| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|
| | EMCF08 due at 9am-key<br>Notes: page, 4-per, video notes, video<br>Homework 3 due in lab/workshop | Practice Test 2 is posted on CourseWare, and it is a required online quiz | EMCF09 due at 9am - key<br>Notes: page, 4-per, video notes, video<br>Homework 4 posted | Review Problems for Test 2 (7.1-8.3)<br>Solutions: notes, videos | EMCF10 due at 9am<br>Live Test 2 Review Part I<br>Notes: page, 4-per (see Monday for the video)<br>Quiz in lab/workshop | Quiz 3 closes (7.6-7.8) |
| 10 | 11<br>EMCF11 due at 9am<br>Live Test 2 Review Part II<br>Blank Slides: page, 4-per<br>Homework 4 due in lab/workshop | 12<br>UH events this week | 13<br>EMCF12 due at 9am<br>Homework 5 posted | 14<br>Test 2 starts (7.1-8.3)<br>Check the dates on CourseWare | 15<br>EMCF13 due at 9am<br>Quiz in lab/workshop | 16<br>Quiz 4 closes (8.1-8.3) |
| 17<br>Practice Test 2 closes | 18<br>EMCF14 due at 9am<br>Homework is NOT DUE until Wednesday | 19 | 20<br>EMCF15 due at 9am<br>Homework 5 due in lab/workshop | 21 | 22<br>EMCF16 due at 9am<br>Quiz in lab/workshop | 23<br>Quiz 5 closes (8.4 & review) |

# Test 2 Review

**(Continued from Friday)**

- Inverse functions
- Logarithmic functions
- Exponential functions
- Logarithmic differentiation
- Exponential growth and decay (word problems)
- Inverse trig functions
- Hyperbolic functions
- Integration by parts
- Integration of powers and products of trig functions

7.1 – 8.3

**Addition Problems:** See the homework, examples given in the class notes, questions from poppers, questions from EMCFs, questions in online quizzes, review problems and videos posted from the lectures page, and questions given on Friday quizzes.

30% MC

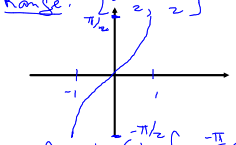
70% Written

## Inverse trigonometric functions

$\arcsin(x)$ $\longleftrightarrow$ inverse for $\sin(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Domain: $[-1, 1]$. Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

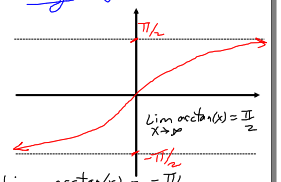
$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$




$\arctan(x)$ $\longleftrightarrow$ inverse for $\tan(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Domain: $(-\infty, \infty)$. Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$




$\lim_{x \to -\infty} \arctan(x) = -\pi/2$

$2x^4 = (\sqrt{2}x^2)^2$ $\quad u = \sqrt{2}x^2$ $\quad du = 2\sqrt{2}x\,dx$

**Example:** $\displaystyle\int \frac{x}{9+2x^4}\,dx = \frac{1}{2\sqrt{2}}\int \frac{2\sqrt{2}x}{3^2+(\sqrt{2}x^2)^2}\,dx = \frac{1}{2\sqrt{2}}\int \frac{du}{3^2+u^2}$

$a>0$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

$$= \frac{1}{6\sqrt{2}}\arctan\left(\frac{u}{3}\right)+C$$

$$= \frac{1}{6\sqrt{2}}\arctan\left(\frac{\sqrt{2}x^2}{3}\right)+C$$

**Example:** $\displaystyle\int \frac{x}{\sqrt{9-2x^4}}\,dx =$ you

$a>0$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right)+C$$

**Example:** $f(x)=\arcsin(\ln(x))$. Find the domain and give the tangent line at the point where $x=\sqrt{e}$.

Domain: $x>0$ and $-1 \le \ln(x) \le 1$

Note: $\ln(x)$ is increasing

$\frac{1}{e} \le x \le e$

T.L.: Point: $(\sqrt{e}, f(\sqrt{e})) = (\sqrt{e}, \pi/6)$

slope: $f'(\sqrt{e}) = \frac{2}{\sqrt{3e}}$

$f'(x) = \frac{1}{\sqrt{1-(\ln(x))^2}}\cdot\frac{1}{x} \Rightarrow f'(\sqrt{e}) = \frac{1}{\sqrt{3/4}}\cdot\frac{1}{\sqrt{e}} = \frac{2}{\sqrt{3e}}$

Equation: $y - \frac{\pi}{6} = \frac{2}{\sqrt{3e}}(x-\sqrt{e})$

**Example:** $f(x)=\sin(\arctan(x))$. Find $f^{-1}\left(\frac{1}{2}\right) = \frac{1}{\sqrt{3}}$

Solve $f(x)=\frac{1}{2}$.

$\sin(\arctan(x)) = \frac{1}{2}$

$\arctan(x) = \pi/6$

$x = \tan(\pi/6) = \frac{1}{\sqrt{3}}$

## Hyperbolic functions

$\cosh(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

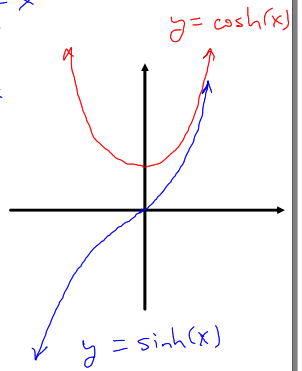
$\sinh(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

Domain: $(-\infty, \infty)$

$\cosh^2(x) - \sinh^2(x) = 1$

$\frac{d}{dx}\cosh(x) = \sinh(x)$

$\frac{d}{dx}\sinh(x) = \cosh(x)$




**Example:** Solve $\cosh(x) - 2\sinh(x) = 0$.

rewrite $\frac{1}{2}e^x + \frac{1}{2}e^{-x} - 2\left(\frac{1}{2}e^x - \frac{1}{2}e^{-x}\right) = 0$

$-\frac{1}{2}e^x + \frac{3}{2}e^{-x} = 0$

$3e^{-x} = e^x$

$3 = e^{2x}$ $\quad \ln(3) = 2x \Rightarrow x = \frac{1}{2}\ln(3)$.

**Example:** Graph $f(x)=\cosh(x)+\sinh(x)$.

$= \frac{1}{2}e^x + \frac{1}{2}e^{-x} + \frac{1}{2}e^x - \frac{1}{2}e^{-x} = e^x$

Graph it.

**Example:** Differentiate $f(x)=\sinh(x+\ln(x))$.

$$f'(x)=\cosh(x+\ln(x))\cdot\left(1+\frac{1}{x}\right)$$

**Example:** Compute $\int \cos(x)\sinh(\sin(x))dx$.

$u=\sin(x)$

$du=\cos(x)dx$

$$=\int \sinh(u)\,du$$

$$=\cosh(u)+C$$

$$=\cosh(\sin(x))+C$$

Integration by parts

$$\int u\,dv = uv-\int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

**Example:** Compute $\int_1^e \ln(x)dx$.

$u=\ln(x)$ $\quad du=\frac{1}{x}dx$

$dv=dx$ $\quad v=x$

$$=x\ln(x)\Big|_1^e-\int_1^e dx$$

$$=e-0-x\Big|_1^e$$

$$=e-(e-1)=1$$

**Example:** Compute $\int_0^1 xe^{-x}dx$.

$u=x$ $\quad du=dx$

$dv=e^{-x}dx$ $\quad v=-e^{-x}$

$$=-xe^{-x}\Big|_0^1-\int_0^1 -e^{-x}dx$$

$$=-\frac{1}{e}-e^{-x}\Big|_0^1$$

$$=-\frac{1}{e}-\left(\frac{1}{e}-1\right)$$

$$=1-\frac{2}{e}$$

Integration of powers and products of trig functions

$$\int \sin^m(x)\cos^n(x)dx$$

$$\int \sec^m(x)\tan^n(x)dx$$

$$\cos^2(x)=\frac{1}{2}+\frac{1}{2}\cos(2x)$$

$$\sin^2(x)=\frac{1}{2}-\frac{1}{2}\cos(2x)$$

$$1+\tan^2(x)=\sec^2(x)$$

**Example:** Compute $\int \sin^4(x)dx$. $= \int (\sin^2(x))^2 dx$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)^2 dx$

$= \int \left(\frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\right) dx$

$= \frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{4}\int\left(\frac{1}{2} + \frac{1}{2}\cos(4x)\right) dx$

$= \frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x) + C$

combine.

**Example:** Compute $\int \sin^2(x)\cos^2(x)dx$. $= \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)\left(\frac{1}{2} + \frac{1}{2}\cos(2x)\right)dx$

similar

multiply + finish

**Example:** Compute $\int \sin^3(x)\cos^3(x)dx$. $= \int \sin^3(x)\cos^2(x)\cos(x)dx$

$= \int \sin^3(x)\left(1 - \sin^2(x)\right)\cos(x)dx$

$= \int \sin^3(x)\cos(x)dx - \int \sin^5(x)\cos(x)dx$

$= \frac{1}{4}\sin^4(x) - \frac{1}{6}\sin^6(x) + C$

**Example:** Compute $\int \tan^4(x)dx$. $= \int \tan^2(x)\tan^2(x)dx$

$= \int \tan^2(x)\left(\sec^2(x) - 1\right)dx$

$= \int \tan^2(x)\sec^2(x)dx - \int \tan^2(x)dx$

$= \frac{1}{3}\tan^3(x) - \int \left(\sec^2(x) - 1\right)dx$

$= \frac{1}{3}\tan^3(x) - \tan(x) + x + C$

**Example:** Compute $\int \tan^3(x)\sec^3(x)dx$.

$= \int \tan^2(x)\sec^2(x)\sec(x)\tan(x)dx$

$= \int \left(\sec^2(x) - 1\right)\sec^2(x)\sec(x)\tan(x)dx$

$= \int \sec^4(x)\sec(x)\tan(x)dx - \int \sec^2(x)\sec(x)\tan(x)dx$

$= \frac{1}{5}\sec^5(x) - \frac{1}{3}\sec^3(x) + C$

**Example:** Compute $\int \tan^2(x)\sec^3(x)dx$.

you