Written Quiz Today in lab/worshop/recitation.

Homework is due Monday.

Online Quiz 2 is due tomorrow night.

EMCFs are posted for next week.

Test 2 is rapidly approaching.
Material: 7.1 - 8.3

Please tell your high school friends and former teachers about our High School Mathematics Contest

February 9th
University of Houston

Free

http://mathcontest.uh.edu

Today...

Integration By Parts

Undoing the Product Rule

(Section 8.2)

Note: Section 8.1 is a review of integration formulas that you should have already encountered. We will not review this section.

Recall the product rule:

$$\frac{d}{dx}(uv) = u\frac{d}{dx}v + v\frac{d}{dx}u$$

$$d(uv) = \underline{u\,dv} + v\,du$$

Question: What does this tell us about

$$u\,dv = d(uv) - v\,du$$

$$\int \underline{udv} \quad ?$$

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$= uv - \int v\,du$$

Using the Integration by Parts Formula

$$\boxed{\int u\,dv = uv - \int v\,du}$$

3 Settings:

1. Reduction to integrate

$$x^n \sin(ax),\ x^n \cos(ax),\ x^n e^{ax},$$

$$\int \underbrace{x}_{u}\ \underbrace{e^{2x}\,dx}_{dv}$$

$$polynomial \cdot \sin(ax),\ polynomial \cdot \cos(ax),\ polynomial \cdot e^{ax}$$

2. Cycling to integrate

$$\cos(ax)\sin(bx),\ \cos(ax)e^{bx},\ \sin(ax)e^{bx}$$

3. Change of Form to integrate

$$\underbrace{\boxed{\ln(x)}}_{u}\,\underbrace{f(x)}_{dv},\ \boxed{\arctan(x)}\,f(x),\ \boxed{\arcsin(x)}\,f(x)$$

(where $f(x)$ has a simple antiderivative)

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

Reduction uses the idea that polynomials can be reduced through differentiation.

Examples: $\int \underbrace{x}_{u} \underbrace{\sin(x)\,dx}_{dv} = -x\cos(x) - \int -\cos(x)\,dx$

$u = x \qquad du = dx$

$dv = \sin(x)\,dx \qquad v = -\cos(x)$

$= -x\cos(x) + \sin(x) + C$

$\int \underbrace{x^2}_{u} \underbrace{\cos(x)\,dx}_{dv} = x^2\sin(x) - \int 2x\sin(x)\,dx$

$u = x^2 \qquad du = 2x\,dx$

$dv = \cos(x)\,dx \qquad v = \sin(x)$

$= x^2\sin(x) - 2\left(-x\cos(x) + \sin(x)\right) + C$

$= x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$

$\int \underbrace{x}_{u} \underbrace{e^{2x}\,dx}_{dv} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.$

$u = x \qquad du = dx$

$dv = e^{2x}\,dx \qquad v = \frac{1}{2}e^{2x}$

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

$\boxed{\text{Cycling}}$ uses the idea that certain functions return when differentiated or integrated repeatedly.

Example: $\boxed{\int \underbrace{e^x}_{u} \underbrace{\sin(2x)\,dx}_{dv}} = -\frac{1}{2}e^x\cos(2x) - \int -\frac{1}{2}e^x\cos(2x)\,dx$

$u = e^x \qquad du = e^x\,dx$

$dv = \sin(2x)\,dx \qquad v = -\frac{1}{2}\cos(2x)$

$$= -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\int \underbrace{e^x}_{u} \underbrace{\cos(2x)\,dx}_{dv}$$

$u = e^x \qquad du = e^x\,dx$

$dv = \cos(2x)\,dx \qquad v = \frac{1}{2}\sin(2x)dx$

$$= -\frac{1}{2}e^x\cos(2x) + \frac{1}{2}\left[\frac{1}{2}e^x\sin(2x) - \int \frac{1}{2}e^x\sin(2x)\,dx\right]$$

$$\Rightarrow \underline{\int e^x\sin(2x)\,dx} = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) - \frac{1}{4}\underline{\int e^x\sin(2x)\,dx}$$

$$\frac{5}{4}\underline{\int e^x\sin(2x)\,dx} = -\frac{1}{2}e^x\cos(2x) + \frac{1}{4}e^x\sin(2x) + \tilde{C}$$

$$\Rightarrow \int e^x\sin(2x)\,dx = -\frac{2}{5}e^x\cos(2x) + \frac{1}{5}e^x\sin(2x) + C$$

Integration by parts formula: $\int u\,dv = uv - \int v\,du$

Change of Form uses the idea that some functions change their form completely when they are differentiated.

Examples: $\int x \underbrace{\arctan(x)} dx = \int \underbrace{\arctan(x)}_{u} \underbrace{x\,dx}_{dv}$

$u = \arctan(x) \qquad du = \frac{1}{1+x^2}dx$

$dv = x\,dx \qquad v = \frac{1}{2}x^2$

$= \frac{1}{2}x^2\arctan(x) - \int \frac{\frac{1}{2}x^2}{1+x^2}dx$

$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}dx$

$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$

$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C$

$\int \underbrace{\ln(x)}_{u}\underbrace{dx}_{dv} = x\ln(x) - \int x\cdot\frac{1}{x}dx$

$u = \ln(x) \qquad du = \frac{1}{x}dx$

$dv = dx \qquad v = x$

$= x\ln(x) - \int dx$

$= x\ln(x) - x + C$

Integration by Parts with Definite Integrals

$$\int_a^b u\,dv = (uv)\Big|_a^b - \int_a^b v\,du$$

Example: $\int_0^1 \underbrace{x}_{u}\, \underbrace{e^x dx}_{dv} = xe^x \Big|_0^1 - \int_0^1 e^x\,dx$

$u = x \qquad du = dx$

$dv = e^x\,dx \qquad v = e^x$

$= e - 0 - e^x \Big|_0^1$

$= e - (e-1) = 1.$