Written Quiz Today in lab/worshop/recitation.

Homework is due Monday.

Online Quiz 2 is due tomorrow night.

EMCFs are posted for next week.

Test 2 is rapidly approaching.
Material: 7.1 - 8.3

Please tell your high school friends and former teachers about our High School Mathematics Contest

February 9th
University of Houston

Free

http://mathcontest.uh.edu

Today...

Integration By Parts
Undoing the Product Rule
(Section 8.2)

Note: Section 8.1 is a review of integration formulas that you should have already encountered. We will not review this section.

Popper Number 03

Popper
Spring 2013
Math 1432 13209

2012-2-13596-1-2-1

Use a No. 2 Pencil. Do Not Write Outside of This Box.

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Bubble your ID

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Popper Number 03

1. $f(x) = \arcsin(2x)$. $f\,'(1/4) =$

2. Give the slope of the tangent line to the graph of $g(x) = \cosh(x^2)$ at $x = 2$.

3. Let $f(x) = \ln(1 + x^2)$. $f\,'(-2) =$

Popper Number 03

1 2 3 4

4. Give the domain $f(x) = \arcsin(x)$

a. $\left\{ x \middle| -\frac{\pi}{2} \le x \le \frac{\pi}{2} \right\}$

b. $\{x | -1 \le x \le 1\}$

c. $\{x | 0 \le x < \infty\}$

d. $\{x | -\infty < x < \infty\}$

5. Give the domain $f(x) = \arctan(x)$

a. $\left\{ x \middle| -\frac{\pi}{2} \le x \le \frac{\pi}{2} \right\}$

b. $\{x | -1 \le x \le 1\}$

c. $\{x | 0 \le x < \infty\}$

d. $\{x | -\infty < x < \infty\}$

Input 1 for *a*, 2 for *b*, 3 for *c*, and 4 for *d*.

Recall the product rule:

$$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$$

$(*)$ $d(uv) = u\,dv + v\,du$

Question: What does this tell us about

$u\,dv = d(uv) - v\,du$

$$\int u\,dv \quad ?$$

$$\int u\,dv = \int \left(d(uv) - v\,du \right)$$

$$= uv - \int v\,du$$

Using the Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du$$

3 Settings:

1. $\boxed{\text{Reduction}}$ to integrate

$$x^n \sin(ax),\ x^n \cos(ax),\ x^n e^{ax},$$

$$polynomial \cdot \sin(ax),\ polynomial \cdot \cos(ax),\ polynomial \cdot e^{ax}$$

2. $\boxed{\text{Cycling}}$ to integrate

$$\cos(ax)\sin(bx),\ \cos(ax)e^{bx},\ \sin(ax)e^{bx}$$

3. $\boxed{\text{Change of Form}}$ to integrate

$$\ln(x)\,f(x),\ \arctan(x)\,f(x),\ \arcsin(x)\,f(x)$$

(where $f(x)$ has a simple antiderivative)

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

Reduction uses the idea that polynomials can be reduced through differentiation.

Examples: $\int \underbrace{x}_{u} \underbrace{\sin(x)\,dx}_{dv} = -x\cos(x) - \int -\cos(x)\,dx$

$u = x$ $\quad du = dx$

$dv = \sin(x)\,dx$ $\quad v = -\cos(x)$

$= -x\cos(x) + \sin(x) + C$

2nd degree

1st degree

$\int \underbrace{x^2}_{u} \underbrace{\cos(x)\,dx}_{dv} = x^2\sin(x) - \int 2x\sin(x)\,dx$

one more time

$u = x^2$ $\quad du = 2x\,dx$

$dv = \cos(x)\,dx$ $\quad v = \sin(x)$

$= x^2\sin(x) - 2\left(-x\cos(x) + \sin(x)\right) + C$

$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$

$\int \underbrace{x^2}_{u} \underbrace{e^x\,dx}_{dv} = x^2e^x - \int 2xe^x\,dx$

$u = x^2$ $\quad du = 2x\,dx$

$dv = e^x\,dx$ $\quad v = e^x$

$= x^2e^x - 2\int \underbrace{x}_{u} \underbrace{e^x\,dx}_{dv}$

2nd time

$u = x$ $\quad du = dx$

$dv = e^x\,dx$ $\quad v = e^x$

$= x^2e^x - 2\left(xe^x - \int e^x\,dx\right)$

$= x^2e^x - 2xe^x + 2e^x + C$

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

Cycling uses the idea that certain functions return when differentiated or integrated repeatedly.

Example: $\int \underbrace{e^x}_{u} \underbrace{\sin(2x)\,dx}_{dv} = -\frac{1}{2}e^x\cos(2x) - \int -\frac{1}{2}e^x\cos(2x)\,dx$

$u = e^x \qquad du = e^x\,dx$

$dv = \sin(2x)\,dx \qquad v = -\frac{1}{2}\cos(2x)$

$= -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\int \underbrace{e^x}_{u}\underbrace{\cos(2x)\,dx}_{dv}$

$u = e^x \qquad du = e^x\,dx$

$dv = \cos(2x)\,dx \qquad v = \frac{1}{2}\sin(2x)$

$= -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\left(\frac{1}{2}e^x\sin(2x) - \int \frac{1}{2}e^x\sin(2x)\,dx\right)$

$$\int e^x\sin(2x)\,dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) - \frac{1}{4}\int e^x\sin(2x)\,dx$$

$$\frac{5}{4}\int e^x\sin(2x)\,dx = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) + \tilde{C}$$

$$\therefore \int e^x\sin(2x)\,dx = -\frac{2}{5}e^x\cos(2x) + \frac{1}{5}e^x\sin(2x) + C$$

Popper Number 03

6. Free Friday!! The answer is $-23/47$.

7. Free Friday!! The answer is 23.52.

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

Change of Form uses the idea that some functions change their form completely when they are differentiated.

Examples: $\int x\arctan(x)\,dx = \int \underbrace{\arctan(x)}_{u}\ \underbrace{x\,dx}_{dv}$

$u = \arctan(x)$ $\quad du = \frac{1}{1+x^2}dx$

$dv = x\,dx$ $\quad v = \frac{1}{2}x^2 + \frac{1}{2}$

$= \left(\frac{1}{2}x^2+\frac{1}{2}\right)\arctan(x) - \int \frac{\frac{1}{2}x^2+\frac{1}{2}}{1+x^2}dx$

$= \left(\frac{1}{2}x^2+\frac{1}{2}\right)\arctan(x) - \int \frac{1}{2}dx$

$= \left(\frac{1}{2}x^2+\frac{1}{2}\right)\arctan(x) - \frac{1}{2}x + C$

$\int \ln(x)\,dx =$

$u = \ln(x)$
$dv = dx$

you do it.

See the video.

Integration by Parts with Definite Integrals

$$\int_a^b u\,dv = (uv)\Big|_a^b - \int_a^b v\,du$$

Example: $\int_0^1 \underbrace{x}_{u}\, \underbrace{e^x dx}_{dv} = xe^x \Big|_0^1 - \int_0^1 e^x dx$

$u = x \qquad du = dx$

$dv = e^x dx \qquad v = e^x$

$= e - 0 - e^x \Big|_0^1$

$= e - (e - 1) = 1.$