Trigonometric Substitution

Section 8.4

Trigonometric Substitution

The following terms can sometimes cause trouble in integration problems:

$$\sqrt{a^2 - x^2} \qquad \sqrt{a^2 + x^2} \qquad \sqrt{x^2 - a^2}$$

Fortunately, these terms can be collapsed by using trigonometric identifies.

Making Appropriate Substitutions

$$a > 0$$

$\sqrt{a^2 - x^2}$ ⟷

$\sqrt{a^2 + x^2}$ ⟷

$\sqrt{x^2 - a^2}$ ⟷

Examples: What substitution would help with the integration of...

$\sqrt{9 - x^2}$ ⟷

$\sqrt{16 + x^2}$ ⟷

$\sqrt{x^2 - 4}$ ⟷

Related, but more complicated...

(The key is completing the square.)

$\sqrt{2-x^2+4x}$

$\sqrt{16+x^2-6x}$

$\sqrt{x^2-x-4}$

Examples:

$\int \frac{x^2}{\sqrt{4+x^2}}\,dx$ $\int \sqrt{9-x^2}\,dx$ $\int \sqrt{2-x^2+4x}\,dx$

$\int x\sqrt{16+x^2-6x}\,dx$ $\int \frac{x}{\sqrt{x^2-x-4}}\,dx$

$$\int \frac{x^2}{\sqrt{4+x^2}}\,dx$$

$$\int \sqrt{9-x^2}\,dx$$

$$\int \sqrt{2-x^2+4x}\,dx$$