Numerical Integration
(an introduction)

Approximating $\int_a^b f(x)dx$

Why bother?

Recall:
1. The left hand endpoint method.
2. The right hand endpoint method.
3. The midpoint method.

New:
4. The trapezoid method.
5. Simpson's method.

General Formulas to approximate $\int_a^b f(x)dx$.

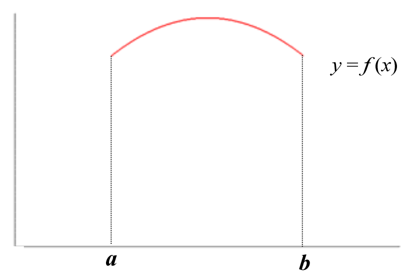
Left Hand Endpoint Method:

Right Hand Endpoint Method:

Midpoint Method:

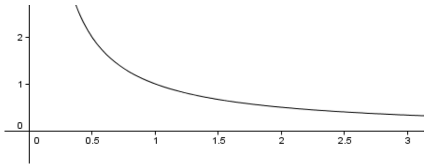
Example: Use the midpoint method with $n = 2$ to approximate $\int_1^2 \frac{1}{x}dx$.

New: Trapezoid Method to approximate $\int_a^b f(x)dx$.



Example: Use the trapezoid method with $n = 2$

to approximate $\int_1^3 \frac{1}{x}\,dx.$



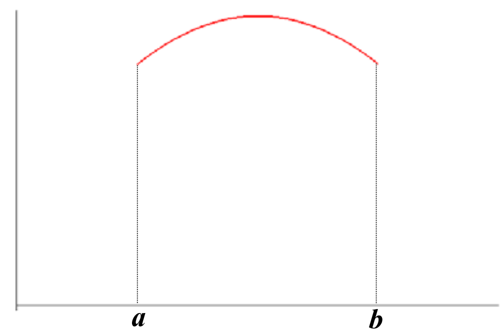
Trapezoid Rule Error Estimate

$$|E_n^T| \le \frac{(b-a)^3}{12\,n^2} M$$

I'll post a video with this information.

Example: Give a value of n that will guarantee the Trapezoid method approximates $\int_0^{\pi/2} \sin(x)\,dx$ within 10^{-4}.

New: Simpson's Method to approximate $\int_a^b f(x)\,dx.$



This leads to...

Simpson's Method:

$$S_n = \frac{b-a}{6n} \left\{ f(x_0) + f(x_n) + 2[f(x_1) + \cdots + f(x_{n-1})] + 4\left[f\left(\frac{x_0+x_1}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right) \right] \right\}$$

Note: This formulation for Simpson's method is different than other texts. Be careful!!

$$\frac{1}{3}T_n + \frac{2}{3}M_n = S_n$$

Relationships Between Formulas

- $\frac{1}{2}(L_n + R_n) = T_n$
- $\frac{1}{3}T_n + \frac{2}{3}M_n = S_n$

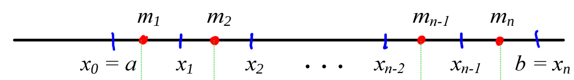
Example: Use Simpson's method with $n = 3$ to approximate $\int_1^4 \frac{1}{x}\,dx$.

Simpson's Method Error Estimate

$$|E_n^S| \le \frac{(b-a)^5}{2880n^4} M$$

Example: **Give a value of n that will guarantee Simpson's method approximates $\int_0^{\pi/2} \sin(x)\,dx$ within 10^{-4}.**

Methods for approximating $\int_a^b f(x)\,dx$



L_n		$\frac{b-a}{n}$
R_n		$\frac{b-a}{n}$
T_n		$\frac{b-a}{2n}$
M_n		$\frac{b-a}{n}$
S_n		$\frac{b-a}{6n}$