Test 2 is Graded
Add the Scores for Test 2 and Test2FR to obtain your total score out of 100.

Class Median = 87
Final Comments on Numerical Integration

Methods for approximating $\int_a^b f(x) \, dx$

$x_0 = a \quad x_1 \quad x_2 \quad \cdots \quad x_{n-2} \quad x_{n-1} \quad b = x_n$

$N$ is a natural number

\( m_i \) = midpoints

\( \chi_i \) = nodes

Left hand endpoint

\[ L_n = \frac{b - a}{n} \left[ f(x_0) + f(x_1) + \cdots + f(x_{n-1}) \right] \]

Right hand endpoint

\[ R_n = \frac{b - a}{n} \left[ f(x_1) + f(x_2) + \cdots + f(x_n) \right] \]

Midpoint method

\[ M_n = \frac{b - a}{n} \left[ f\left(\frac{x_0 + x_1}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right) \right] \]

Trapezoid method

\[ T_n = \frac{b - a}{2n} \left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \]

Simpson's method

\[ S_n = \frac{b - a}{6n} \left\{ f(x_0) + f(x_n) + 2[f(x_1) + \cdots + f(x_{n-1})] + 4 \left[f\left(\frac{x_0 + x_1}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right)\right] \right\} \]

Note:

\[ T_n = \frac{1}{2} L_n + \frac{1}{2} R_n \]

\[ S_n = \frac{1}{3} T_n + \frac{2}{3} M_n \]
Methods for approximating $\int_a^b f(x) \, dx$

See the videos posted on the course homepage.
Example: Use Simpson's method with \( n = 2 \) to approximate \( \int_{0}^{1} \sin\left(x^2\right) \, dx \).

\[
\begin{align*}
\int_{0}^{1} \sin\left(x^2\right) \, dx & \approx \frac{1-0}{6 \cdot 2} \left[ f(0) + 4f(m_1) + 2f(x_1) + 4f(m_2) + f(1) \right] \\
& = \frac{1}{12} \left[ f(0) + 4f(\frac{1}{4}) + 2f(\frac{1}{2}) + 4f(\frac{3}{4}) + f(1) \right] \\
& = \frac{1}{12} \left[ \sin(0) + 4 \sin\left(\frac{1}{4}\right) + 2 \sin\left(\frac{1}{2}\right) + 4 \sin\left(\frac{3}{4}\right) + \sin(1) \right]
\end{align*}
\]

\( S_2 = 0.3099439057 \)

Value from calculator:

\( 0.3102683017 \)

Very close!!
Polar Coordinates
(9.3 and 9.4)

**Question:** How is the point \((a, b)\) represented in Cartesian coordinates?

Polar Coordinates afford another mechanism for visualizing points in the xy-plane.
The graphs below are easy to describe in polar coordinates.
What are polar coordinates?

\[
[r, \theta] = \left[ \sqrt{a^2 + b^2}, \theta \right]
\]

One polar representation is:

\[
[r, \theta] = \left[ -\sqrt{a^2 + b^2}, \theta \right]
\]

Not the same
**Question:** Can we give more than one polar representation for the same point?

Yes

Infinitely many.
**Example:** How many ways can we represent the cartesian point (0,1) in polar coordinates? *Infinitely many*

\[(x, y) \quad \text{rectangular} \quad [r, \theta] \quad \text{polar}\]

(0,1)

\[
\begin{align*}
[1, \pi/2] \\
[-1, -\pi/2] \\
[1, \pi/2 + 2\pi] \\
[-1, -\pi/2 - 2\pi] \\
\vdots \\
[1, -3\pi/2]
\end{align*}
\]
Standard representation:

\[ r = \sqrt{x^2 + y^2} \]

\[ \theta = \arctan \left( \frac{y}{x} \right) \]

(x, y) in Q_1, Q_4

\[ x = r \cos(\theta) \]
\[ y = r \sin(\theta) \]
Example: Determine whether the polar graph of \( r = \cos(\theta) \) is symmetric about the origin... the \( x \)-axis.

About the origin?: 2 questions

\([r, \theta] \Rightarrow [-r, \theta]\)

Check: does \( r = \cos(\theta) \)

\[ \implies -r = \cos(\theta) \]

No!

About the \( x \)-axis: Yes

\([r, \theta] \Rightarrow [r, -\theta]\)

Check: does \( r = \cos(\theta) \)

\[ \implies r = \cos(-\theta) \]

Yes. \( \because \) Cosine is even.
Rewrite the following curve in polar coordinates.

\[ y = -\frac{1}{2}x^2 + 3x - 1 \]
Example: Rewrite \( r = \sin(\theta) \tan(\theta) \) in cartesian coordinates.
**Question:** How can we describe lines in polar coordinates?

**Cases:** Horizontal, Vertical, General
Polar Equations for 3 Types of Circles

I.  

II.  

III.