Test 2 is Graded
Add the Scores for Test 2 and Test2FR to obtain your total score out of 100.

Class Median = 87

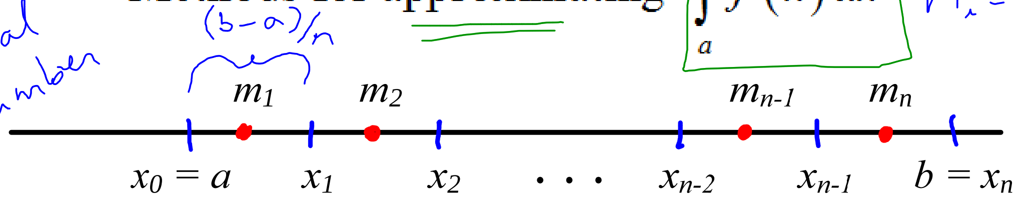
Final Comments on Numerical Integration

$x_i \equiv$ nodes

$m_i \equiv$ midpoints

n is a natural number

Methods for approximating $\boxed{\int_a^b f(x)\,dx}$



left hand endpoint • $L_n = \dfrac{b-a}{n}\,[f(x_0)+f(x_1)+\cdots+f(x_{n-1})]$ Crummy

right hand endpoint • $R_n = \dfrac{b-a}{n}\,[f(x_1)+f(x_2)+\cdots+f(x_n)]$ Crummy

midpoint method • $M_n = \dfrac{b-a}{n}\left[f\left(\dfrac{x_0+x_1}{2}\right)+\cdots+f\left(\dfrac{x_{n-1}+x_n}{2}\right)\right]$ fair

trapezoid method • $T_n = \dfrac{b-a}{2n}\,[f(x_0)+2f(x_1)+\cdots+2f(x_{n-1})+f(x_n)]$ fair

Simpson's • $S_n = \dfrac{b-a}{6n}\left\{f(x_0)+f(x_n)+2[f(x_1)+\cdots+f(x_{n-1})]\right.$

$$\left.+4\left[f\left(\frac{x_0+x_1}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right]\right\}$$

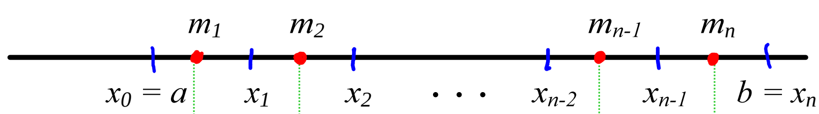
very good

Note:

$$T_n = \tfrac{1}{2}L_n + \tfrac{1}{2}R_n$$

$$S_n = \tfrac{1}{3}T_n + \tfrac{2}{3}M_n$$

Methods for approximating $\int_a^b f(x)\,dx$

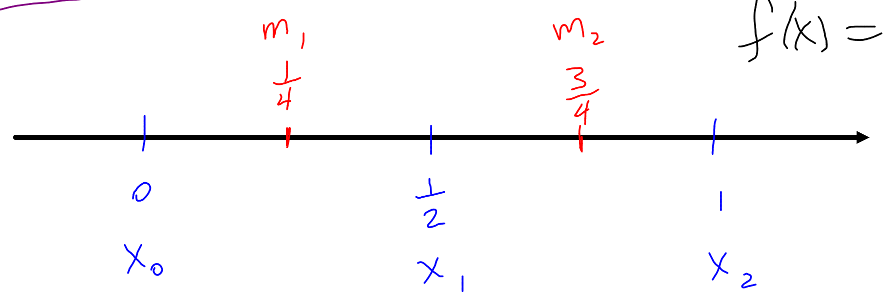


	$x_0 = a$	m_1	x_1	m_2	x_2	$\cdots$	x_{n-2}	m_{n-1}	x_{n-1}	m_n	$b = x_n$	
L_n	1		1		1	$\cdots$	1		1			$\frac{b-a}{n}$
R_n			1		1	$\cdots$	1		1		1	$\frac{b-a}{n}$
T_n	1		2		2	$\cdots$	2		2		1	$\frac{b-a}{2n}$
M_n		1		1		$\cdots$		1		1		$\frac{b-a}{n}$
S_n	1	4	2	4	2	$\cdots$	2	4	2	4	1	$\frac{b-a}{6n}$

See the videos posted on the course homepage.

Example: Use Simpson's method with $n = 2$ to approximate $\int_0^1 \sin\left(x^2\right) dx$.

$f(x) = \sin(x^2)$



$$S_2 = \frac{1-0}{6 \cdot 2}\left[f(x_0) + 4f(m_1) + 2f(x_1) + 4f(m_2) + f(x_2)\right]$$

$$= \frac{1}{12}\left[f(0) + 4f\left(\tfrac{1}{4}\right) + 2f\left(\tfrac{1}{2}\right) + 4f\left(\tfrac{3}{4}\right) + f(1)\right]$$

$$= \frac{1}{12}\left[\sin(0) + 4\sin\left(\tfrac{1}{16}\right) + 2\sin\left(\tfrac{1}{4}\right) + 4\sin\left(\tfrac{9}{16}\right) + \sin(1)\right]$$

Only $n = 2$.

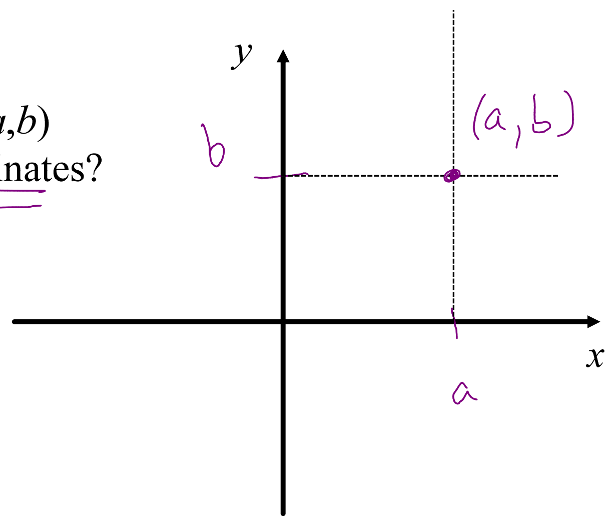
S2 = 0.3099439057

0.3102683017 ← value from calculator

Very Close!!

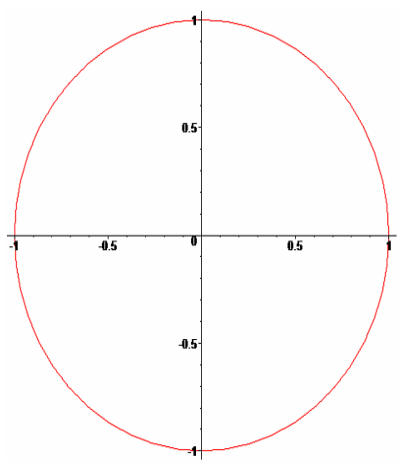
Polar Coordinates
(9.3 and 9.4)

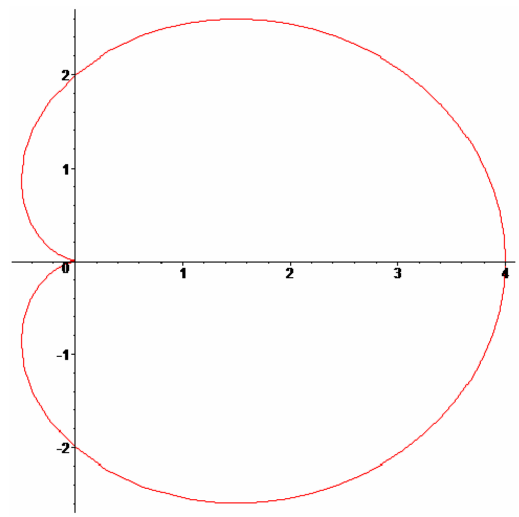
Question: How is the point (a,b) represented in cartesian coordinates?

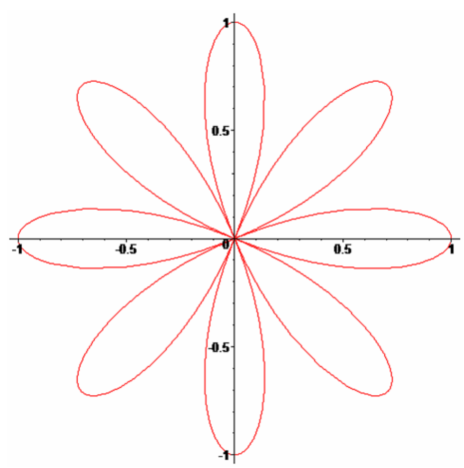


Polar Coordinates afford another mechanism for visualizing points in the xy-plane.

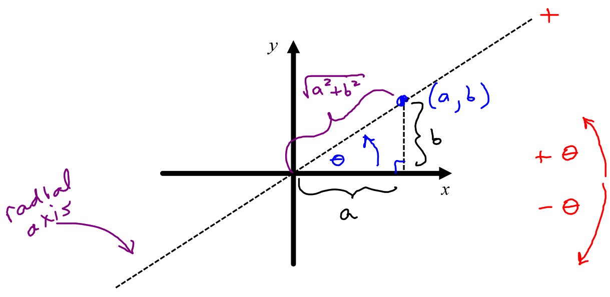
The graphs below are easy to describe in polar coordinates.

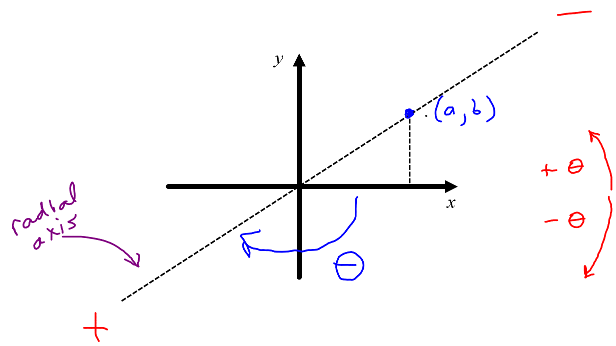






What are polar coordinates?





One polar representation is

$[r, \theta] = [\sqrt{a^2+b^2}, \theta]$

$[r, \theta] = [-\sqrt{a^2+b^2}, \theta]$

not the same

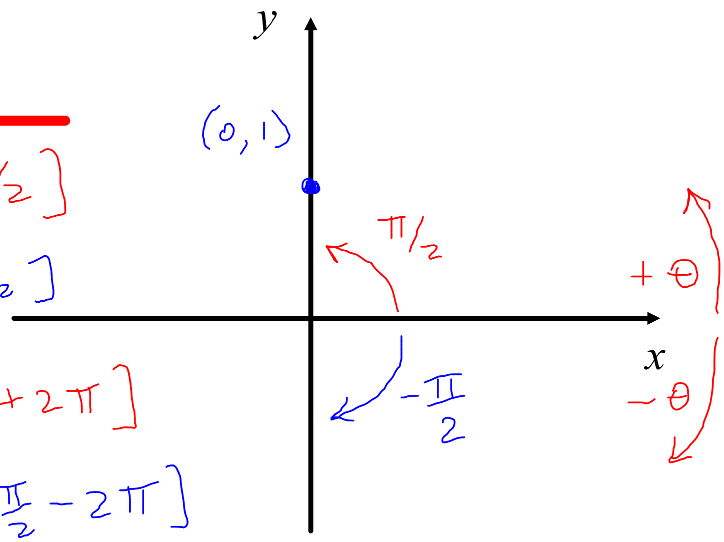
Question: Can we give more than one **polar representation** for the same point?

yes

Infinitely many.

Example: How many ways can we represent the cartesian point (0,1) in polar coordinates? Infinitely many

(x, y) rectangular	$[r, \theta]$ polar
(0,1)	$[1, \pi/2]$
	$[-1, -\pi/2]$
	$[1, \frac{\pi}{2} + 2\pi]$
	$[-1, -\frac{\pi}{2} - 2\pi]$
	$\vdots$
	$[1, -\frac{3\pi}{2}]$



Standard representation:

$r^2 = x^2 + y^2$

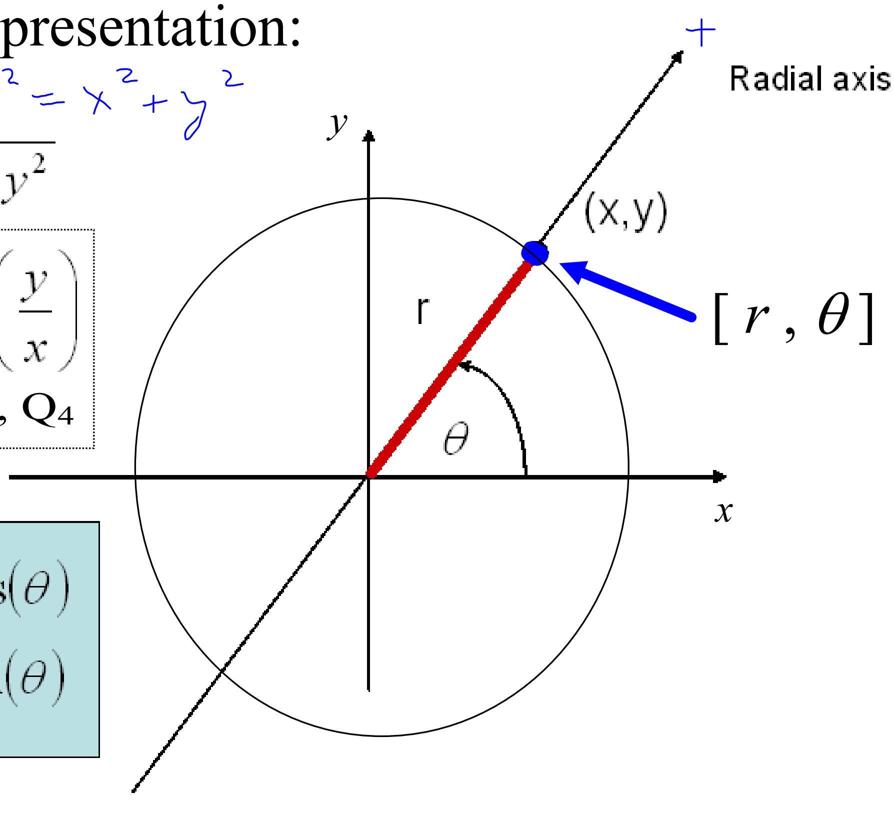
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

(x,y) in Q_1, Q_4

$$x = r\cos(\theta)$$

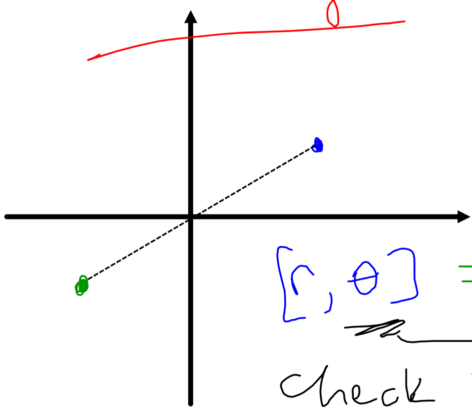
$$y = r\sin(\theta)$$



Example: Determine whether the polar graph of $r = \cos(\theta)$ is symmetric about the origin... the x-axis.

2 questions

About the origin?

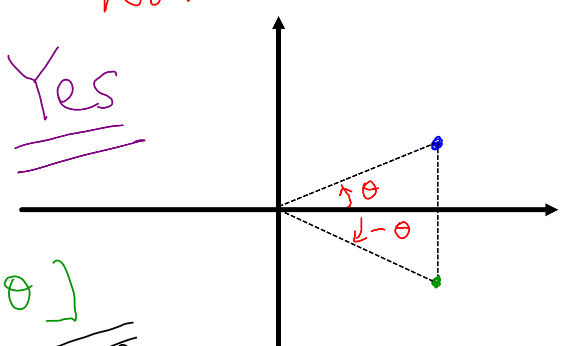


$[r, \theta] \Rightarrow [-r, \theta]$

Check: Does $r = \cos(\theta)$ imply $-r = \cos(\theta)$

No!

About the x-axis: Yes



$[r, \theta] \Rightarrow [r, -\theta]$

Check: does $r = \cos(\theta)$ imply $r = \cos(-\theta)$?

yes. b/c cosine is even.

Rewrite the following curve in polar coordinates.

$$y = -\frac{1}{2}x^2 + 3x - 1$$

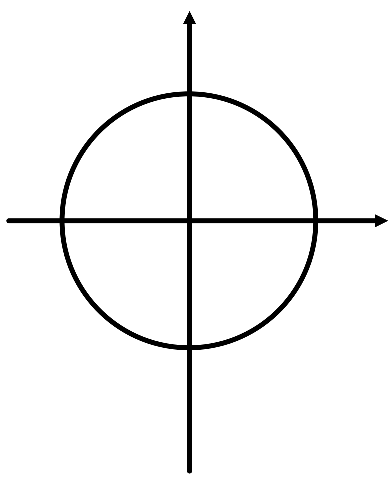
Example: Rewrite $r = \sin(\theta)\tan(\theta)$ in cartesian coordinates.

Question: How can we describe lines in polar coordinates?

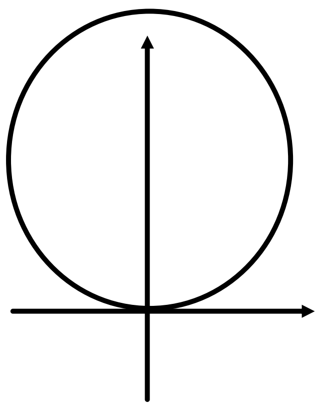
Cases: Horizontal, Vertical, General

Polar Equations for 3 Types of Circles

I.



II.



III.