Recall: Polar Coordinates

\[ r = \sqrt{x^2 + y^2} \]
\[ \theta = \arctan \left( \frac{y}{x} \right) \]
\[ (x, y) \text{ in Q1 or Q4} \]
\[ r = x \cos(\theta) \]
\[ y = r \sin(\theta) \]

\[ (r, \theta) \text{ (polar)} \]

Polar equations for 3 different types of circles.

I. \( x^2 + y^2 = a^2 \)
   \[ r = a \]
   Centered at \((0, 0)\)
   \[ r = a \cos(\theta) \text{ in polar coord.} \]

II. \( (x-a)^2 + y^2 = a^2 \)
   \[ r = a \]
   \[ r^2 \cos^2(\theta) + (r \sin(\theta) - a)^2 = a^2 \]
   \[ r = a \sin(\theta) - 2a \cos(\theta) + a^2 \]
   \[ r = 2a \cos(\theta) \text{ in polar coord.} \]

III. \( x^2 + (y-a)^2 = a^2 \)
   \[ r = a \]
   \[ r^2 \cos^2(\theta) + r^2 \sin^2(\theta) - 2ar \sin(\theta) + a^2 \]
   \[ r = 2a \sin(\theta) \text{ in polar coord.} \]

Polar Flowers

**Example:**
Plot the polar curve

\[ r = \sin(3\theta) \]

We are not in any more

**Step 1**
Create a plot in the \( \theta r \) plane.
(this is NOT the polar plot)

**Step 2**
Interpret the plot above to create the polar plot.
Geogebra Exploration:

\[ \begin{cases} r = a \cos(m\theta) \\ r = a \sin(m\theta) \end{cases} \]

See the applets linked from the course homepage after class.

Polar graphs that produce flowers

\[ \begin{cases} r = a \cos(m\theta) \\ r = a \sin(m\theta) \end{cases} \]

is a real \( a \neq 0 \), and \( m \) is a positive integer.

Fundamental Question: How do the values \( a \) and \( m \) effect the graph?

- \( m \) odd \( \Rightarrow \) \( m \) petals
- \( m \) even \( \Rightarrow \) \( 2m \) petals.

\( |a| \) gives the distance of each tip from the origin.

The petals are equally spaced.

\[ r = 2 \cos(2\theta) \quad r = 2 \cos(5\theta) \]

\[ r = 2 \sin(4\theta) \quad r = 2 \sin(3\theta) \]

**Popper 09**

3. Give the number of petals on the flower \( r = 3 \cos(4\theta) \).

4. Give the number of petals on the flower \( r = 4 \cos(5\theta) \).

5. Give the furthest distance to a tip of a petals on the flower \( r = 6 \cos(4\theta) \).
Example:

Plot the polar curve

\[ r = 3 + 4 \cos(\theta) \]

Step 1
Create a plot in the \( \theta r \) plane.
(this is NOT the polar plot)

Step 2
Interpret the plot above to create the polar plot.

Example: Geogebra Investigation

Investigate

\[ r = 3 + b \cos(\theta) \]

for \( b \) between 1 and 5.

Geogebra Exploration:

\[ r = a + b \cos(\theta) \]

and

\[ r = a + b \sin(\theta) \]

See the applets linked from the course homepage after class.

You can create similar graphs.
Polar curves of the form

\[ r = a + b \cos(\theta) \]

and

\[ r = a + b \sin(\theta) \]

are Cardioids, Limacons with dimples, and Limacons with inner loops.

\(|a| < |b|\)  \( |a| > |b| \)

Note: The cosine versions can be reflected across the \( y \)-axis if \( b \) is negative.