Recall: Polar Coordinates

Standard form

$r = \sqrt{x^2 + y^2}$

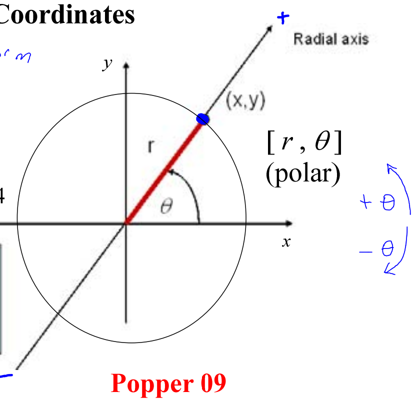
$\theta = \arctan\left(\frac{y}{x}\right)$

(x, y) in Q1 or Q4

Always

$x = r\cos(\theta)$

$y = r\sin(\theta)$



$[r, \theta]$ (polar)

$+\theta$

$-\theta$

Popper 09

1. 0.125

Polar equations for 3 different types of circles.

I. "$x^2 + y^2 = a^2$ centered at $(0,0)$"

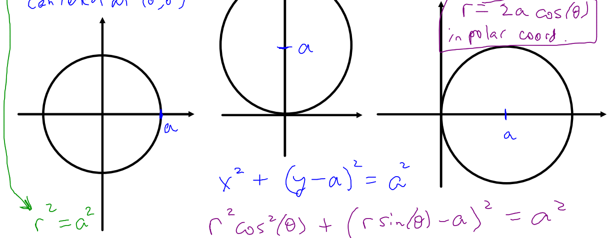
$r^2 = a^2$

$\boxed{r = a}$ in polar coord.

II. $r = 2a\sin(\theta)$

III. $(x-a)^2 + y^2 = a^2$

$\boxed{r = 2a\cos(\theta) \text{ in polar coord.}}$



$x^2 + (y-a)^2 = a^2$

$r^2\cos^2(\theta) + (r\sin(\theta) - a)^2 = a^2$

$\underline{r^2\cos^2(\theta) + r^2\sin^2(\theta)} - 2ar\sin(\theta) + \cancel{a^2} = \cancel{a^2}$

$r^2 - 2ar\sin(\theta) = 0$

$r^2 = 2ar\sin(\theta)$

$\boxed{r = 2a\sin(\theta) \text{ in polar coord.}}$

Ex. Graph the polar curve

$r = 3\sin(\theta)$

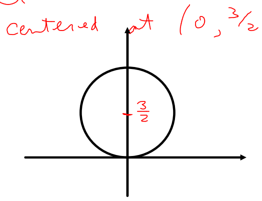
$\Downarrow$

Circle of radius $\frac{3}{2}$ centered at $(0, 3/2)$

$r = 2a\sin(\theta)$

$\updownarrow$

circle of radius $|a|$ centered at $(0, a)$ $\Rightarrow$



Polar Flowers

Example:

Plot the polar curve

$r = \sin(3\theta)$ ↦ We are not in Kansas any more.

Popper 09

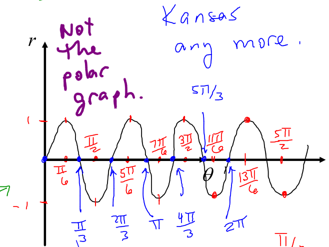
2. 0

Step 1

Create a plot in the θr plane.
(this is NOT the polar plot)

Graph $r = \sin(3\theta)$ here

Not the polar graph.

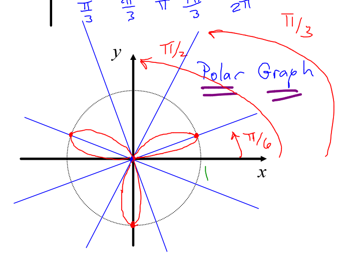


• ⇔ $r = 0$

Step 2

Interpret the plot above to create the polar plot.

Polar Graph



3 petal flower

Geogebra Exploration:

$$\begin{cases} r = a\cos(m\theta) \\ r = a\sin(m\theta) \end{cases}$$

See the applets linked from the course homepage after class.

Polar graphs that produce flowers

$$\begin{cases} r = a\cos(m\theta) \\ r = a\sin(m\theta) \end{cases}$$

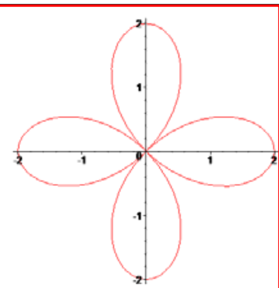
a is a real #, $a \neq 0$, and m is a positive integer.

Fundamental Question: How do the values a and m effect the graph?

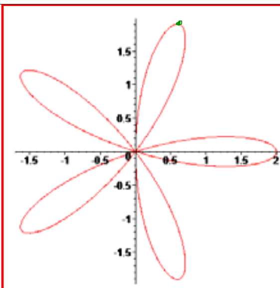
m odd $\Rightarrow$ m petals | m even $\Rightarrow$ $2m$ petals.

$|a|$ gives the distance of each tip from the origin.

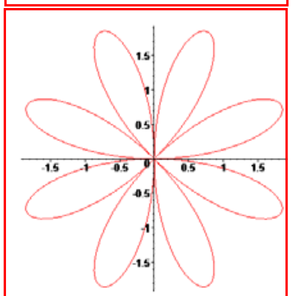
The petals are equally spaced.



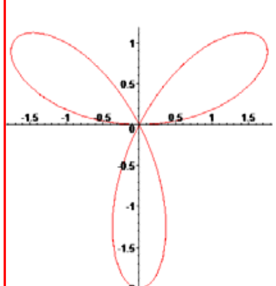
$r = 2\cos(2\theta)$



$r = 2\cos(5\theta)$



$r = 2\sin(4\theta)$



$r = 2\sin(3\theta)$

Popper 09

3. Give the number of petals on the flower $r = 3\cos(4\theta)$.

4. Give the number of petals on the flower $r = 4\cos(5\theta)$.

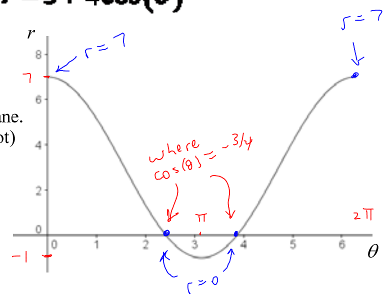
5. Give the furthest distance from the origin to a tip of a petals on the flower $r = 6\cos(4\theta)$.

Example:

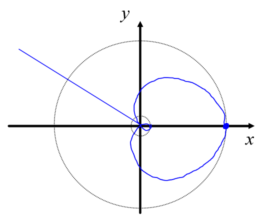
Plot the polar curve

$$r = 3 + 4\cos(\theta)$$

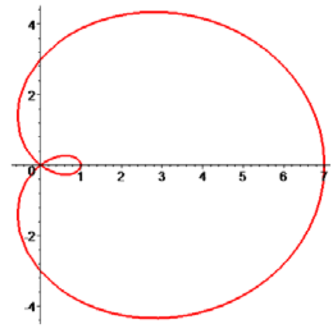
Step 1
Create a plot in the θr plane.
(this is NOT the polar plot)



Step 2
Interpret the plot above to create the polar plot.



$$r = 3 + 4\cos(\theta)$$



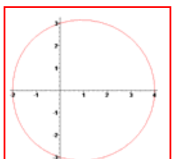
Example: Geogebra Investigation

Investigate

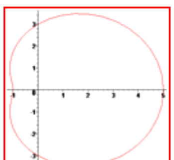
$$r = 3 + b\cos(\theta)$$

See the link from the course homepage after class.

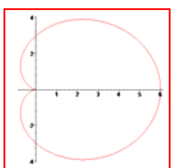
for b between 1 and 5.



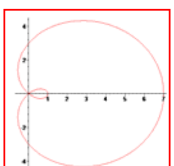
$r = 3 + \cos(\theta)$



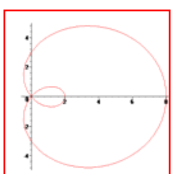
$r = 3 + 2\cos(\theta)$



$r = 3 + 3\cos(\theta)$



$r = 3 + 4\cos(\theta)$



$r = 3 + 5\cos(\theta)$

You can create similar graphs.

Geogebra Exploration:

$$r = a + b\cos(\theta)$$

you

and

$$r = a + b\sin(\theta)$$

See the applets linked from the course homepage after class.

Polar curves of the form

$$r = a + b\cos(\theta)$$

and

$$r = a + b\sin(\theta)$$

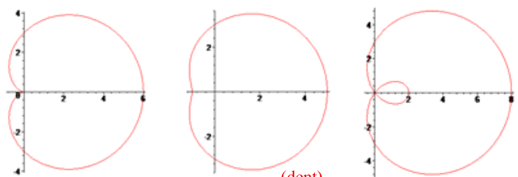
$|a| = |b|$ $|a| > |b|$

are Cardioids, Limacons with dimples, and Limaçons with inner loops.

$|a| < |b|$

Look Below...

$$r = a + b\cos(\theta)$$



Cardioid
$|a|=|b|$

(dent)
Limacon with dimple
$|a|>|b|$

Limacon with inner loop
$|a|<|b|$

Note: The cosine versions can be reflected across the y axis if b is negative.