Info...

- New homework and EMCFs are posted.

- Video help is posted for selected problems in 9.4 and 9.5.
Review of Polar Coordinates:

\[ x = r \cos(\theta) \]

\[ y = r \sin(\theta) \]

Standard Representation for \( r = \sqrt{x^2 + y^2} \)

Standard Representation for \( \theta = \arctan \left( \frac{y}{x} \right) \)

for \((x, y)\) in \(Q_1\) or \(Q_4\).
More Review:

Overview of Polar Graphs:

\[ r = \cos(3\theta) \] is a \( 3 \) petal flower

\[ r = \sin(4\theta) \] is a \( 8 \) petal flower

\[ r = 3\cos(\theta) \] is a circle of radius 3/2 centered at (3/2,0)

\[ r = 4\sin(\theta) \] is a circle of radius 2 centered at (0,2)

\[ r = a + b\cos(\theta) \] is a limaçon, with the actual shape and placement dependent on \( a \) and \( b \).

\[ |a| = |b| \] \( \leftarrow \) cardioid

\[ |a| > |b| \] \( \leftarrow \) dimple/dent

\[ |a| < |b| \] \( \leftarrow \) inner loop

\[ r = -2\cos(\theta) \] circle of radius 1 centered at \((-1, 0)\)
\[ r = a + b \cos(\theta) \quad |a| < |b| \]

\[ r = a + b \sin(\theta) \]

Some Limacons with Inner Loops

- Identical plots, but different starting points with \( \theta = 0 \).

- \( r = 1 + 2 \sin(\theta) \) and \( r = -1 + 2 \sin(\theta) \) have sine plots that align along the \( y \) axis.

- \( r = 1 - 2 \sin(\theta) \) and \( r = -1 - 2 \sin(\theta) \) have identical plots with different starting points.

- \( r = 1 + 2 \cos(\theta) \) and \( r = -1 + 2 \cos(\theta) \) have cosine plots aligning along the \( x \) axis.

- \( r = 1 - 2 \cos(\theta) \) and \( r = -1 - 2 \cos(\theta) \) have identical plots with different starting points.
\[ r = a + b \cos(\theta) \]
\[ r = a + b \sin(\theta) \quad |a| > |b| \]

Some Limacons with Dimples (Dents)

Identical plots, but different starting points with \( \theta = 0 \).

sine plots align along the y axis

Very slight dent.

Identical plots, but different starting points with \( \theta = 0 \).

cosine plots align along the x axis

Very slight dent.

Identical plots, but different starting points with \( \theta = 0 \).
Some Cardioids

- \( r = a + b \cos(\theta) \)  
  \(|a| = |b|\)

- \( r = a + b \sin(\theta) \)

Sine plots align along the y-axis.

Cosine plots align along the x-axis.

Identical plots, but different starting points with \( \theta = 0 \).
Popper 10

1. Give the $x$ coordinate of the polar point $[2, 2.71]$.
2. Give the $y$ coordinate of the polar point $[2, 2.71]$.

**Area In Polar Coordinates**

Our Goal: Find the area of the region between the origin and the polar graph of $r = r(\theta)$ for $\theta$ between $a$ and $b$.

\[
\begin{align*}
\text{Area} &= \sum_{i=1}^{n} \text{Area (sector } i) \\
&= \sum_{i=1}^{n} \frac{1}{2} r(\theta_i)^2 \Delta \theta_i \\
&= \int_{a}^{b} \frac{1}{2} r(\theta)^2 d\theta
\end{align*}
\]
**Area Formula:** The area of the region between the origin and the polar graph of $r = r(\theta)$ for $\theta$ between $a$ and $b$ is given by

$$\frac{1}{2} \int_a^b (r(\theta))^2 \, d\theta$$
Example: Find the area inside one petal of the flower given by $r = \cos(4\theta)$.

$$\text{Area} = 2 \int_0^{\frac{\pi}{8}} \frac{1}{2} (\cos(4\theta))^2 \, d\theta$$

Find the first positive $\theta$ for which $\cos(4\theta) = 0$.

$4 \theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{8}$

$$= \left[ \frac{1}{16} + \frac{1}{16} \sin(8\theta) \right]_0^{\frac{\pi}{8}}$$

$$= \frac{\pi}{16} + 0 - 0 = \frac{\pi}{16}.$$
Popper 10

3. Give the number of petals in the polar flower \( r = 2 \sin(3\theta) \).
4. Give the number of petals in the polar flower \( r = 3 \cos(2\theta) \).
5. Give the positive value of \( \alpha \) so that the polar graph of \( r = -2 + \alpha \cos(\theta) \) is a cardioid.

Example: Find the area in the upper half of the cardioid \( r = 2 + 2\cos(\theta) \).

\[
\text{Area} = \frac{1}{2} \int_{0}^{\pi} (2 + 2\cos(\theta))^2 \, d\theta
\]

\[
= \frac{1}{2} \int_{0}^{\pi} (4 + 8\cos(\theta) + 4\cos^2(\theta)) \, d\theta
\]

\[
= \left[ 2\theta + 4\sin(\theta) + 2\left( \frac{\theta}{2} + \frac{\sin(2\theta)}{4} \right) \right]_{0}^{\pi}
\]

\[
= 2\pi + \left( \pi + \frac{\pi}{2} \right) - 0
\]

\[
= 2\pi + \frac{3\pi}{2}
\]

\[
= 3\pi
\]
**Example:** Find the area inside the outer loop of \( r = 1 + 2\cos(\theta) \).

\[
\text{Area} = 2 \cdot \frac{1}{2} \int_0^{\frac{2\pi}{3}} \left(1 + 2\cos(\theta)\right)^2 \, d\theta
\]

Find \( \theta \) between \( \frac{\pi}{2} \) and \( \pi \)

where \( 1 + 2\cos(\theta) = 0 \)

\[\cos(\theta) = -\frac{1}{2}\]

\[\theta = \frac{2\pi}{3}\]

**Popper 10**

6. Give the value.