Reminders...

Online Quizzes are open. **Online Quiz 3** closes Saturday.

Test 2 is coming soon.

Homework 04 is posted for next Monday.

EMCFs are due each MWF.

Please tell your high school friends and former teachers about our High School Mathematics Contest

February 9th
University of Houston

Free

http://mathcontest.uh.edu

More Integration by Parts

Comments Through Examples...

$$\int \underbrace{\ln(x+1)}_{u}\underbrace{dx}_{dv} = (x+1)\ln(x+1) - \int dx$$

$u = \ln(x+1)$ $\quad$ $du = \frac{1}{x+1}\,dx$

$dv = dx$ $\quad$ $v = x+1$

$$= (x+1)\ln(x+1) - x + C$$

$$\int \underbrace{x\,\underbrace{\arctan(x)}_{u}\,dx}_{dv} = \left(\frac{x^2}{2} + \frac{1}{2}\right)\arctan(x) - \int \frac{1}{2}\,dx$$

$u = \arctan(x)$ $\quad$ $du = \frac{1}{1+x^2}\,dx$

$dv = x\,dx$ $\quad$ $v = \frac{x^2}{2} + \frac{1}{2}$

$$= \left(\frac{x^2}{2} + \frac{1}{2}\right)\arctan(x) - \frac{1}{2}x + C$$

Today...

Integrating Powers and Products of Trigonometric Functions

Section 8.3 - Part I

Products of Sine and Cosine

Integrals of

$$\cos^m(x),\ \sin^m(x),\ \cos^m(x)\sin^n(x)$$

Examples: $\int \sin^2(x)dx =$

$\int \cos^4(x)dx =$

$\int \cos^4(x)\sin^3(x)dx =$

$\int \cos^4(x)\sin^2(x)dx =$

Identities:

$$\cos^2(x)+\sin^2(x)=1$$

$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$

$$\sin^2(x)=\frac{1-\cos(2x)}{2}$$

Example: $\int \sin^2(x)dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)dx$

$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ $\quad = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

$\int \cos^2(x)\,dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)dx$

$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$ $\quad = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

Example: $\int \cos^4(x)dx = \int \left(\cos^2(x)\right)^2 dx$

$$\boxed{\cos^2(u) = \frac{1}{2} + \frac{1}{2}\cos(2u)} \qquad = \int \left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)^2 dx$$

$$= \int \left(\frac{1}{4} + 2\cdot\frac{1}{2}\cdot\frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\right) dx$$

$$= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{4}\int\left(\frac{1}{2} + \frac{1}{2}\cos(4x)\right)dx$$

$$= \frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x) + C$$

$$= \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$$

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

Example: $\int \cos^4(x)\sin^2(x)dx =$

$$= \int (\cos^2(x))^2 \sin^2(x)\,dx$$

$$= \int \left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)^2 \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)dx$$

$$= \int \left(\frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\right)\left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)dx$$

$$= \int \left(\frac{1}{8} + \frac{1}{4}\cos(2x) + \frac{1}{8}\cos^2(2x) - \frac{1}{8}\cos(2x) - \frac{1}{4}\cos^2(2x) - \frac{1}{8}\cos^3(2x)\right)dx$$

$$= \int \left(\frac{1}{8} + \frac{1}{8}\cos(2x) - \frac{1}{8}\cos^2(2x) - \frac{1}{8}\cos^3(2x)\right)dx$$

$$= \frac{1}{8}x + \frac{1}{16}\sin(2x) - \frac{1}{8}\int\left(\frac{1}{2} + \frac{1}{2}\cos(4x)\right)dx - \frac{1}{8}\int \cos^2(2x)\cos(2x)\,dx$$

$$= \frac{1}{8}x + \frac{1}{16}\sin(2x) - \frac{1}{16}x - \frac{1}{64}\sin(4x) - \frac{1}{8}\int (1 - \sin^2(2x))\cos(2x)\,dx$$

$$= \frac{1}{16}x + \cancel{\frac{1}{16}\sin(2x)} - \frac{1}{64}\sin(4x) - \cancel{\frac{1}{16}\sin(2x)} + \frac{1}{8}\int \sin^2(2x)\cos(2x)\,dx$$

$$= \frac{1}{16}x - \frac{1}{64}\sin(4x) + \frac{1}{48}\sin^3(2x) + C$$

Example: $\int \cos^4(x)\sin^3(x)\,dx =$ odd power

$$= \int \cos^4(x)\sin^2(x)\,\underline{\sin(x)\,dx}$$

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$$= \int \cos^4(x)\left(1-\cos^2(x)\right)\underline{\sin(x)\,dx}$$

$$= \int \left(\cos^4(x)-\cos^6(x)\right)\underline{\sin(x)\,dx}$$

$$= \int \cos^4(x)\,\underline{\sin(x)}\,dx - \int \cos^6(x)\,\underline{\sin(x)}\,dx$$

$$= -\frac{\cos^5(x)}{5} + \frac{\cos^7(x)}{7} + C.$$

Strategy for products of sine and cosine with even powers:

use $\cos^2(u) = \frac{1}{2} + \frac{1}{2}\cos(2u)$

$\sin^2(u) = \frac{1}{2} - \frac{1}{2}\cos(2u)$

Strategy for products of sine and cosine with at least **one odd power:**

pull one power off of the odd power term.

Then, change everything else to the "other" trig function.