Homework is posted.

Poppers started Monday!!
You must have the correct popper form.

Finish Quiz 2 and Start Quiz 3 asap.

Test 2 registration starts at 12:01am Thursday.

Please tell you high school friends and former teachers about our

High School Mathematics Contest

February 9th

University of Houston

http://mathcontest.uh.edu

Review: What is the solution to $y' = ky$?

constant

$$y = \underline{\underline{C}}\, e^{kt}$$

value of $y(0)$

Give the solution to $\underbrace{u'(t) + 3u(t) = 0}$, $\boxed{u(0) = 2.}$

initial data

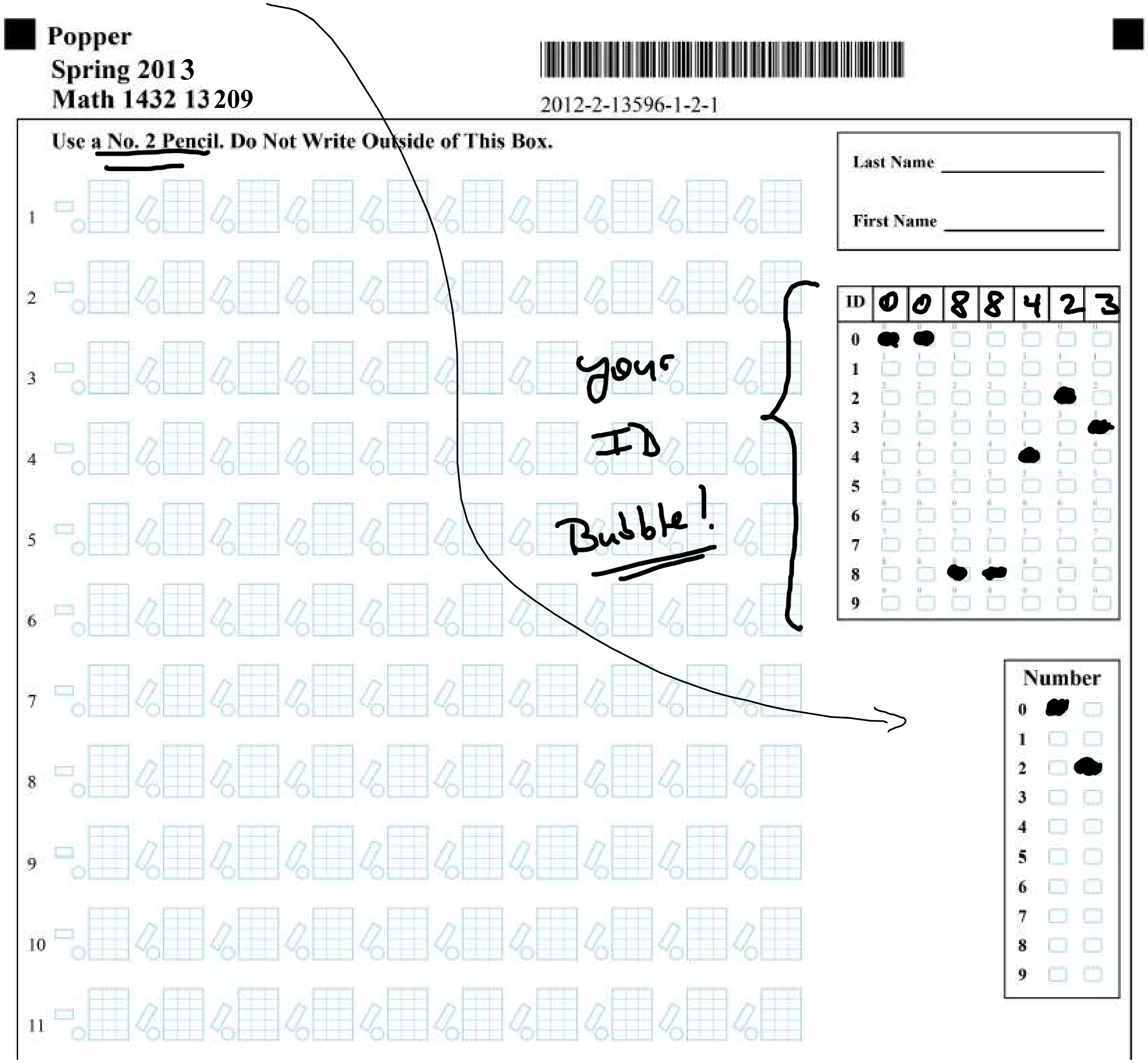
$$u'(t) = -3\,u(t)$$

$$u(t) = C\,e^{-3t}$$

$$2 = C$$

$$\Rightarrow \boxed{u(t) = 2e^{-3t}}$$

Popper P02



Popper
Spring 2013
Math 1432 13209

2012-2-13596-1-2-1

Use a No. 2 Pencil. Do Not Write Outside of This Box.

Last Name ____________

First Name ____________

1
2
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10
11

your ID Bubble!

ID 0 0 8 8 4 2 3

Number

Recall:

Chain Rule Derivative Formulas	Consequences
$\dfrac{d}{dx}\arcsin(u) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$	$\displaystyle\int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C$
$\dfrac{d}{dx}\arctan(u) = \dfrac{1}{1+u^2}\dfrac{du}{dx}$	$\displaystyle\int \frac{1}{1+u^2}\,du = \arctan(u) + C$

Aside on inverse trig integration formulas...

Assume $a > 0$.

"Looks like" $\int \frac{1}{1+u^2}\,du.$

$w = \frac{u}{a}$

$dw = \frac{1}{a}\,du$

$$\int \frac{1}{a^2+u^2}\,du = \int \frac{1}{a^2\left(1+\frac{u^2}{a^2}\right)}\,du = \frac{a}{a^2}\int \frac{1/a}{1+\left(\frac{u}{a}\right)^2}\,du$$

$$= \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du =$$ similar $= \arcsin\left(\frac{u}{a}\right) + C$

Examples:

$$\int \frac{1}{4+x^2}\,dx = \int \frac{1}{2^2+x^2}\,dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$\int \frac{\cos(x)}{\sqrt{3-\sin^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{(\sqrt{3})^2 - \sin^2(x)}}\,dx \quad \begin{array}{|l} u = \sin(x) \\ du = \cos(x)\,dx \end{array}$$

$$= \int \frac{du}{\sqrt{(\sqrt{3})^2 - u^2}} = \arcsin\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \arcsin\left(\frac{\sin(x)}{\sqrt{3}}\right) + C$$

New

Hyperbolic Functions

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

hyperbolic cosine of x

formulas

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

hyperbolic sine of x

hyperbolic tangent

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \qquad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \qquad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh(x) = \text{hyperbolic cosine of } x$$

$$\sinh(x) = \text{hyperbolic sine of } x$$

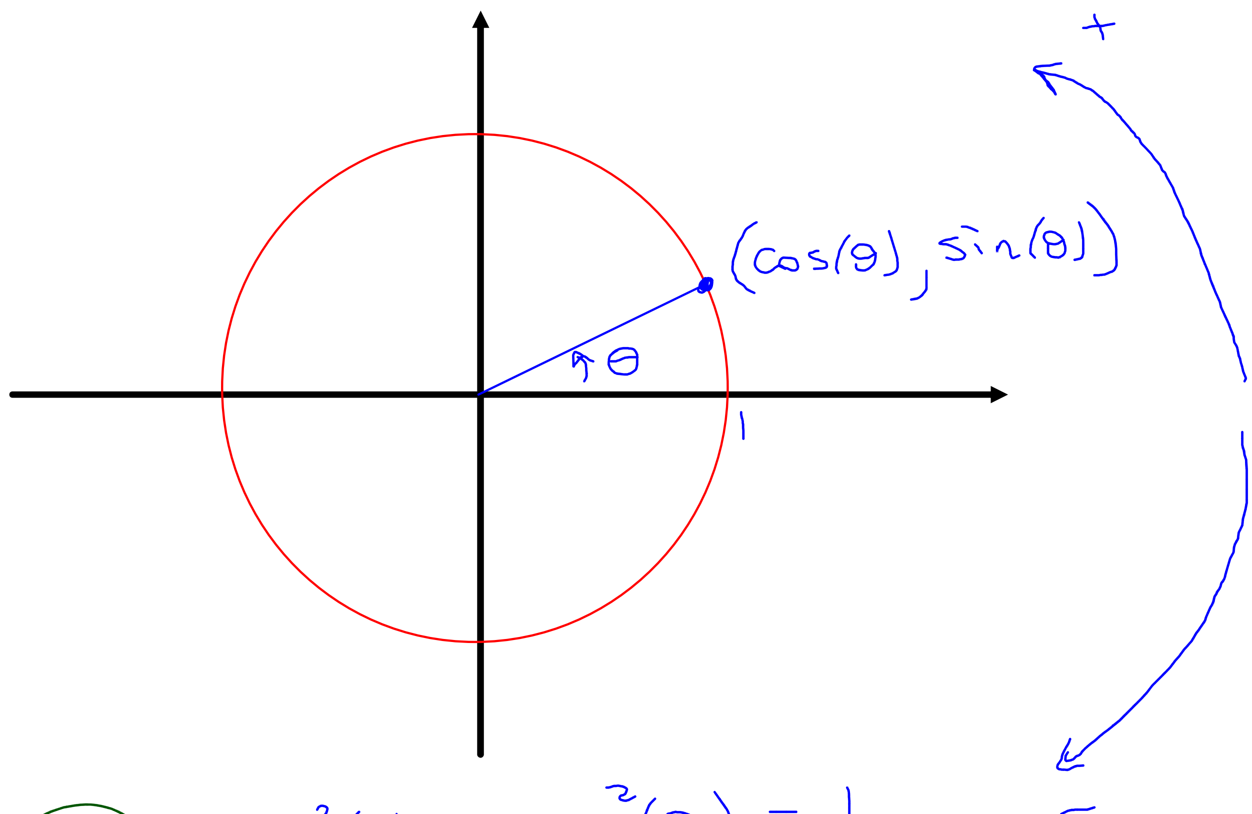
$\tanh(x) = \text{hyperbolic tangent of } x$ $\coth(x) = \text{hyperbolic cotangent of } x$

$\operatorname{sech}(x) = \text{hyperbolic secant of } x$ $\operatorname{csch}(x) = \text{hyperbolic cosecant of } x$

Where do these names come from?

First, recall that sine and cosine are called circular functions.

Do you recall why?



$$\cos^2(\theta) + \sin^2(\theta) = 1$$

Now let's do some computations with cosh and sinh.

$$\cosh^2(t) = \left(\frac{e^t + e^{-t}}{2}\right)^2 = \frac{1}{4}\left(e^{2t} + \underline{\underline{2}} + e^{-2t}\right)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\sinh^2(t) = \left(\frac{e^t - e^{-t}}{2}\right)^2 = \frac{1}{4}\left(e^{2t} - \underline{\underline{2}} + e^{-2t}\right)$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

<u>Note:</u> $\cosh^2(t) - \sinh^2(t) = \frac{1}{2} - -\frac{1}{2} = 1$

Hyperbolic Identity

$$\cosh^2(t) - \sinh^2(t) = 1$$

why?

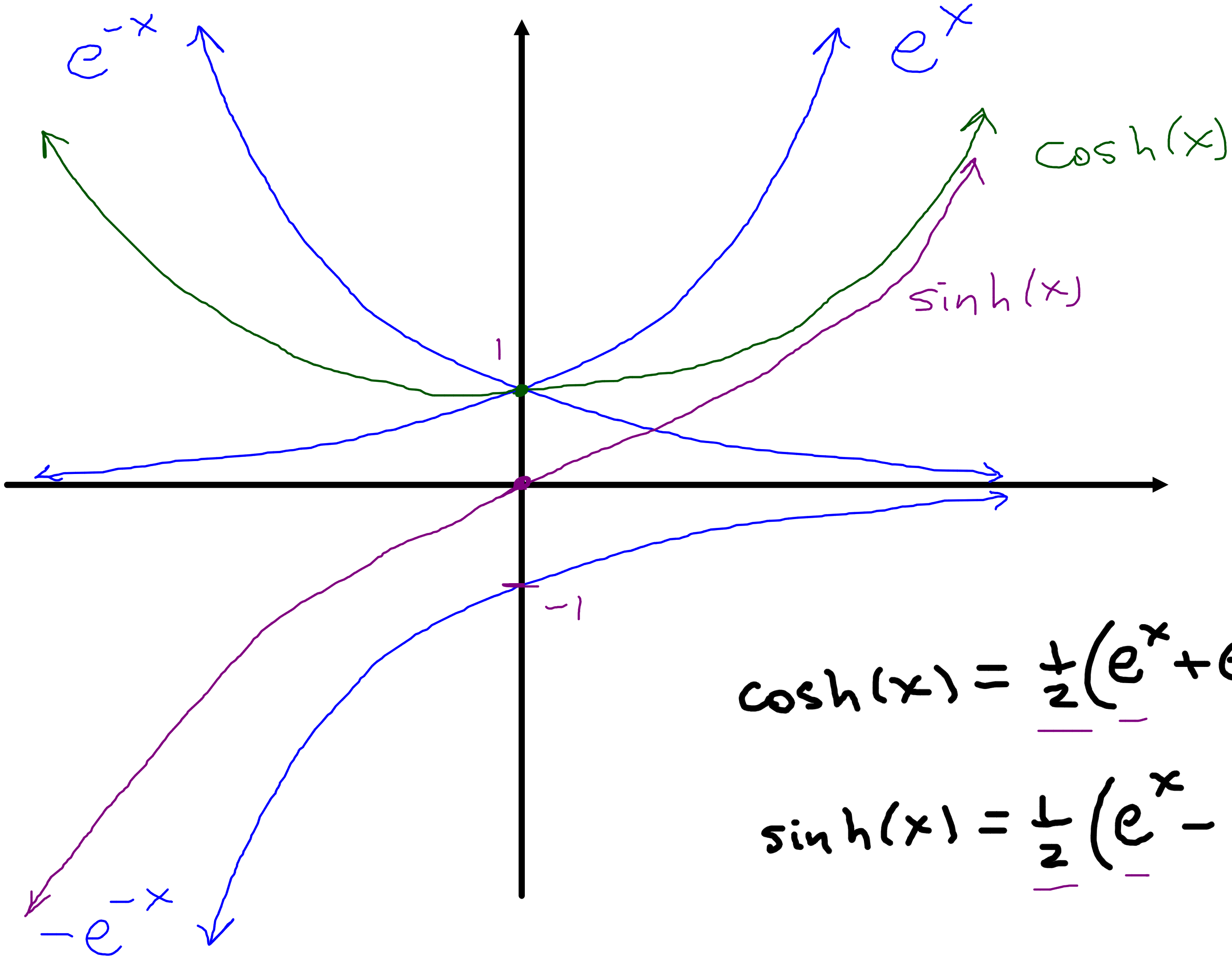
$x = \cosh(t)$

$y = \sinh(t)$

$x^2 - y^2 = 1$

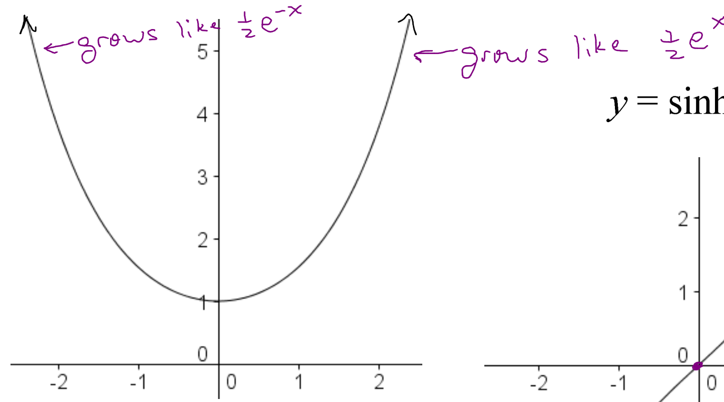
hyperbola

Graphs of sinh(x) and cosh(x)?

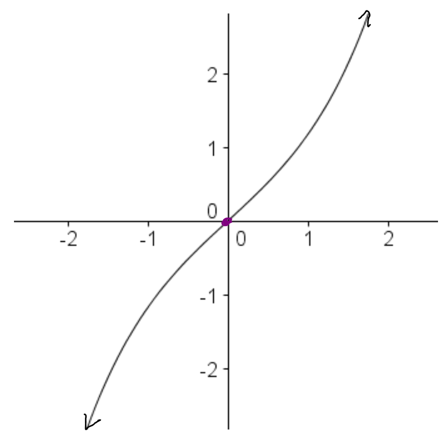


$$\cosh(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

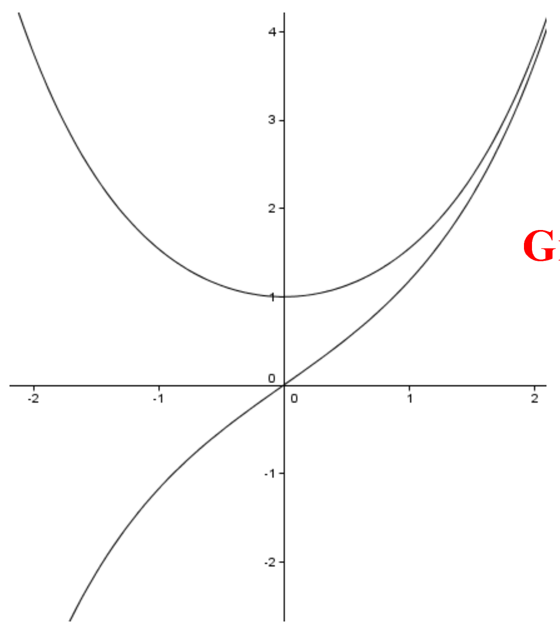
$$\sinh(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$



$y = \sinh(x)$



$y = \cosh(x)$



Graphed together!!

Derivatives

$$\frac{d}{dx}\cosh(x) = \frac{d}{dx}\left(\frac{1}{2}e^{x} + \frac{1}{2}e^{-x}\right) = \underbrace{\frac{1}{2}e^{x} - \frac{1}{2}e^{-x}}_{= \sinh(x)}$$

$$\frac{d}{dx}\sinh(x) = \frac{d}{dx}\left(\frac{1}{2}e^{x} - \frac{1}{2}e^{-x}\right) = \underbrace{\frac{1}{2}e^{x} + \frac{1}{2}e^{-x}}_{= \cosh(x)}$$

Chain Rule Formula and Consequences

$\frac{d}{dx}\cosh(u) = \sinh(u)\frac{du}{dx}$	$\int \cosh(u)\,du = \sinh(u) + C$
$\frac{d}{dx}\sinh(u) = \cosh(u)\frac{du}{dx}$	$\int \sinh(u)\,du = \cosh(u) + C$

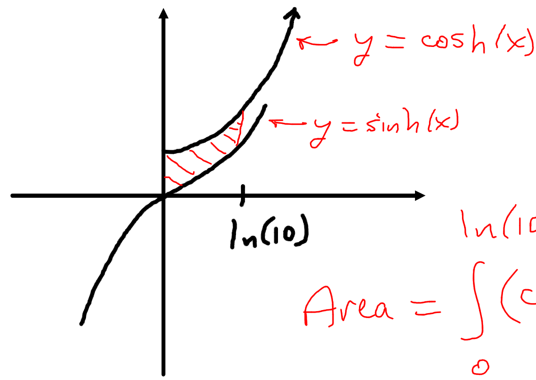
$*$ **Learn the derivative formulas for tanh, coth, sech and csch!!**

Example: Compute $\frac{d}{dx}\cosh\Big(\underbrace{\ln\big(\exp(x)+2x\big)}_{u}\Big)$

$$= \sinh(u)\,\frac{du}{dx}$$

$$= \sinh\Big(\underbrace{\ln\big(e^{x}+2x\big)}\Big)\cdot\frac{1}{e^{x}+2x}\cdot\left(e^{x}+2\right)$$

Example: Find the area bounded by the graphs of $f(x) = \cosh(x)$ and $g(x) = \sinh(x)$ for $0 \le x \le \ln(10)$.



$$\text{Area} = \int_0^{\ln(10)} (\cosh(x) - \sinh(x))\,dx$$

$$= (\sinh(x) - \cosh(x))\Big|_0^{\ln(10)}$$

$\sinh(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$\cosh(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

$$= -e^{-x}\Big|_0^{\ln(10)}$$

$$= -\left(e^{-\ln(10)} - e^0\right)$$

$$= -e^{\ln(\frac{1}{10})} + 1$$

$$= -\frac{1}{10} + 1 = \frac{9}{10}.$$