Reminders...

Online Quiz 3 closes this Saturday. Other **Online Quizzes** are available.

Test 2 is coming soon. **Review problems** and **video solutions** are posted, and there is a **Practice Test 2** on CourseWare.

EMCFs are due on each MWF.

Homework is posted and due on Monday.

There is a written **Quiz** in lab on Friday.

Please tell your high school friends and former teachers about our Mathematics Contest

(middle school and high school students are welcome)

February 9th

University of Houston

Free

http://mathcontest.uh.edu

Today

Final comments for integrating products
of sine and cosine.

Integrating Powers and Products of
Secant and Tangent Functions

Popper Number 05

1. –3.14

2. $\frac{1}{\pi}\int_{0}^{\pi}\sin^{2}(x)\,dx =$ **1/2**

Examples of simple products with different arguments:

- $\int \sin(2x)\cos(3x)\,dx =$

- $\int \cos(2x)\cos(3x)\,dx =$

- $\int \sin(2x)\sin(3x)\,dx =$

Different arguments

Techniques:

1. Parts (cyclic) (twice)
2. Trig identities

$\frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) = \sin(A)\cos(B)$

$= \int \frac{1}{2}\left(\sin(5x) + \sin(-x)\right)dx$

$= \frac{1}{2}\int \left(\sin(5x) - \sin(x)\right)dx$

$= -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x) + C.$

$\frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right) = \cos(A)\cos(B)$

$= \frac{1}{2}\int \left(\cos(5x) + \cos(-x)\right)dx$

$= \frac{1}{2}\int \left(\cos(5x) + \cos(x)\right)dx$

$= \frac{1}{10}\sin(5x) + \frac{1}{2}\sin(x) + C$

Useful Identities:

$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(-B) - \sin(A)\sin(-B)$$

$$= \cos(A)\cos(B) + \sin(A)\sin(B)$$

Add

$$\frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right) = \cos(A)\cos(B)$$

$$\sin(A+B)=\sin(A)\cos(B)+\sin(B)\cos(A)$$

$$\sin(A-B) = \sin(A)\cos(-B) + \sin(-B)\cos(A)$$

$$= \sin(A)\cos(B) - \sin(B)\cos(A)$$

Add

$$\sin(A+B) + \sin(A-B) = 2\sin(A)\cos(B)$$

$$\frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) = \sin(A)\cos(B)$$

Even Powers of secant and tangent:

Examples: $\int \sec^4(x)\,dx =$

$$\int \tan^4(x)\,dx =$$

Fundamental Identity:

$$\boxed{1+\tan^2(x)=\sec^2(x)}$$

Example: $\int \tan^2(x)\sec^4(x)\,dx =$ even

$$= \int \tan^2(x)\underbrace{\sec^2(x)}_{1+\tan^2(x)}\underbrace{\sec^2(x)\,dx}_{\text{derivative of }\tan(x)}$$

$$= \int \tan^2(x)\left(1+\tan^2(x)\right)\underbrace{\sec^2(x)\,dx}$$

$$= \int \tan^2(x)\underbrace{\sec^2(x)\,dx} + \int \tan^4(x)\underbrace{\sec^2(x)\,dx}$$

$$= \frac{1}{3}\tan^3(x) + \frac{1}{5}\tan^5(x) + C$$

Examples: $\int \sec^4(x)\,dx =$

$$\int \tan^4(x)\,dx = \int \left(\tan^2(x)\right)^2 dx$$

$$= \int \left(\sec^2(x) - 1\right)^2 dx$$

$$= \int \left(\sec^4(x) - 2\sec^2(x) + 1\right) dx$$

$$= \int \sec^4(x)\,dx - 2\int \sec^2(x)\,dx + \int dx$$

$$= \int \sec^2(x)\,\underbrace{\sec^2(x)\,dx}$$

$$= \int \left(1 + \tan^2(x)\right)\sec^2(x)\,dx$$

$$= \underbrace{\int \sec^2(x)\,dx} + \int \tan^2(x)\,\underbrace{\sec^2(x)\,dx}$$

$$= \tan(x) + \frac{1}{3}\tan^3(x) + C$$

YOU

General strategy for calculating

$$\int \tan^{\mathrm{n}}(x)\sec^{m}(x)\,dx$$

when m is even.

(peel off $\sec^2(x)$, and then turn everything else into powers of $\tan(x)$)

Popper Number 05

3. $\int_{0}^{\pi/4} \tan^2(x)\,dx =$ **(1 - ln(2))/2**

= 0.1534264097

Odd Powers $\rightarrow$ Parts

Example: $\int \sec^3(x)\,dx = \int \underbrace{\sec(x)}_{u}\,\underbrace{\sec^2(x)\,dx}_{dv}$

$$\boxed{\begin{array}{ll} u = \sec(x) & du = \sec(x)\tan(x)\,dx \\ dv = \sec^2(x)\,dx & v = \tan(x) \end{array}}$$

$$= \sec(x)\tan(x) - \int \sec(x)\tan^2(x)\,dx$$

$$= \sec(x)\tan(x) - \int \sec(x)\left(\sec^2(x) - 1\right)dx$$

$$\int \sec^3(x)\,dx = \sec(x)\tan(x) - \int \sec^3(x)\,dx + \int \sec(x)\,dx$$

$$2\int \sec^3(x)\,dx = \sec(x)\tan(x) + \ln\left(|\sec(x)+\tan(x)|\right) + \tilde{C}$$

$$\Rightarrow \int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln\left(|\sec(x)+\tan(x)|\right) + C$$

Odd Powers

Example: $\int \tan^2(x)\underline{\sec^5(x)}dx =$

$$\int \left[\sec^7(x) - \sec^5(x)\right] dx$$

odd powers of sec(x)

Parts

$dv = \sec^2(x)dx$

General strategy for calculating

$$\int \tan^{n}(x)\sec^{m}(x)\,dx$$

when n is even and m is odd.

(the hardest case - this typically involves parts)

u = everything else $\quad dv = \sec^2(x)\,dx$

General strategy for calculating

$$\int \tan^n(x)\sec^m(x)\,dx$$

when n is odd and m is odd.

(peel off a sec(x)tan(x), and then turn everything else into powers of sec(x))

Example: $\int \tan^3(x)\sec^3(x)\,dx =$

$$= \int \underbrace{\tan^2(x)\sec^2(x)}_{\text{change to secants}} \underbrace{\sec(x)\tan(x)\,dx}_{\text{deriv. of } \sec(x)}$$

$$= \int \left(\sec^2(x)-1\right)\sec^2(x)\,\underbrace{\sec(x)\tan(x)\,dx}$$

$$= \int \sec^4(x)\,\underbrace{\sec(x)\tan(x)\,dx} - \int \sec^2(x)\,\underbrace{\sec(x)\tan(x)\,dx}$$

$$= \frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C.$$