Info...

- New homework and EMCFs are posted.

- Video help is posted for selected problems in 9.4 and 9.5.

Review of Polar Coordinates:

$x =$ $r\cos(\theta)$

$y =$ $r\sin(\theta)$

Standard Representation for $r =$ $\sqrt{x^2+y^2}$

Standard Representation for $\theta =$ $\arctan\left(\frac{y}{x}\right)$

for (x, y) in Q1 or Q4.

More Review:

Overview of Polar Graphs:

$r = -2\cos(\theta)$
circle of radius 1
centered at (−1, 0)

$r = \cos(3\theta)$ is a — 3 petal flower

$r = \sin(4\theta)$ is a — 8 petal flower

$r = 3\cos(\theta)$ is a — circle of radius 3/2 centered at (3/2,0)

$r = 4\sin(\theta)$ is a — circle of radius 2 centered at (0,2)

$r = a + b\cos(\theta)$ is a
$r = a + b\sin(\theta)$ is a

limaçon, with the actual shape and placement dependent on *a* and *b*.

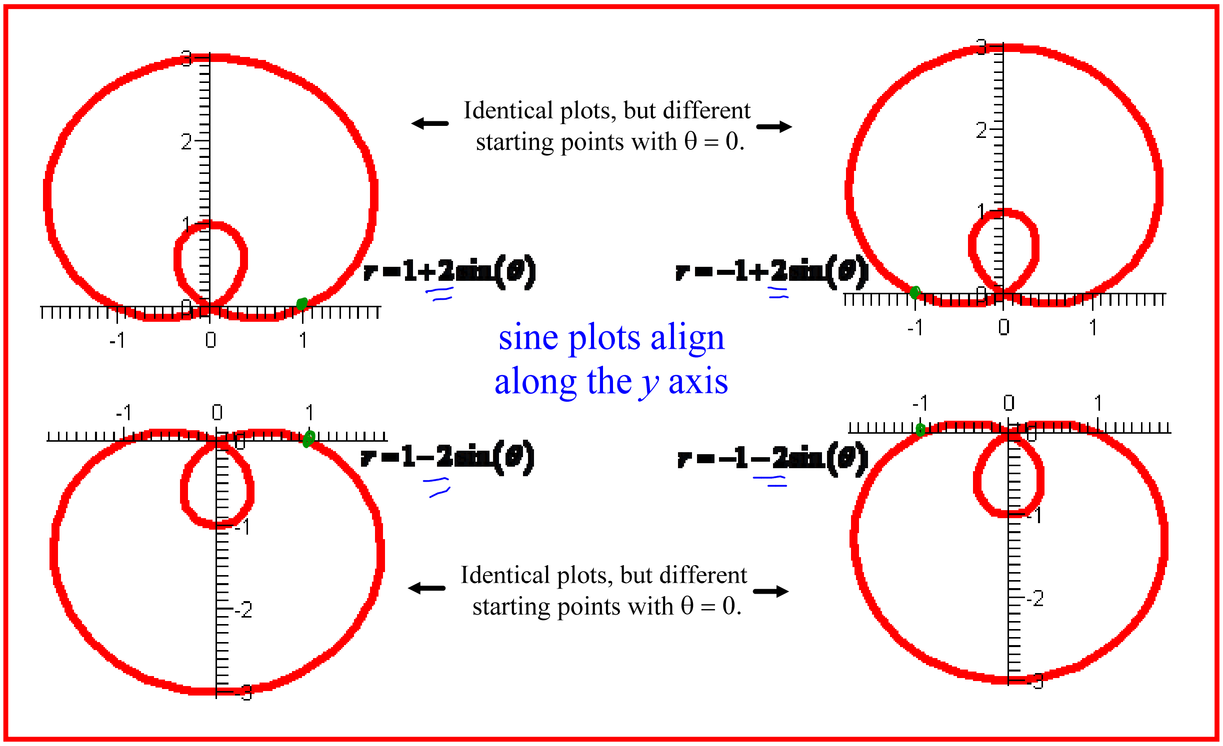
$|a| = |b|$ ← cardioid

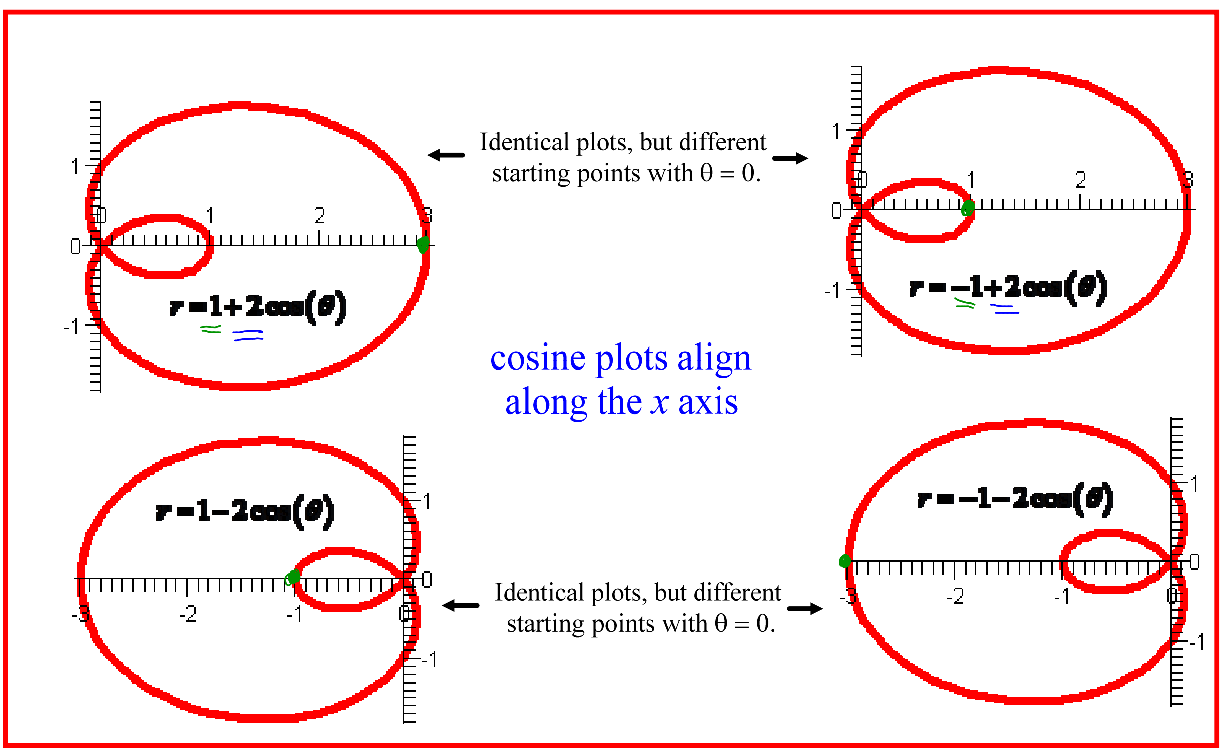
$|a| > |b|$ ← dimple/dent

$|a| < |b|$ ← inner loop

$$r = a + b\cos(\theta) \qquad |a| < |b|$$
$$r = a + b\sin(\theta)$$

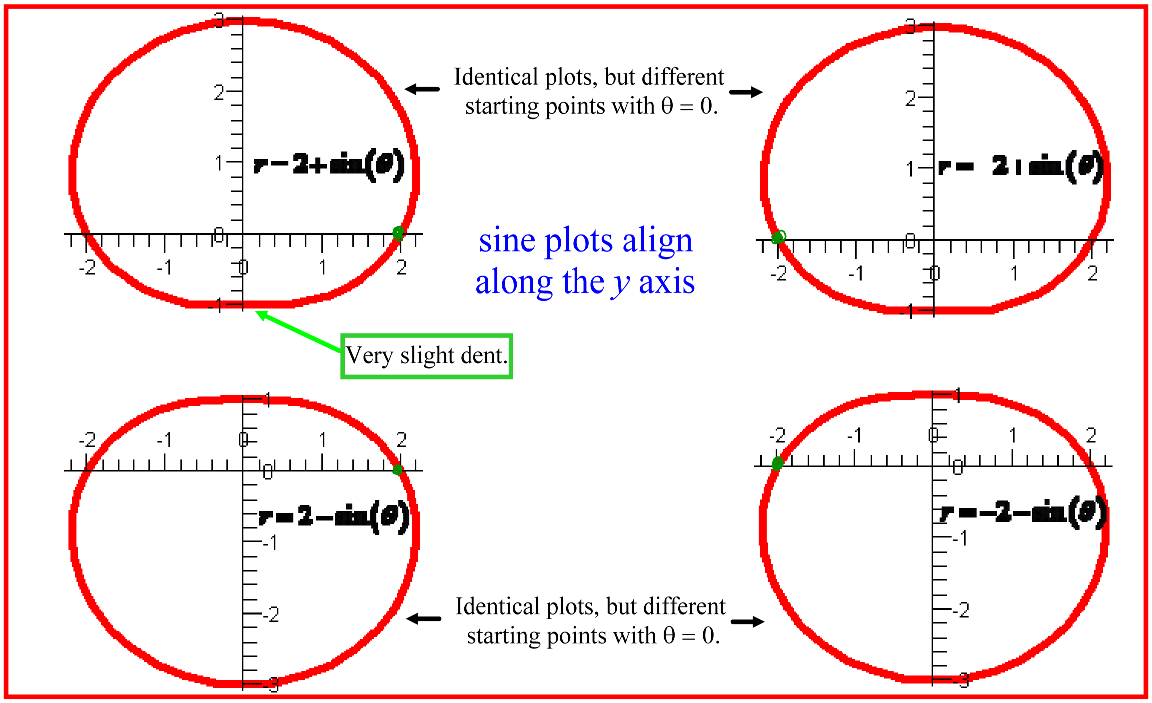
Some Limacons with Inner Loops

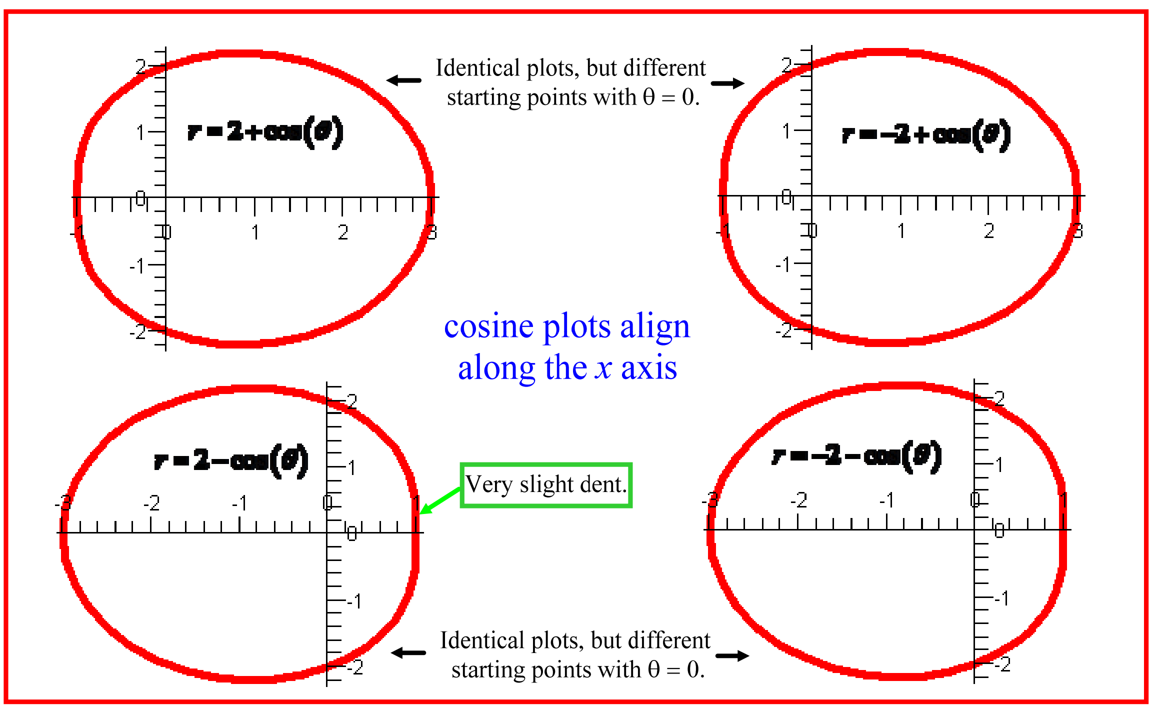




$$r = a + b\cos(\theta) \qquad r = a + b\sin(\theta) \qquad |a| > |b|$$

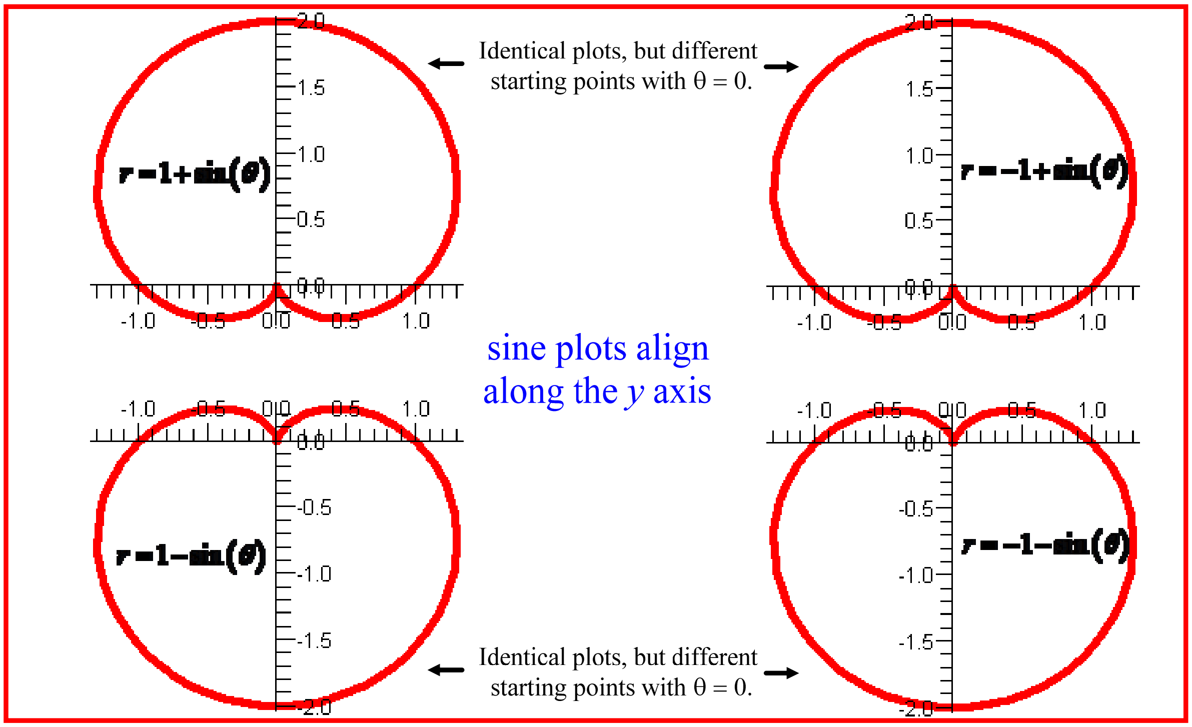
Some Limacons with Dimples (Dents)

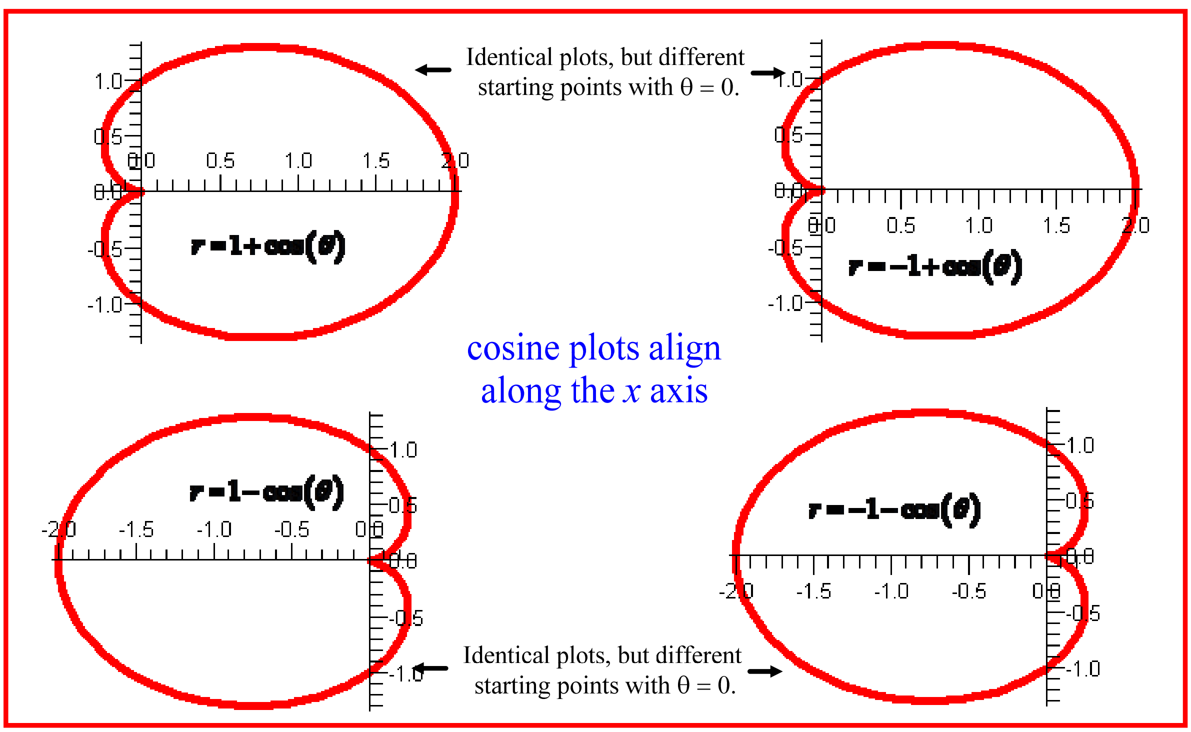




$$r = a + b\cos(\theta)$$
$$r = a + b\sin(\theta)$$
$$|a| = |b|$$

Some Cardioids



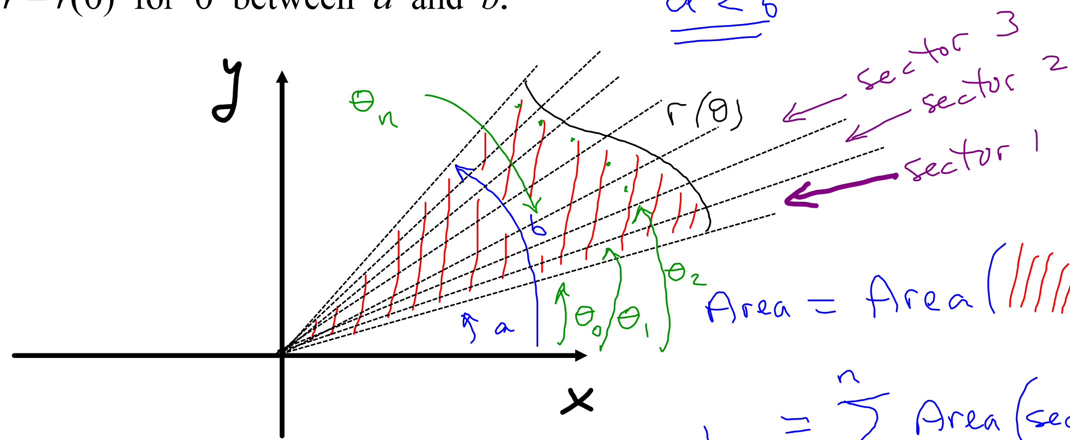


Popper 10

1. Give the x coordinate of the polar point [2 , 2.71].
2. Give the y coordinate of the polar point [2 , 2.71].

Area In Polar Coordinates

Our Goal: Find the area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b. $a < b$



$$\text{Area} = \text{Area}(\text{shaded region})$$

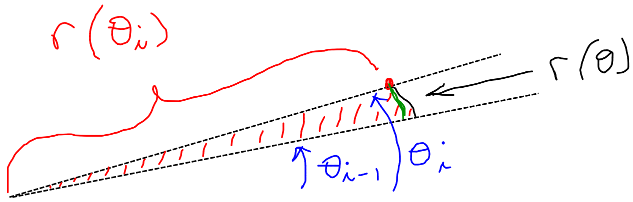
$$= \sum_{i=1}^{n} \text{Area}(\text{sector } i)$$

$$\approx \sum_{i=1}^{n} \frac{1}{2} r(\theta_i)^2 \Delta\theta_i$$

In the limit,

$$\int_a^b \frac{1}{2} r(\theta)^2 \, d\theta$$

Blown up version of sector i



Approx sector i with a circular sector of radius $r(\theta_i)$.

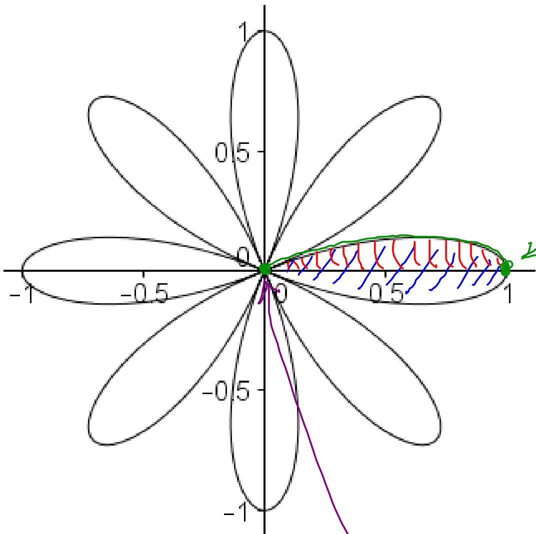
$$\pi r(\theta_i)^2 \frac{(\theta_i - \theta_{i-1})}{2\pi} = \frac{1}{2} r(\theta_i)^2 (\theta_i - \theta_{i-1})$$

$(\theta_i - \theta_{i-1}) = \Delta\theta_i$

Area Formula: The area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b is given by

$$\frac{1}{2}\int_a^b (r(\theta))^2 \, d\theta$$

Example: Find the area inside one petal of the flower given by $r = \cos(4\theta)$.



$\theta = 0$

Area = Area(////)

$= 2\,$Area(\\\\\\)

$$= 2 \cdot \frac{1}{2} \int_0^{\pi/8} (\cos(4\theta))^2 \, d\theta$$

$$= \int_0^{\pi/8} \left(\frac{1}{2} + \frac{1}{2}\cos(8\theta) \right) d\theta$$

$r = 0$, $\theta = \pi/8$

Find the first positive θ for which $\cos(4\theta) = 0$.

$$4\theta = \frac{\pi}{2} \Rightarrow \theta = \pi/8$$

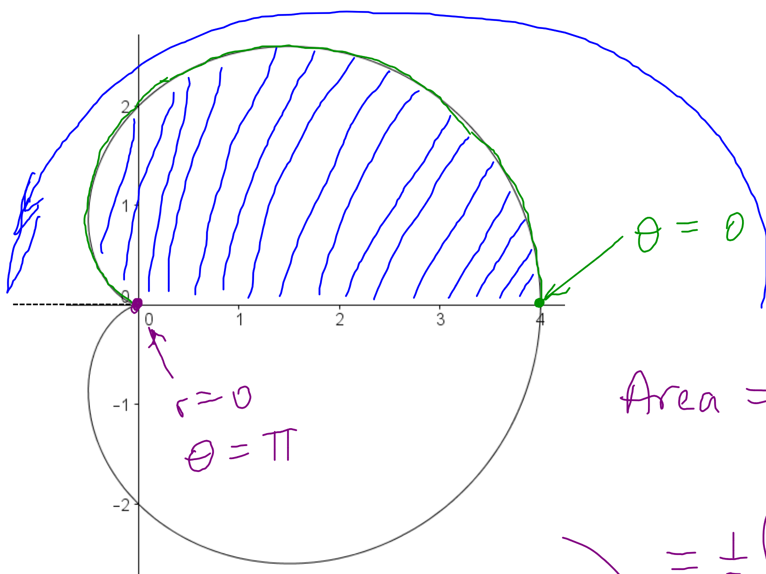
$$= \left(\frac{1}{2}\theta + \frac{1}{16}\sin(8\theta) \right) \Big|_0^{\pi/8}$$

$$= \frac{\pi}{16} + 0 - 0 = \pi/16.$$

Popper 10

3. Give the number of petals in the polar flower $r = 2\sin(3\theta)$.
4. Give the number of petals in the polar flower $r = 3\cos(2\theta)$.
5. Give the positive value of a so that the polar graph of $r = -2 + a\cos(\theta)$ is a cardioid.

Example: Find the area in the upper half of the cardioid $r = 2 + 2\cos(\theta)$.



$$\text{Area} = \frac{1}{2}\int_0^{\pi} (2 + 2\cos(\theta))^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \left(4 + 8\cos(\theta) + 4\cos^2(\theta)\right) d\theta$$

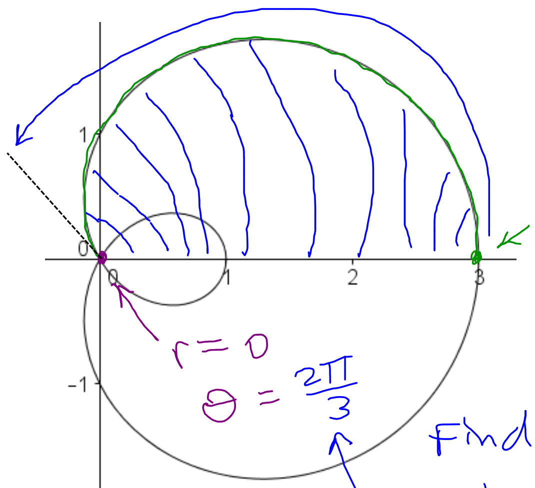
$$= \left(2\theta + 4\sin(\theta)\right)\Big|_0^{\pi} + 2\int_0^{\pi}\left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right) d\theta$$

$$= 2\pi + \left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi}$$

$$= 2\pi + \pi = 3\pi$$

$$2 + 2\cos(\theta) = 0$$
$$\cos(\theta) = -1$$
$$\theta = \pi$$

Example: Find the area inside the outer loop of $r = 1 + 2\cos(\theta)$.



$$\text{Area} = 2\,\text{Area}(\text{////})$$

$$= 2 \cdot \frac{1}{2}\int_0^{2\pi/3} \left(1+2\cos(\theta)\right)^2 d\theta$$

Find θ between $\frac{\pi}{2}$ and π where $1 + 2\cos(\theta) = 0$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

Popper 10

6. Give the value.