - \frac{2}{4} = 1
\]

Hyperbolic Identity

\[
\cosh^2(t) - \sinh^2(t) = 1
\]

\[
x = \cosh(t), \quad y = \sinh(t)
\]

\[
x^2 - y^2 = 1
\]
**Derivatives**

\[
\frac{d}{dx} \cosh(x) = \frac{d}{dx} \left( \frac{1}{2} e^x + \frac{1}{2} e^{-x} \right) = \frac{1}{2} e^x - \frac{1}{2} e^{-x} = \sinh(x)
\]

\[
\frac{d}{dx} \sinh(x) = \frac{d}{dx} \left( \frac{1}{2} e^x - \frac{1}{2} e^{-x} \right) = \frac{1}{2} e^x + \frac{1}{2} e^{-x} = \cosh(x)
\]

**Chain Rule Formula and Consequences**

\[
\frac{d}{dx} \cosh(u) = \sinh(u) \frac{du}{dx} \quad \int \cosh(u) \, du = \sinh(u) + C
\]

\[
\frac{d}{dx} \sinh(u) = \cosh(u) \frac{du}{dx} \quad \int \sinh(u) \, du = \cosh(u) + C
\]

Learn the derivative formulas for tanh, coth, sech and csch!!

**Example: Compute** \( \frac{d}{dx} \cosh(\ln(e^x + 2x)) \)

\[
\frac{d}{dx} \cosh(\ln(e^x + 2x)) = \sinh(u) \frac{du}{dx}
\]

\[
= \sinh(\ln(e^x + 2x)) \cdot \frac{1}{e^x + 2x} \cdot (e^x + 2)
\]

**Popper P02**

4. Give the slope of the tangent line to \( f(x) = \cosh(2x) + \sinh(-3x) \) at \( x = 0 \).

   \(-3\)

5. The answer is 1/2.

6. The answer is -13/7.
Example: Find the area bounded by the graphs of \( f(x) = \cosh(x) \) and \( g(x) = \sinh(x) \) for \( 0 \leq x \leq \ln(10) \).

\[
\text{Area} = \int_{0}^{\ln(10)} (\cosh(x) - \sinh(x)) \, dx
\]

\[
= \left[ \sinh(x) - \cosh(x) \right]_{0}^{\ln(10)}
\]

\[
= \left( e^{\ln(10)} - e^{0} \right) - \left( e^{0} - e^{0} \right)
\]

\[
= e^{\ln(10)} - 1
\]

\[
= e^{10} - 1
\]

\[
= \frac{10}{10} = 9
\]