Recall: **Polar Coordinates**

Standard

$$r = \sqrt{x^2 + y^2}$$

$r^2 = x^2 + y^2$

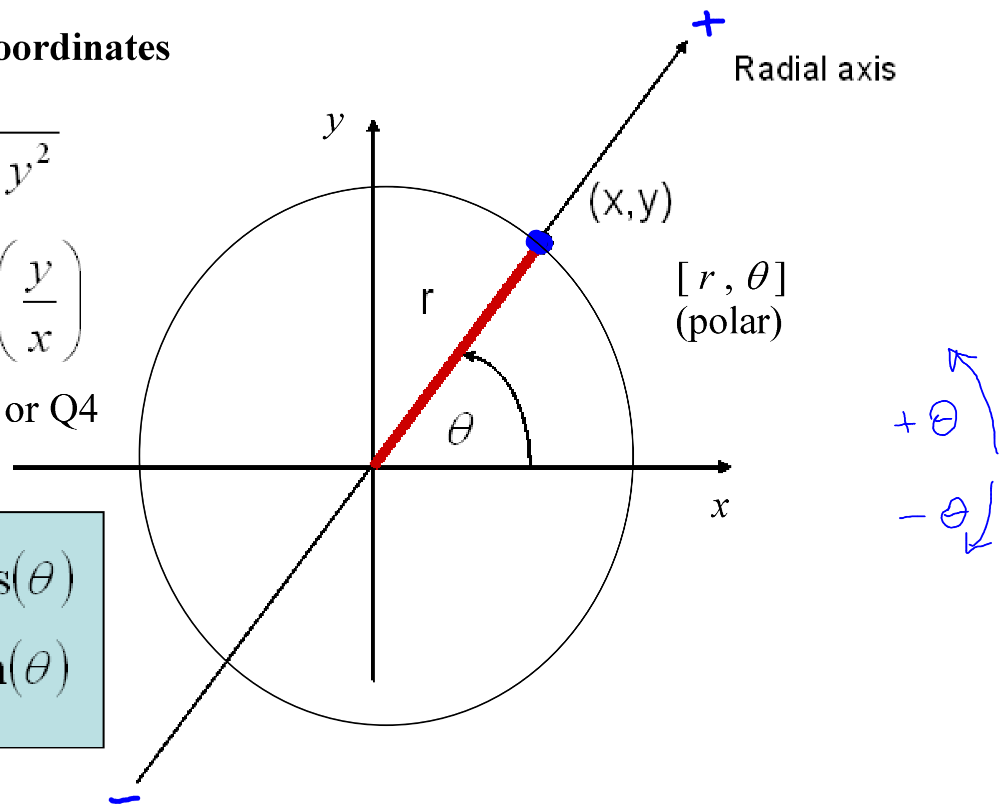
$$\theta = \arctan\left(\frac{y}{x}\right)$$

(x, y) in Q1 or Q4

Always

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

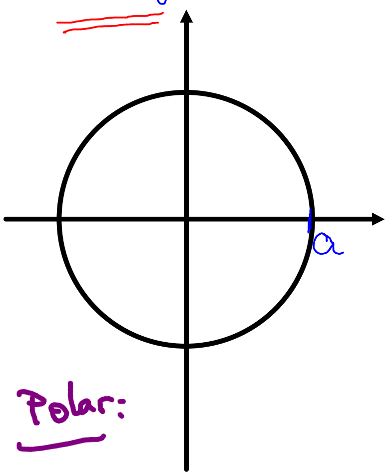


Polar equations for 3 different types of circles.

$a > 0$

I.

$x^2 + y^2 = a^2$



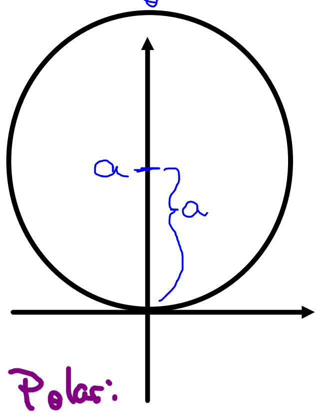
Polar:

$r^2 = a^2$

$\boxed{r = a}$

II.

$x^2 + (y-a)^2 = a^2$



Polar:

$\underbrace{x^2 + y^2}_{r^2} - 2ay + a^2 = a^2$

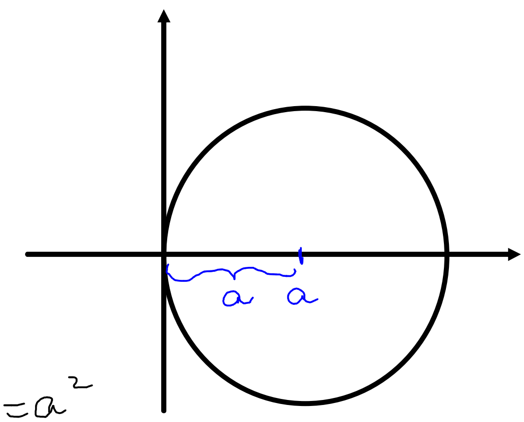
$r^2 - 2ar\sin(\theta) = 0$

$r^2 = 2ar\sin(\theta)$

$\boxed{r = 2a\sin(\theta)}$

III.

$(x-a)^2 + y^2 = a^2$



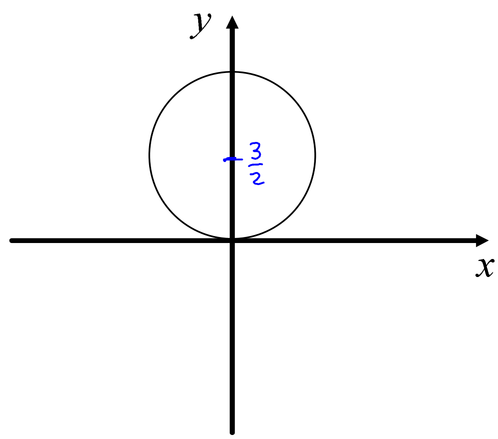
Polar:

$\boxed{r = 2a\cos(\theta)}$

Note: If $a < 0$, we get the same with the radius being $|a|$.

Example: Plot the polar curve $r = 3\sin(\theta)$.

Recall: $r = 2a\sin(\theta)$ is the circle of radius a centered at $(0, a)$.



$\therefore$ $r = 3\sin(\theta)$ is the circle with radius $\frac{3}{2}$ centered at $(0, \frac{3}{2})$.

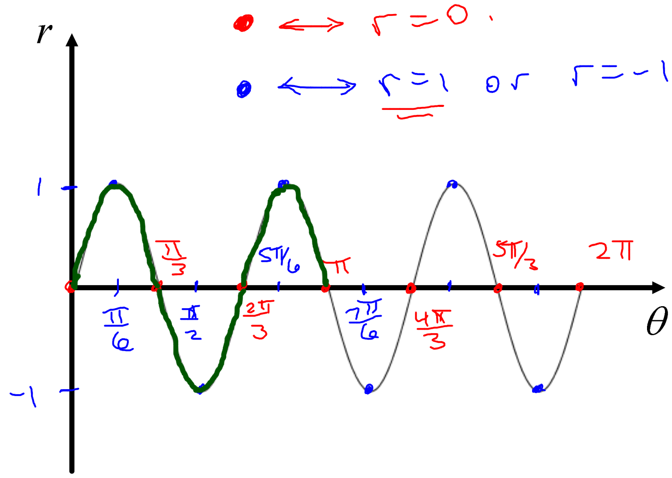
Polar Flowers

Example:

Plot the polar curve

$$r = \sin(3\theta)$$

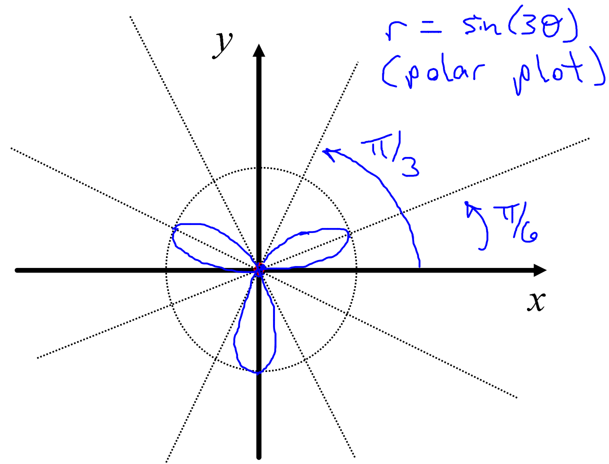
Step 1
Create a plot in the θr plane.
(this is NOT the polar plot)



Step 2
Interpret the plot above to create the polar plot.

Note: $r = 0 \iff$ origin

$r = \pm 1 \iff$ circle of radius 1 centered at $(0, 0)$



flower with 3 petals

Geogebra Exploration:

$$\begin{cases} r = a\cos(m\theta) \\ r = a\sin(m\theta) \end{cases} \quad *$$

See the applets linked from the course homepage after class.

Polar graphs that produce flowers

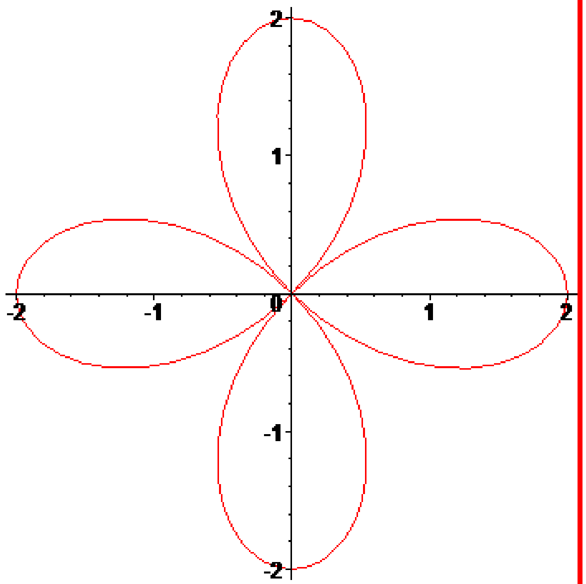
$$\begin{cases} r = a\cos(m\theta) \\ r = a\sin(m\theta) \end{cases}$$

a is a real number and m is a positive integer.

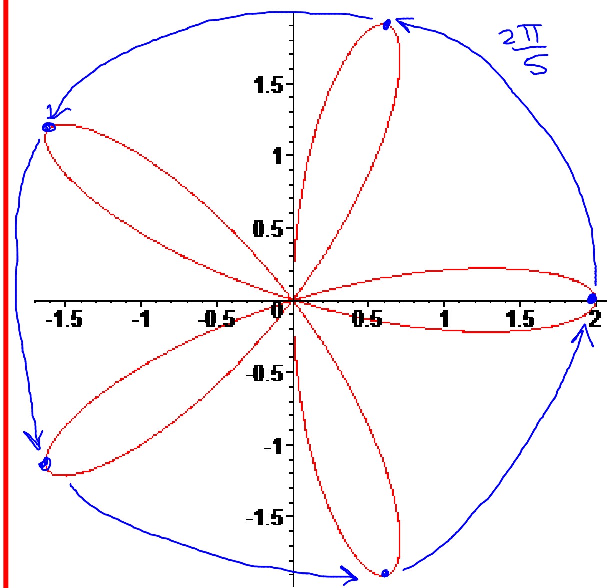
Fundamental Question: How do the values a and m effect the graph?

If $a \neq 0$, then this is a flower with $2m$ petals if m is even and m petals if m is odd. a magnifies, shrinks and reflects.

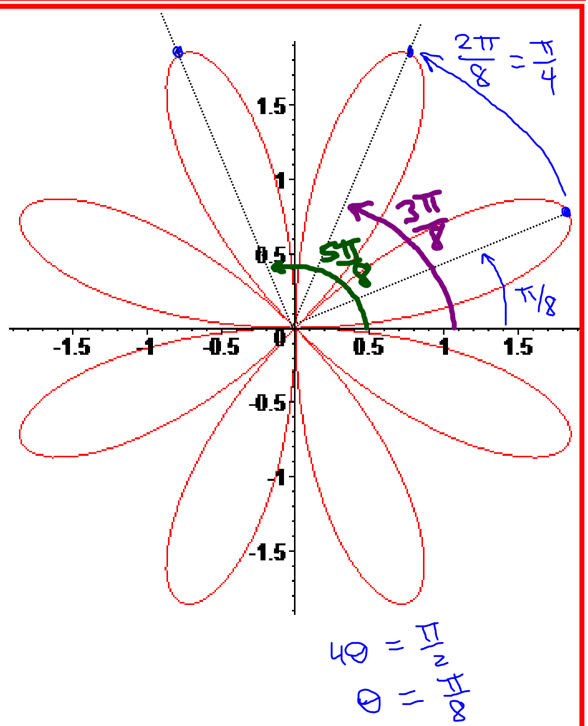
The tips of the petals on the flowers always lie on the circle of radius $|a|$. Also, the petals are evenly spaced around this circle.



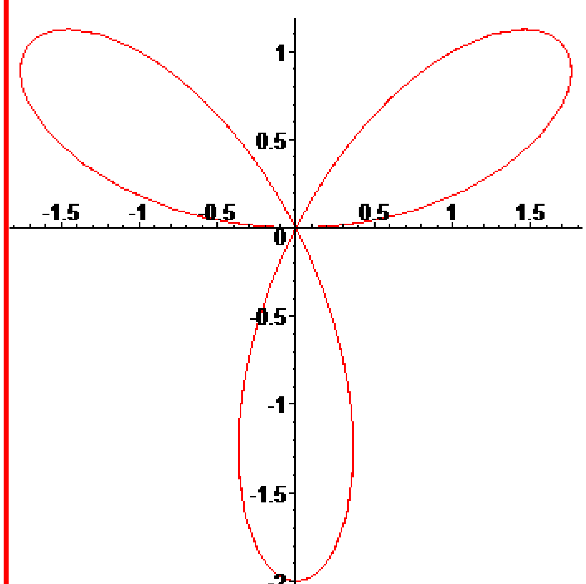
$$r = 2\cos(2\theta)$$



$$r = 2\cos(5\theta)$$



$$r = 2\sin(4\theta)$$



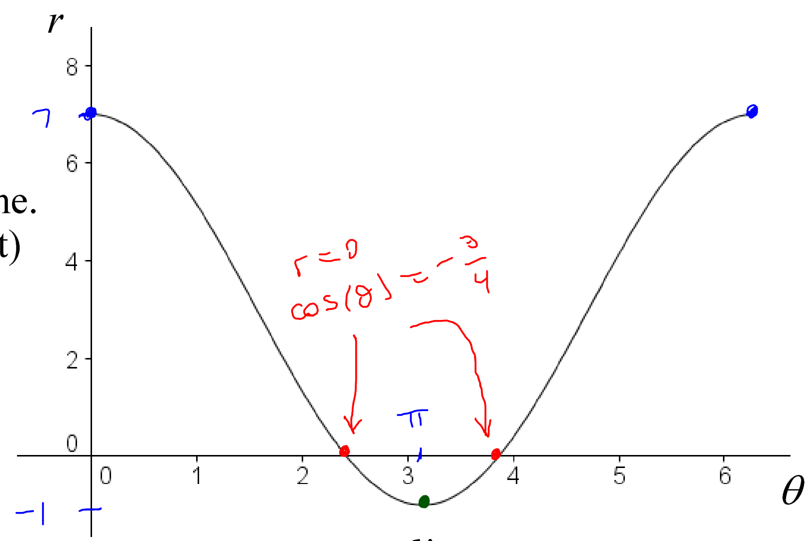
$$r = 2\sin(3\theta)$$

Example:

Plot the polar curve

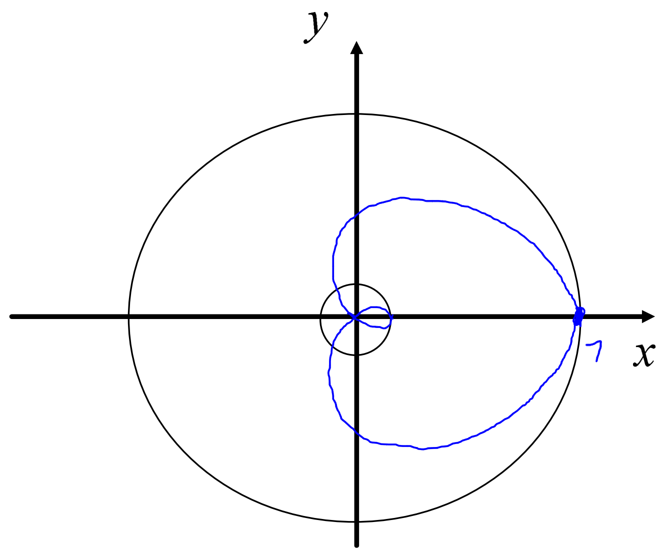
$$r = 3 + 4\cos(\theta)$$

Step 1
Create a plot in the θr plane.
(this is NOT the polar plot)

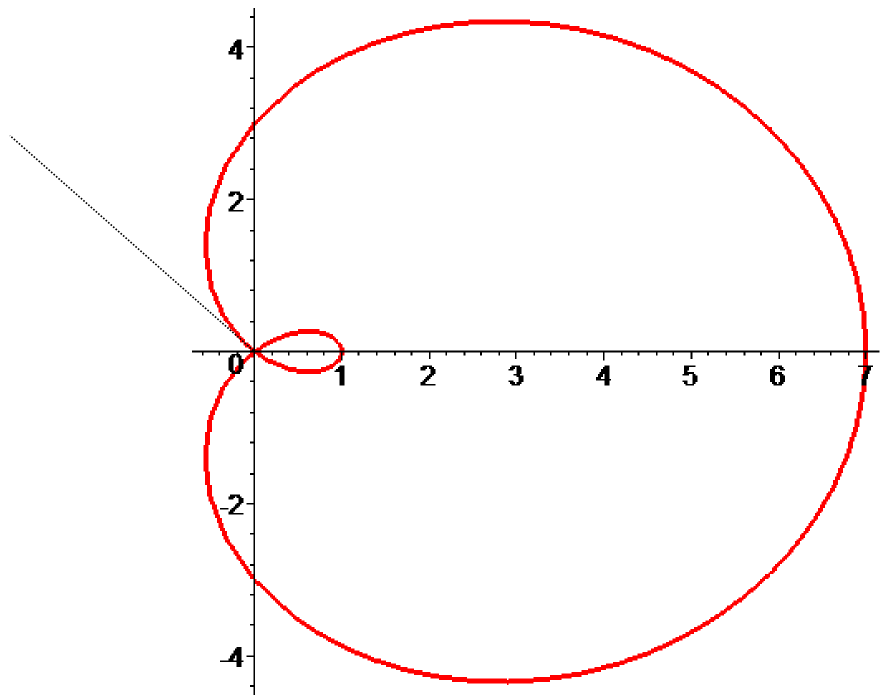


• ↔ r = 7
• ↔ r = -1
• ↔ r = 0

Step 2
Interpret the plot above to create the polar plot.



$$r = 3 + 4\cos(\theta)$$



Example: Geogebra Investigation

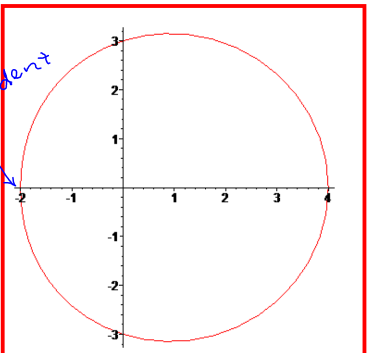
Investigate

$$r = 3 + b\cos(\theta)$$

See the link from the course homepage after class.

for b between 1 and 5.

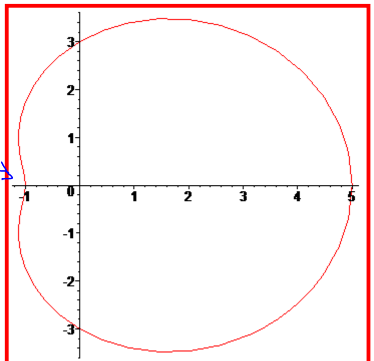
mini dent



$r = 3 + \cos(\theta)$

$|3| > |1|$

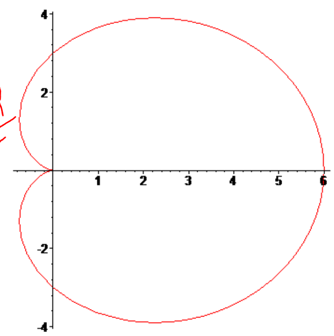
dent



$r = 3 + 2\cos(\theta)$

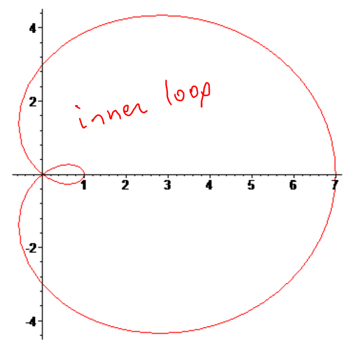
$|3| > |2|$

Cardioid



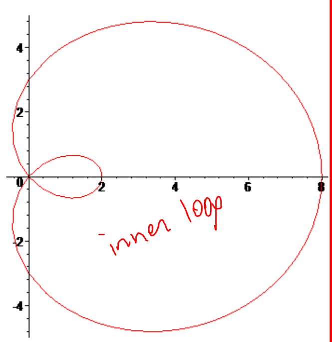
$r = 3 + 3\cos(\theta)$

$|3| = |3|$



$r = 3 + 4\cos(\theta)$

$|3| < |4|$



$r = 3 + 5\cos(\theta)$

$|3| < |5|$

You can create similar graphs.

Geogebra Exploration:

$|a| > |b|$ dent

$|a| = |b|$ cardioid

aligned on x-axis

$$r = a + b\cos(\theta)$$

and

$|a| < |b|$ inner loop

$$r = a + b\sin(\theta)$$

aligned on y-axis.

See the applets linked from the course homepage after class.

Polar curves of the form

$$r = a + b\cos(\theta)$$

and

$$r = a + b\sin(\theta)$$

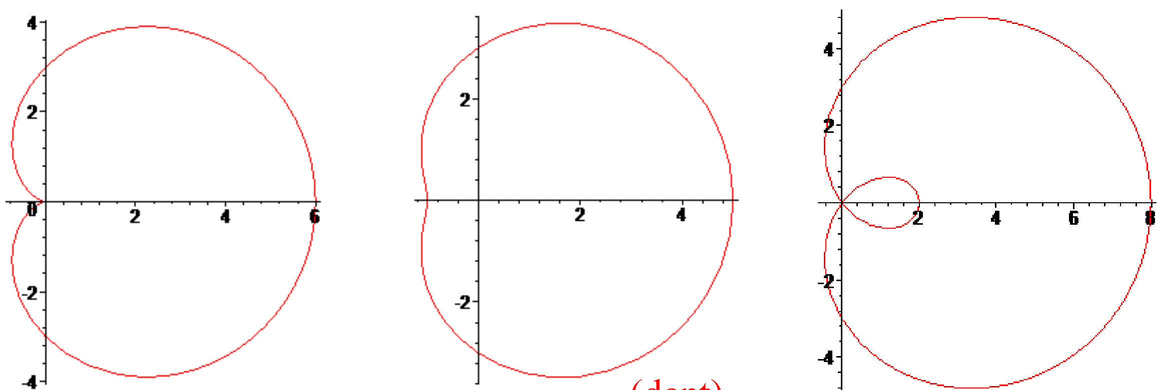
$|a| = |b|$ $\quad$ $|a| > |b|$ $\quad$ dents

are Cardioids, Limacons with dimples, and Limaçons with inner loops.

$|a| < |b|$

Look Below...

$$r = a + b\cos(\theta)$$



Cardioid
$|a|=|b|$

(dent)
Limacon with dimple
$|a|>|b|$

Limacon with inner loop
$|a|<|b|$

Note: The cosine versions can be reflected across the y axis if b is negative.