Written Quiz Today in lab/worship/recitation.

Homework is due Monday.

Online Quiz 2 is due tomorrow night.

EMCFs are posted for next week.

Test 2 is rapidly approaching.
   Material: 7.1 - 8.3
Please tell your high school friends and former teachers about our High School Mathematics Contest

February 9th
University of Houston

http://mathcontest.uh.edu
Today...

Integration By Parts

_undoing the Product Rule_

(Section 8.2)

Note: Section 8.1 is a review of integration formulas that you should have already encountered. We will not review this section.
Popper Number 03

Use a No. 2 Pencil. Do Not Write Outside of This Box.

ID

Bubble your ID

Number
0 1 2 3 4 5 6 7 8 9
Popper Number 03

1. \( f(x) = \arcsin(2x) \). \( f'(1/4) = \)

2. Give the slope of the tangent line to the graph of \( g(x) = \cosh(x^2) \) at \( x = 2 \).

3. Let \( f(x) = \ln(1 + x^2) \). \( f'(-2) = \)
Popper Number 03

4. Give the domain $f(x) = \arcsin(x)$
   a. $\left\{ x \mid -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \right\}$
   b. $\left\{ x \mid -1 \leq x \leq 1 \right\}$
   c. $\left\{ x \mid 0 \leq x < \infty \right\}$
   d. $\left\{ x \mid -\infty < x < \infty \right\}$

5. Give the domain $f(x) = \arctan(x)$
   a. $\left\{ x \mid -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \right\}$
   b. $\left\{ x \mid -1 \leq x \leq 1 \right\}$
   c. $\left\{ x \mid 0 \leq x < \infty \right\}$
   d. $\left\{ x \mid -\infty < x < \infty \right\}$

Input 1 for a, 2 for b, 3 for c, and 4 for d.
Recall the product rule:

\[
\frac{d}{dx} (uv) = u \frac{d}{dx} v + v \frac{d}{dx} u
\]

\[d(uv) = u \, dv + v \, du\]

Question: What does this tell us about

\[\int u \, dv = \int (d(uv) - v \, du)\]

\[
= uv - \int v \, du
\]
Using the Integration by Parts Formula

\[\int u \, dv = uv - \int v \, du\]

3 Settings:

1. **Reduction** to integrate
   \[x^n \sin(ax), \ x^n \cos(ax), \ x^n e^{ax},\]
   \[\text{polynomial} \cdot \sin(ax), \ \text{polynomial} \cdot \cos(ax), \ \text{polynomial} \cdot e^{ax}\]

2. **Cycling** to integrate
   \[\cos(ax) \sin(bx), \ \cos(ax)e^{bx}, \ \sin(ax)e^{bx}\]

3. **Change of Form** to integrate
   \[\ln(x)\ f(x), \ \arctan(x)\ f(x), \ \arcsin(x)\ f(x)\]

   (where \(f(x)\) has a simple antiderivative)
Integration by parts formula: \[ \int u \, dv = uv - \int v \, du \]

**Reduction** uses the idea that polynomials can be reduced through differentiation.

**Examples:**

\[ \int x \sin(x) \, dx \]

**\[ u = x \quad du = dx \]
\[ dv = \sin(x) \, dx \quad v = -\cos(x) \]

\[ uv = -x \cos(x) \]

\[ \Rightarrow \int x \sin(x) \, dx = -x \cos(x) + \int \cos(x) \, dx \]

**\[ +C \]

\[ \int x^2 \cos(x) \, dx \]

**\[ u = x^2 \quad du = 2x \, dx \]
\[ dv = \cos(x) \, dx \quad v = \sin(x) \]

\[ uv = x^2 \sin(x) \]

\[ \Rightarrow \int x^2 \cos(x) \, dx = x^2 \sin(x) - \int 2x \sin(x) \, dx \]

**\[ +C \]

**\[ +C \]

\[ \int \frac{x^2 e^x}{u} \, dv \]

**\[ u = x^2 \quad du = 2x \, dx \]
\[ dv = e^x \, dx \quad v = e^x \]

\[ 2 \text{nd time} \]

**\[ u = x \quad du = dx \]
\[ dv = e^x \, dx \quad v = e^x \]

\[ uv = x e^x \]

\[ \Rightarrow \int \frac{x^2 e^x}{u} \, dv = x e^x - 2 \left( x e^x - \int e^x \, dx \right) \]

\[ = x e^x - 2x e^x + 2e^x + C \]
Integration by parts formula: \[ \int u \, dv = uv - \int v \, du \]

**Cycling** uses the idea that certain functions return when differentiated or integrated repeatedly.

**Example:**
\[ \int e^x \sin(2x) \, dx = \frac{1}{2} e^x \cos(2x) - \frac{1}{2} \int e^x \cos(2x) \, dx \]

\[ u = e^x \]
\[ du = e^x \, dx \]
\[ dv = \sin(2x) \, dx \]
\[ v = -\frac{1}{2} \cos(2x) \]

\[ \frac{1}{2} e^x \cos(2x) - \frac{1}{2} \left( \int e^x \cos(2x) \, dx \right) \]

\[ \int e^x \sin(2x) \, dx = \frac{1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x) - \frac{1}{4} \int e^x \sin(2x) \, dx \]

\[ \int e^x \sin(2x) \, dx = \frac{1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x) + \frac{1}{4} \left( \int e^x \sin(2x) \, dx \right) \]

\[ \int e^x \sin(2x) \, dx = -\frac{1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x) + C \]

\[ \int e^x \sin(2x) \, dx = -\frac{1}{2} e^x \cos(2x) + \frac{1}{5} e^x \sin(2x) + C \]
Popper Number 03

6. Free Friday!! The answer is $\frac{-23}{47}$.

7. Free Friday!! The answer is $23.52$. 
Integration by parts formula: \[ \int u \, dv = uv - \int v \, du \]

**Change of Form** uses the idea that some functions change their form completely when they are differentiated.

**Examples:**

\[ \int x \arctan(x) \, dx = \int \frac{\arctan(x)}{x} \, dx \]

\[ u = \arctan(x) \quad du = \frac{1}{1+x^2} \, dx \]

\[ dv = x \, dx \quad v = \frac{1}{2} x^2 + \frac{1}{2} \]

\[ \int \ln(x) \, dx = \int \ln(x) \, dx \]

\[ u = \ln(x) \quad du = \frac{1}{x} \, dx \]

See the video.
Integration by Parts with Definite Integrals

\[ \int_a^b u \, dv = (uv)|_a^b - \int_a^b v \, du \]

Example:

\[ \int_0^1 \frac{1}{x} e^x \, dx = xe^x|_0^1 - \int_0^1 e^x \, dx \]

\[ u = x, \quad du = dx \]
\[ dv = e^x \, dx, \quad v = e^x \]

\[ e - 0 - e^1|_0 = e - (e - 1) = 1. \]