## Reminders...

**Online Quiz 3** closes this Saturday. Other **Online Quizzes** are available.

**Test 2** is coming soon. **Review problems** and **video solutions** are posted, and there is a **Practice Test 2** on CourseWare.

**EMCFs** are due on each MWF.

**Homework** is posted and due on Monday.

There is a written **Quiz** in lab on Friday.

**Please tell your high school friends and former teachers about our Mathematics Contest**

(middle school and high school students are welcome)

Free

**February 9th**
**University of Houston**

**http://mathcontest.uh.edu**

## Today

Final comments for integrating products of sine and cosine.

Integrating Powers and Products of Secant and Tangent Functions

## Popper Number 05

1. $-3.14$

2. $\frac{1}{\pi}\int_0^{\pi} \sin^2(x)\,dx =$ **1/2**

Examples of simple products with different arguments:

$$\int \sin(2x)\cos(3x)\,dx =$$

Different arguments

$$\int \cos(2x)\cos(3x)\,dx =$$

Techniques:

1. Parts (cyclic) (twice)
2. Trig identities

$$\int \sin(2x)\sin(3x)\,dx =$$

$$\tfrac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) = \sin(A)\cos(B)$$

$$= \int \tfrac{1}{2}\left(\sin(5x) + \sin(-x)\right)dx$$

$$= \tfrac{1}{2}\int\left(\sin(5x) - \sin(x)\right)dx$$

$$= -\tfrac{1}{10}\cos(5x) + \tfrac{1}{2}\cos(x) + C.$$

$$\tfrac{1}{2}\left(\cos(A+B) + \cos(A-B)\right) = \cos(A)\cos(B)$$

$$= \tfrac{1}{2}\int\left(\cos(5x) + \cos(-x)\right)dx$$

$$= \tfrac{1}{2}\int\left(\cos(5x) + \cos(x)\right)dx$$

$$= \tfrac{1}{10}\sin(5x) + \tfrac{1}{2}\sin(x) + C$$

**Useful Identities:**

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(-B) - \sin(A)\sin(-B)$$

$$= \cos(A)\cos(B) + \sin(A)\sin(B)$$

Add

$$\tfrac{1}{2}\left(\cos(A+B) + \cos(A-B)\right) = \cos(A)\cos(B)$$

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

$$\sin(A-B) = \sin(A)\cos(-B) + \sin(-B)\cos(A)$$

$$= \sin(A)\cos(B) - \sin(B)\cos(A)$$

Add

$$\sin(A+B) + \sin(A-B) = 2\sin(A)\cos(B)$$

$$\tfrac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) = \sin(A)\cos(B)$$

**Even Powers of secant and tangent:**

**Examples:** $\int \sec^4(x)\,dx =$

$$\int \tan^4(x)\,dx =$$

**Fundamental Identity:**

$$\boxed{1 + \tan^2(x) = \sec^2(x)}$$

**Example:** $\int \tan^2(x)\sec^4(x)\,dx =$ even

$$= \int \tan^2(x)\underbrace{\sec^2(x)}_{1+\tan^2(x)}\underbrace{\sec^2(x)\,dx}_{\text{derivative of } \tan(x)}$$

$$= \int \tan^2(x)\left(1+\tan^2(x)\right)\sec^2(x)\,dx$$

$$= \int \tan^2(x)\sec^2(x)\,dx + \int \tan^4(x)\sec^2(x)\,dx$$

$$= \tfrac{1}{3}\tan^3(x) + \tfrac{1}{5}\tan^5(x) + C$$

**Examples:** $\int \sec^4(x)\,dx =$

$$= \int \sec^2(x)\,\underline{\sec^2(x)\,dx}$$
$$= \int (1+\tan^2(x))\sec^2(x)\,dx$$
$$= \underline{\int \sec^2(x)\,dx} + \int \underline{\tan^2(x)\sec^2(x)\,dx}$$
$$= \tan(x) + \frac{1}{3}\tan^3(x) + C$$

$$\int \tan^4(x)\,dx = \int (\tan^2(x))^2\,dx$$
$$= \int (\sec^2(x) - 1)^2\,dx$$
$$= \int (\sec^4(x) - 2\sec^2(x) + 1)\,dx$$
$$= \int \sec^4(x)\,dx \checkmark - 2\int \sec^2(x)\,dx \checkmark + \int dx \checkmark.$$

You

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General strategy for calculating

$$\int \tan^n(x)\sec^m(x)\,dx$$

when $m$ is even.

( peel off $\sec^2(x)$, and then turn everything else into powers of $\tan(x)$ )

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**Popper Number 05**

3. $\int_0^{\pi/4} \tan^2(x)\,dx =$ **(1 - ln(2))/2**

**= 0.1534264097**

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**Odd Powers** → Parts

**Example:** $\int \sec^3(x)\,dx = \int \underbrace{\sec(x)}_{u}\,\underbrace{\sec^2(x)\,dx}_{dv}$

$u = \sec(x)$ $\quad du = \sec(x)\tan(x)\,dx$

$dv = \sec^2(x)\,dx$ $\quad v = \tan(x)$

$$= \sec(x)\tan(x) - \int \sec(x)\,\underline{\tan^2(x)}\,dx$$
$$= \sec(x)\tan(x) - \int \sec(x)(\sec^2(x) - 1)\,dx$$
$$\int \sec^3(x)\,dx = \sec(x)\tan(x) - \underline{\int \sec^3(x)\,dx} + \int \sec(x)\,dx$$
$$2\int \sec^3(x)\,dx = \sec(x)\tan(x) + \ln(|\sec(x)+\tan(x)|) + \tilde{C}$$
$$\Rightarrow \int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln(|\sec(x)+\tan(x)|) + C$$

## Odd Powers

**Example:** $\int \tan^2(x)\sec^5(x)dx =$

$$\int \left[\sec^7(x) - \sec^5(x)\right] dx$$

odd powers of sec(x)

Parts $\quad dv = \sec^2(x)dx$

Feb 6-6:59 AM

General strategy for calculating

$$\int \tan^n(x)\sec^m(x)\,dx$$

when $n$ is even and $m$ is odd.

( the hardest case - this typically involves parts )

$u =$ everything else $\quad dv = \sec^2(x)dx$

General strategy for calculating

$$\int \tan^n(x)\sec^m(x)\,dx$$

when $n$ is odd and $m$ is odd.

( peel off a sec($x$)tan($x$), and then turn everything else into powers of sec($x$) )

**Example:** $\int \tan^3(x)\sec^3(x)dx =$

$$= \int \tan^2(x)\sec^2(x)\underbrace{\sec(x)\tan(x)}_{\text{deriv. of }\sec(x)}dx$$

change to secants

$$= \int (\sec^2(x)-1)\sec^2(x)\,\underbrace{\sec(x)\tan(x)\,dx}$$

$$= \int \sec^4(x)\,\underbrace{\sec(x)\tan(x)dx} - \int \sec^2(x)\,\underbrace{\sec(x)\tan(x)dx}$$

$$= \frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C.$$