Info...

● New homework and EMCFs are posted.

● Video help is posted for selected problems in 9.4 and 9.5.

Review of Polar Coordinates:

$x = r\cos(\theta)$

$y = r\sin(\theta)$

Standard Representation for $r = \sqrt{x^2 + y^2}$

Standard Representation for $\theta = \arctan\left(\frac{y}{x}\right)$

for (x, y) in Q1 or Q4.

More Review:

Overview of Polar Graphs:

$r = -2\cos(\theta)$
circle of radius 1
centered at $(-1, 0)$

$r = \cos(3\theta)$ is a   3 petal flower

$r = \sin(4\theta)$ is a   8 petal flower

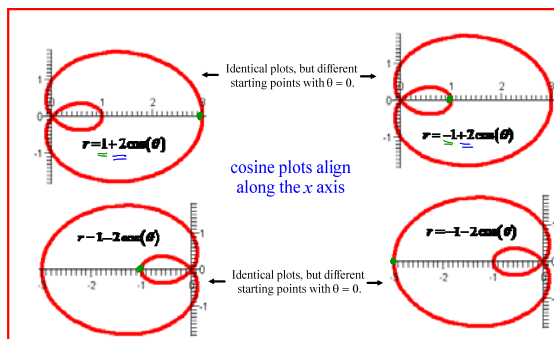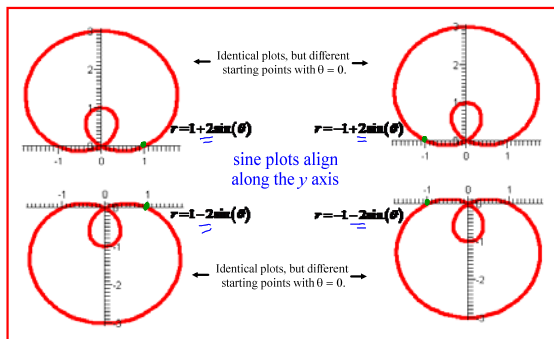$r = 3\cos(\theta)$ is a   circle of radius 3/2 centered at (3/2,0)

$r = 4\sin(\theta)$ is a   circle of radius 2 centered at (0,2)

$\left.\begin{array}{l} r = a + b\cos(\theta) \text{ is a} \\ r = a + b\sin(\theta) \text{ is a} \end{array}\right\}$ limaçon, with the actual shape and placement dependent on a and b.

$|a| = |b| \leftarrow$ cardioid

$|a| > |b| \leftarrow$ dimple/dent

$|a| < |b| \leftarrow$ inner loop

$$r = a + b\cos(\theta)$$
$$r = a + b\sin(\theta)$$
$$|a| < |b|$$

Some Limacons with Inner Loops

Identical plots, but different starting points with θ = 0.

$r=1+2\sin(\theta)$ $r=-1+2\sin(\theta)$

sine plots align along the y axis

$r=1-2\sin(\theta)$ $r=-1-2\sin(\theta)$

Identical plots, but different starting points with θ = 0.

Identical plots, but different starting points with θ = 0.

$r=1+2\cos(\theta)$ $r=-1+2\cos(\theta)$

cosine plots align along the x axis

$r=1-2\cos(\theta)$ $r=-1-2\cos(\theta)$

Identical plots, but different starting points with θ = 0.
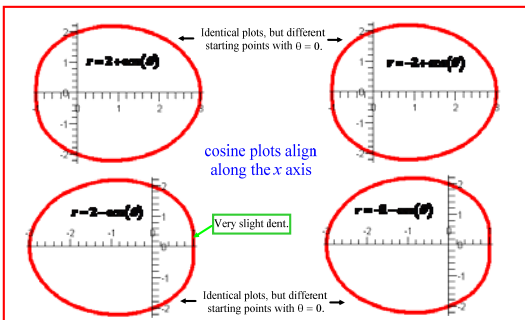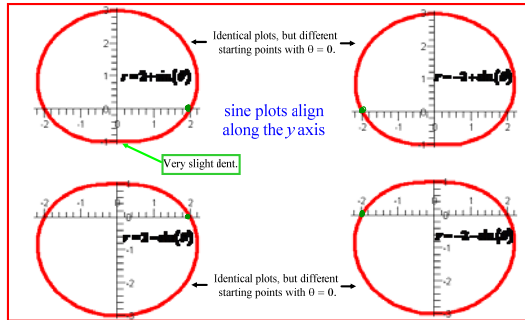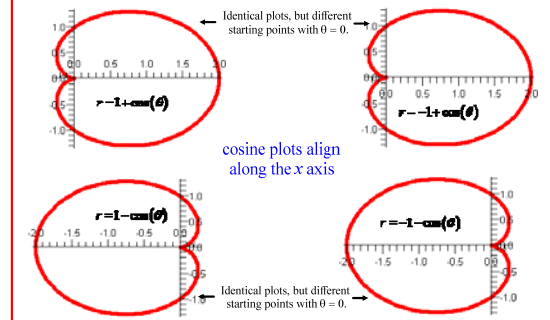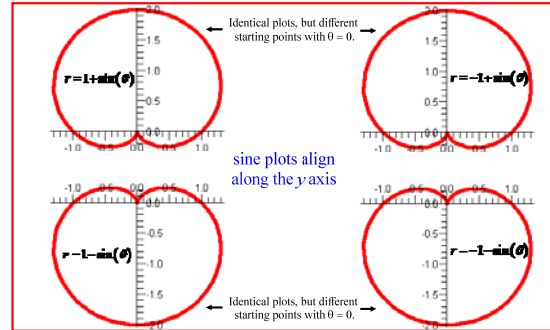
Popper 10

1. Give the x coordinate of the polar point $[\,2\,,\,2.71\,]$.
2. Give the y coordinate of the polar point $[\,2\,,\,2.71\,]$.

Area In Polar Coordinates

Our Goal: Find the area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b.

$a < b$

θ_n $r(\theta)$ Sector 3 Sector 2 Sector 1

$\theta_0 | \theta_1$ θ_2 θ_a

$\text{Area} = \text{Area}(|||||)$

$$= \sum_{i=1}^{n} \text{Area}(\text{sector } i)$$

$$\approx \sum_{i=1}^{n} \frac{1}{2} r(\theta_i)^2 \, \Delta\theta_i$$
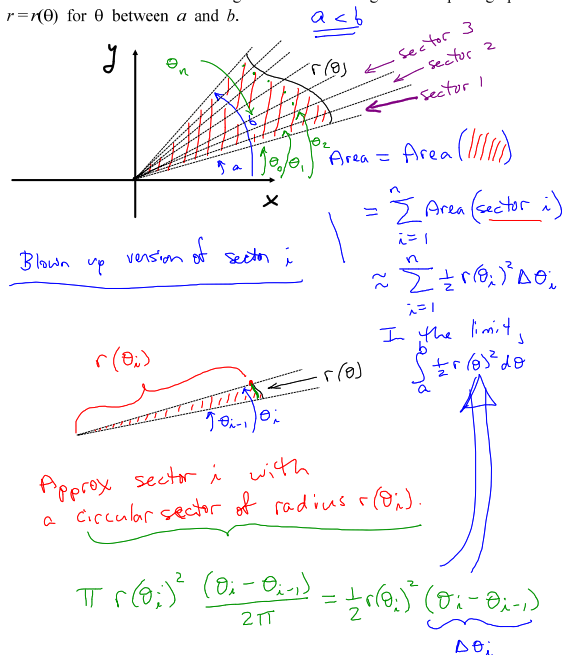
In the limit,
$$\int_a^b \frac{1}{2} r(\theta)^2 \, d\theta$$

Blown up version of sector i

$r(\theta_i)$ $r(\theta)$

$\theta_{i-1} | \theta_i$

Approx sector i with a circular sector of radius $r(\theta_i)$.

$$\pi \, r(\theta_i)^2 \, \frac{(\theta_i - \theta_{i-1})}{2\pi} = \frac{1}{2} r(\theta_i)^2 \, (\theta_i - \theta_{i-1})$$

$\Delta\theta_i$

Area Formula: The area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b is given by
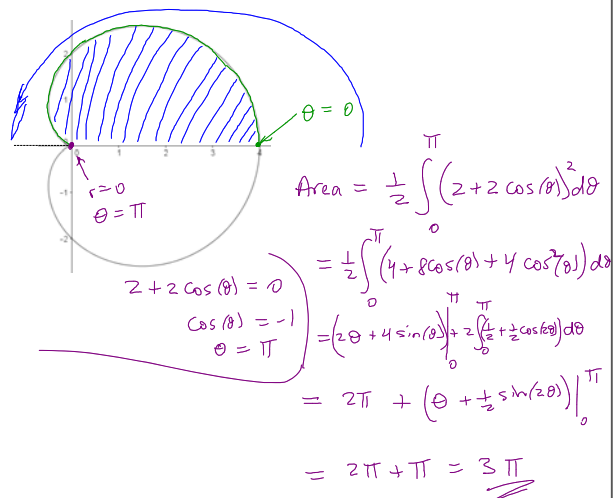
$$\frac{1}{2}\int_a^b (r(\theta))^2 \, d\theta$$

2

Example: Find the area inside one petal of the flower given by $r = \cos(4\theta)$.
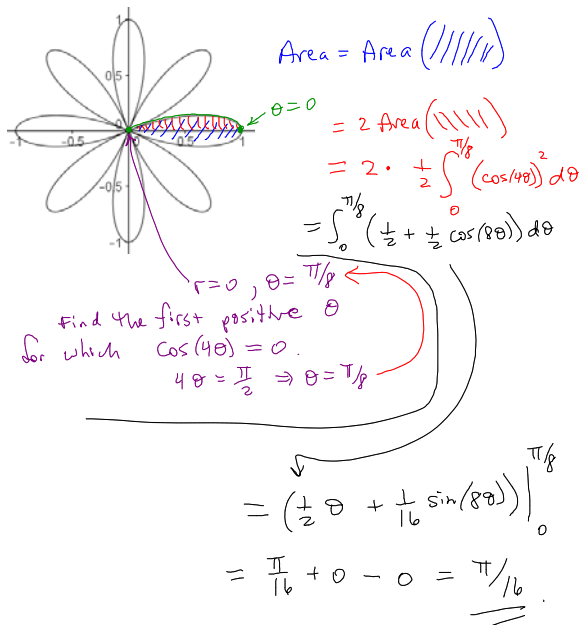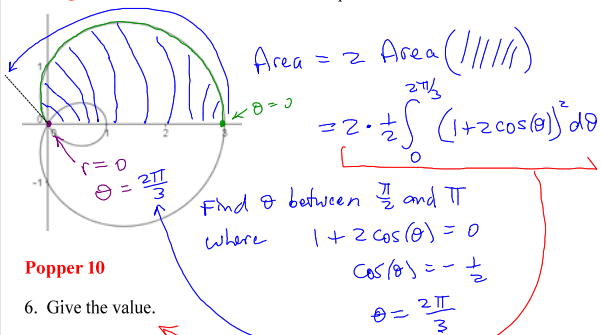
Area = Area $\left(/\!/\!/\!/\!/\!/\!/ \right)$

$\theta = 0$

$= 2$ Area $\left(\backslash\backslash\backslash\backslash\backslash \right)$

$= 2 \cdot \frac{1}{2} \int_0^{\pi/8} \left(\cos(4\theta) \right)^2 d\theta$

$= \int_0^{\pi/8} \left(\frac{1}{2} + \frac{1}{2} \cos(8\theta) \right) d\theta$

$r = 0, \theta = \pi/8$

Find the first positive θ for which $\cos(4\theta) = 0$.

$4\theta = \frac{\pi}{2} \Rightarrow \theta = \pi/8$

$= \left(\frac{1}{2}\theta + \frac{1}{16}\sin(8\theta) \right) \Big|_0^{\pi/8}$

$= \frac{\pi}{16} + 0 - 0 = \pi/16$.

Example: Find the area in the upper half of the cardioid $r = 2 + 2\cos(\theta)$.

$\theta = 0$

$r = 0$
$\theta = \pi$

$2 + 2\cos(\theta) = 0$
$\cos(\theta) = -1$
$\theta = \pi$

Area $= \frac{1}{2} \int_0^\pi \left(2 + 2\cos(\theta) \right)^2 d\theta$

$= \frac{1}{2} \int_0^\pi \left(4 + 8\cos(\theta) + 4\cos^2(\theta) \right) d\theta$

$= \left(2\theta + 4\sin(\theta) \right) \Big|_0^\pi + 2 \int_0^\pi \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta) \right) d\theta$

$= 2\pi + \left(\theta + \frac{1}{2}\sin(2\theta) \right) \Big|_0^\pi$

$= 2\pi + \pi = 3\pi$

Example: Find the area inside the outer loop of $r = 1 + 2\cos(\theta)$.

Area $= 2$ Area $\left(/\!/\!/\!/\!/\!/ \right)$

$\theta = 0$

$= 2 \cdot \frac{1}{2} \int_0^{2\pi/3} \left(1 + 2\cos(\theta) \right)^2 d\theta$

$r = 0$
$\theta = \frac{2\pi}{3}$

Find θ between $\frac{\pi}{2}$ and π where $1 + 2\cos(\theta) = 0$

$\cos(\theta) = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$