4 **New Rule:** You can not "save" seats in the front sections.

3	4 **EMCF08 due at 9am-key** Notes: **page**, **4-per**, **video notes**, **video** Homework 3 due in lab/workshop	5 **Practice Test 2 is posted on CourseWare**, and it is a required online quiz	6 **EMCF09 due at 9am - key** Notes: **page**, **4-per**, **video notes**, **video** **Homework 4 posted**	7 **Review Problems for Test 2** (7.1-8.3) **Solutions: notes, videos**	8 **EMCF10 due at 9am** **Live Test 2 Review Part I** Notes: **page**, **4-per** (see Monday for the video) Quiz in lab/workshop	9 **Quiz 3 closes** (7.6-7.8)
10	11 **EMCF11 due at 9am** **Live Test 2 Review Part II** Notes: **page**, **4-per** **video notes**, **video** Homework 4 due in lab/workshop	12 **UH events this week**	13 **EMCF12 due at 9am** Blank Slides: **page**, **4-per** **Homework 5 posted**	14 **Test 2 starts** (7.1-8.3) **Check the dates on CourseWare**	15 **EMCF13 due at 9am** Quiz in lab/workshop	16 **Quiz 4 closes** (8.1-8.3)
17 **Practice Test 2 closes**	18 EMCF14 due at 9am Homework is NOT DUE until Wednesday	19	20 EMCF15 due at 9am **Homework 5 due in lab/workshop**	21	22 EMCF16 due at 9am Quiz in lab/workshop	23 **Quiz 5 closes** (8.4 & review)

Trigonometric Substitution

Section 8.4

Popper Number 07

1. $$\int_1^e \frac{\ln\left(x^2\right)}{x}\,dx =$$

2. Give the slope of the tangent line to the graph of $f(x) = \arctan(2x - 1) + 3^{-3x}$ at $x = 0$.

Trigonometric Substitution

The following terms can sometimes cause trouble in integration problems:

$a > 0$

$x = a\tan(\theta)$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$\sqrt{a^2 - x^2} \qquad \sqrt{a^2 + x^2} \qquad \sqrt{x^2 - a^2}$$

$x = a\sin(\theta)$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \Rightarrow \cos(\theta) \ge 0$

$\sqrt{a^2 - a^2\sin^2(\theta)} = a\sqrt{1-\sin^2(\theta)} = a\sqrt{\cos^2(\theta)} = a\cos(\theta)$

Fortunately, these terms can be collapsed by using trigonometric identifies.

e.g. $\sqrt{4-x^2}$, $\sqrt{2+x^2}$

$a=2$ $\qquad$ $a=\sqrt{2}$

Spse $a=1$. $\sqrt{1-x^2} = \sqrt{1-\sin^2(\theta)}$

$x = \sin(\theta)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $\qquad = \sqrt{\cos^2(\theta)}$

$\sqrt{(-2)^2} = 2$ $\qquad$ $\cos(\theta) \ge 0$ $\qquad = \cos(\theta)$

Making Appropriate Substitutions

$$a > 0$$

$$\sqrt{a^2 - x^2} \quad \longleftrightarrow$$

$x = a\sin(\theta)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\cos(\theta) \geq 0$

$$\sqrt{a^2 + x^2} \quad \longleftrightarrow$$

$x = a\tan(\theta)$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

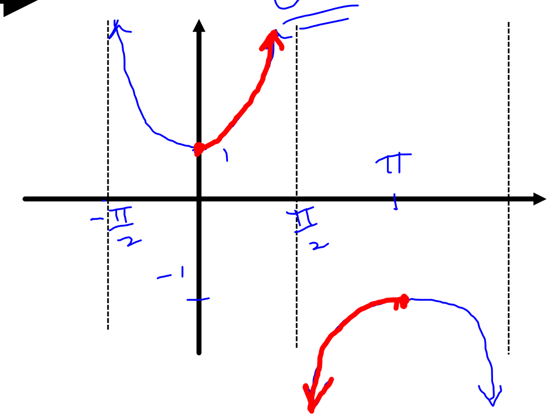
$\sec(\theta) > 0$

$\underline{\underline{a > 0}}$

$$\sqrt{x^2 - a^2} \quad \longleftrightarrow$$

$x = a\sec(\theta)$

$\underline{\theta}$??



$= \sqrt{a^2\sec^2(\theta) - a^2}$

$= a\sqrt{\sec^2(\theta) - 1}$

$= a\sqrt{\tan^2(\theta)}$

"restricted secant requires

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

$0 \le \theta < \frac{\pi}{2}$

$\tan(\theta) \geq 0$

$\frac{\pi}{2} < \theta \le \pi$

$\tan(\theta) \le 0$

$a\tan(\theta)$

$-a\tan(\theta)$.

WOW!

Examples: What substitution would help with the integration?

$\sqrt{9-x^2}$ $\longleftrightarrow$ $x = 3\sin(\theta)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\sqrt{16+x^2}$ $\longleftrightarrow$ $x = 4\tan(\theta)$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sqrt{x^2-4}$ $\longleftrightarrow$ $x = 2\sec(\theta)$

$0 \le \theta < \frac{\pi}{2}$ or $\frac{\pi}{2} < \theta \le \pi$

Related, but more complicated...

(The key is completing the square.)

$$\sqrt{2-x^2+4x} = \sqrt{2+4-(x^2-4x+4)}$$

$$= \sqrt{6-(x-2)^2}$$

$$\sqrt{16+x^2-6x} = \sqrt{16-9+(x^2-6x+9)}$$

$$= \sqrt{7+(x-3)^2}$$

$$\sqrt{x^2-x-4} = \sqrt{x^2-x+\frac{1}{4}-4-\frac{1}{4}}$$

$$= \sqrt{\left(x-\frac{1}{2}\right)^2-\frac{17}{4}}$$

Popper Number 07

3. The substitution $x = a\sin(\theta)$ would help with the integration of $\sqrt{15 - x^2}$. Give the value of a.

4. The substitution $x = a\tan(\theta)$ would help with the integration of $\sqrt{16 + x^2}$. Give the value of a.

Examples:

$$\int \frac{x^2}{\sqrt{4+x^2}}\,dx \qquad \int \sqrt{9-x^2}\,dx \qquad \int \sqrt{2-x^2+4x}\,dx$$

$$\int x\sqrt{16+x^2-6x}\,dx \qquad \int \frac{x}{\sqrt{x^2-x-4}}\,dx$$

$$\int \sqrt{9-x^2}\,dx = \int \sqrt{9 - 9\sin^2(\theta)}\; 3\cos(\theta)\,d\theta$$

$x = 3\sin(\theta)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = 3\cos(\theta)\,d\theta$

$\cos(\theta) \ge 0$

$$= \int 9\sqrt{1-\sin^2(\theta)}\,\cos(\theta)\,d\theta$$

$$= \int 9\sqrt{\cos^2(\theta)}\,\cos(\theta)\,d\theta$$

$$= 9\int \cos^2(\theta)\,d\theta$$

$$= 9\int\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)d\theta$$

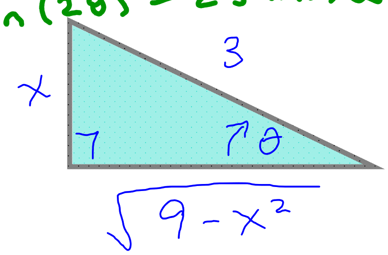
$$= \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta) + C$$

$$= \frac{9}{2}\theta + \frac{9}{2}\sin(\theta)\cos(\theta) + C$$

?

- $\frac{x}{3} = \sin(\theta)$
- $\theta = \arcsin\left(\frac{x}{3}\right)$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$



- $\cos(\theta) = \frac{1}{3}\sqrt{9-x^2}$

$$= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + \frac{9}{6}x\cdot\frac{1}{3}\sqrt{9-x^2} + C$$

$$= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + \frac{1}{2}x\sqrt{9-x^2} + C$$

$$\int \frac{x^2}{\sqrt{4+x^2}}\,dx = \int \frac{4\tan^2(\theta)}{\sqrt{4+4\tan^2(\theta)}} \cdot 2\sec^2(\theta)\,d\theta$$

$x = 2\tan(\theta)$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 2\sec^2(\theta)\,d\theta$

$\sec(\theta) > 0$

$$= \int \frac{4\tan^2(\theta)\sec^2(\theta)}{\sqrt{\sec^2(\theta)}}\,d\theta$$

$$= \int \frac{4\tan^2(\theta)\sec^2(\theta)}{\sec(\theta)}\,d\theta$$

$$= 4\int \tan^2(\theta)\sec(\theta)\,d\theta$$

$$= 4\int (\sec^2(\theta)-1)\sec(\theta)\,d\theta$$

$$= 4\int (\sec^3(\theta) - \sec(\theta))\,d\theta$$

parts. Ouch.

you know

Finish it.

$$\int \sqrt{2-x^2+4x}\,dx = \int \sqrt{2+4-(x^2-4x+4)}\,dx$$

$$= \int \sqrt{6-(x-2)^2}\,dx$$

$x-2=\sqrt{6}\sin(\theta)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$dx = \sqrt{6}\cos(\theta)\,d\theta$

$\cos(\theta) \ge 0$

$$= \int \sqrt{6-6\sin^2(\theta)}\,\sqrt{6}\cos(\theta)\,d\theta$$

$$= 6\int \sqrt{\cos^2(\theta)}\,\cos(\theta)\,d\theta$$

$$= 6\int \cos^2(\theta)\,d\theta$$

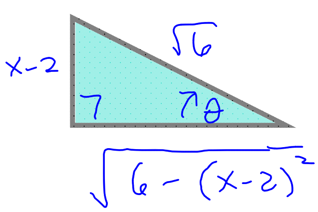
$$= 6\int \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right)d\theta$$

$\theta = \arcsin\left(\frac{x-2}{\sqrt{6}}\right)$

$\frac{x-2}{\sqrt{6}} = \sin(\theta)$

$$= 3\theta + \frac{3}{2}\sin(2\theta) + C$$

$$= 3\theta + 3\sin(\theta)\cos(\theta) + C$$



$\cos(\theta) = \frac{\sqrt{6-(x-2)^2}}{\sqrt{6}}$

$$= 3\arcsin\left(\frac{x-2}{\sqrt{6}}\right) + \frac{3(x-2)}{\sqrt{6}}\,\frac{\sqrt{6-(x-2)^2}}{\sqrt{6}} + C$$

Clean it up.