Info...

- New homework and EMCFs are posted.

- Video help is posted for selected problems in 9.4 and 9.5.

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Review of Polar Coordinates:

$x =$ $r\cos(\theta)$

$y =$ $r\sin(\theta)$

Standard Representation for $r =$ $\sqrt{x^2+y^2}$

Standard Representation for $\theta =$ $\arctan\left(\frac{y}{x}\right)$
(x,y) in Q1 or Q4.

More Review:

Overview of Polar Graphs:

$r = \cos(3\theta)$ is a — 3 petal flower

$r = \sin(4\theta)$ is a — 8 petal flower

$r = 3\cos(\theta)$ is a — circle of radius 3/2 centered at (3/2,0)

$r = 4\sin(\theta)$ is a — circle of radius 2 centered at (0,2)

$r = a + b\cos(\theta)$ is a
$r = a + b\sin(\theta)$ is a

limacon, with the actual shape and placement dependent on *a* and *b*.

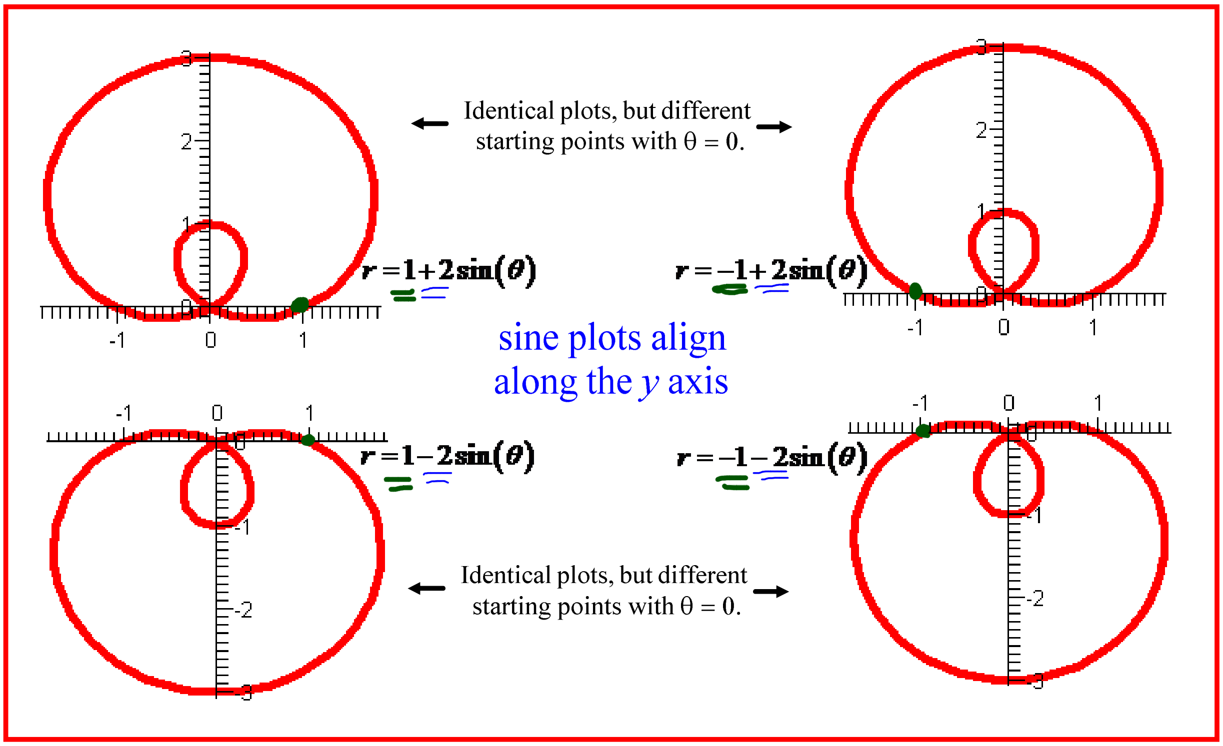
$|a| = |b| \longleftrightarrow$ cardioid

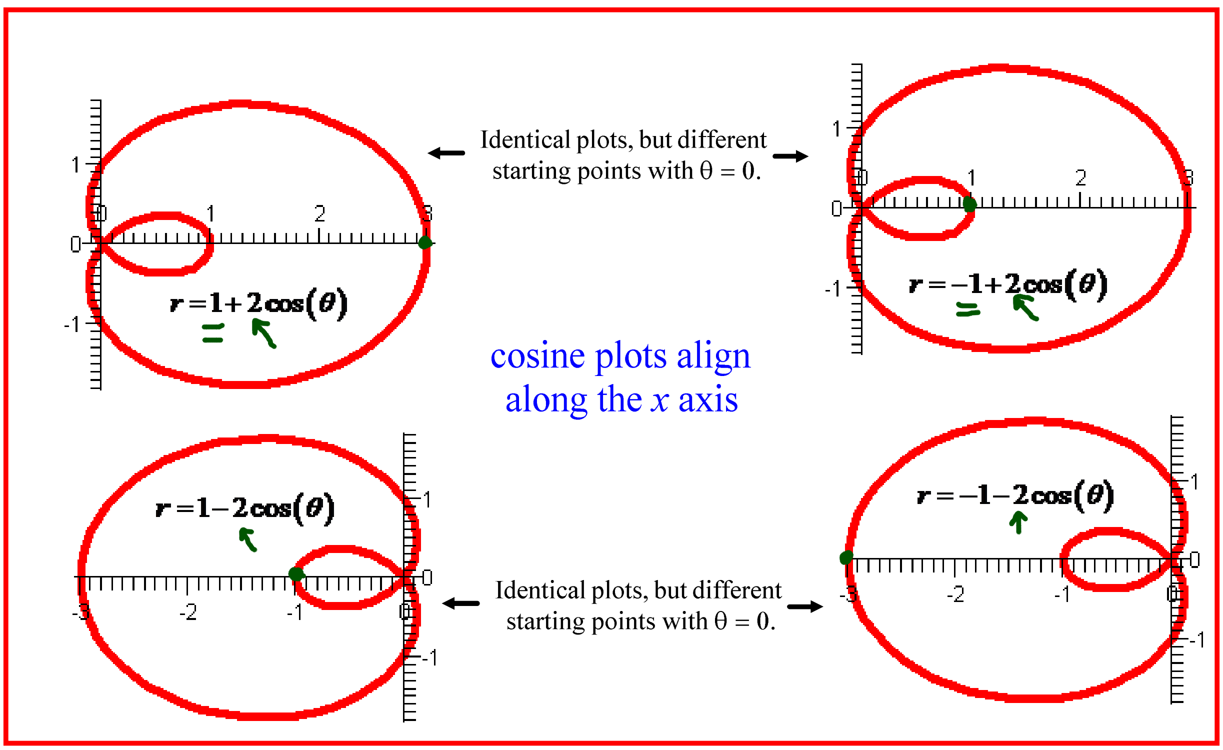
$|a| > |b| \longleftrightarrow$ dent / dimple

$|a| < |b| \longleftrightarrow$ inner loop

$$r = a + b\cos(\theta) \qquad |a| < |b|$$
$$r = a + b\sin(\theta)$$

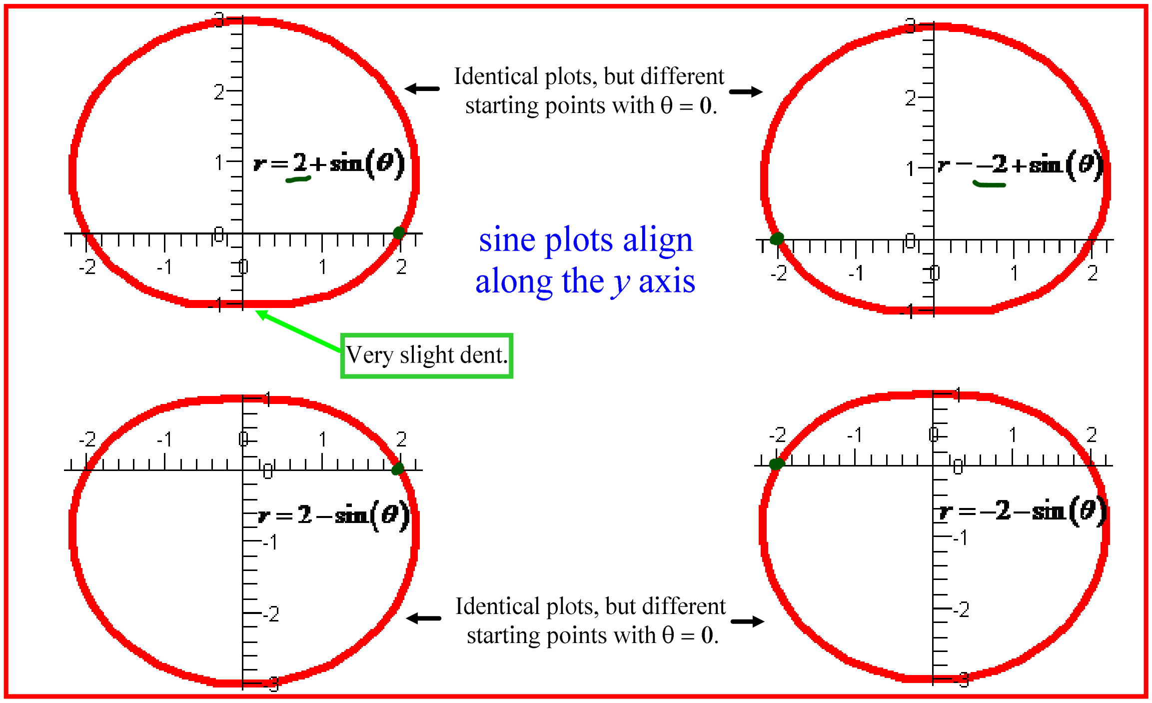
Some Limacons with Inner Loops

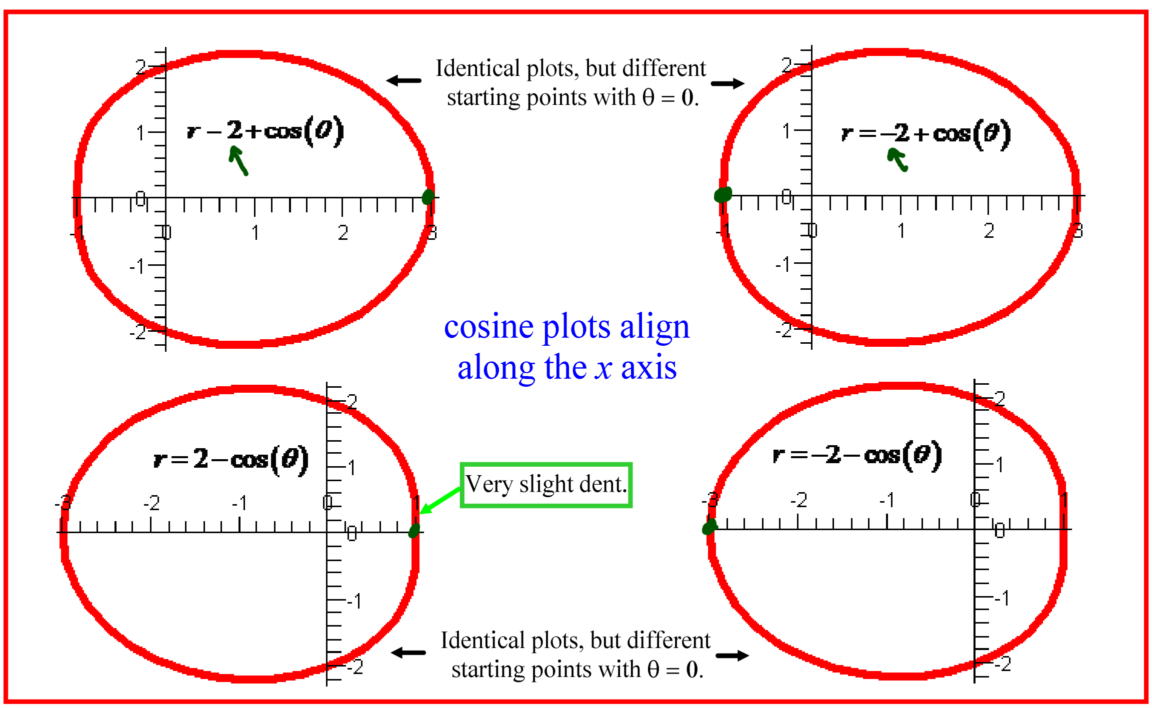




$$r = a + b\cos(\theta)$$
$$r = a + b\sin(\theta)$$
$$|a| > |b|$$

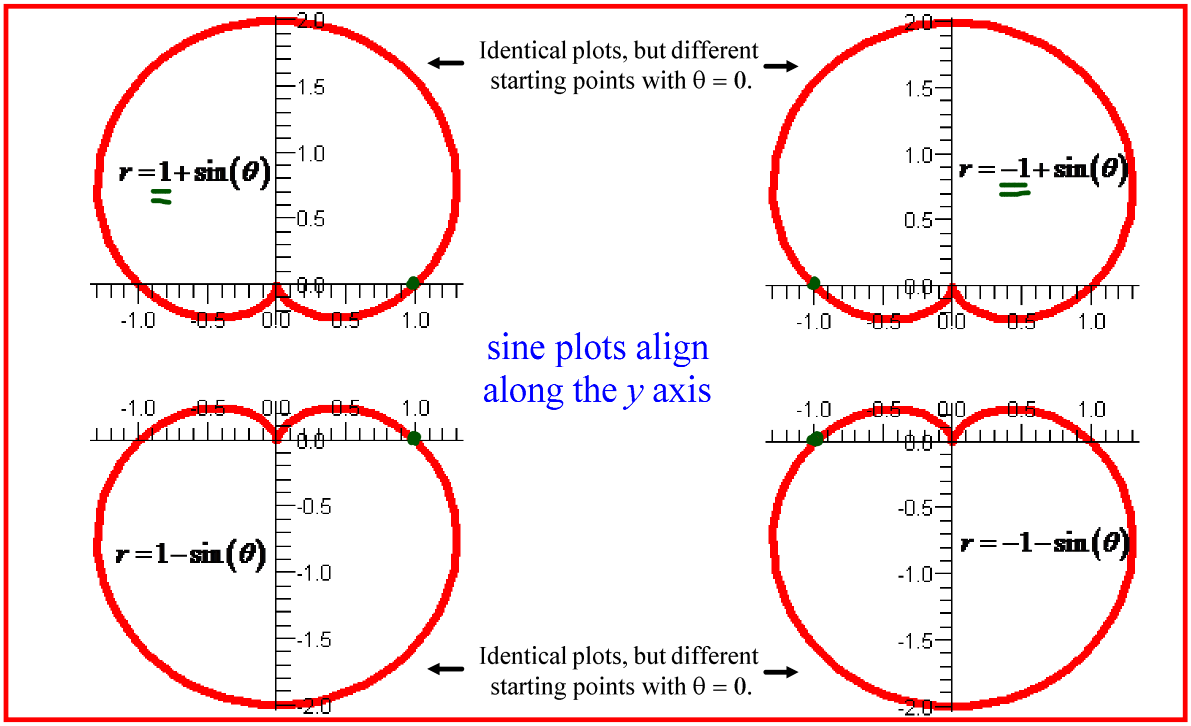
Some Limacons with Dimples (Dents)

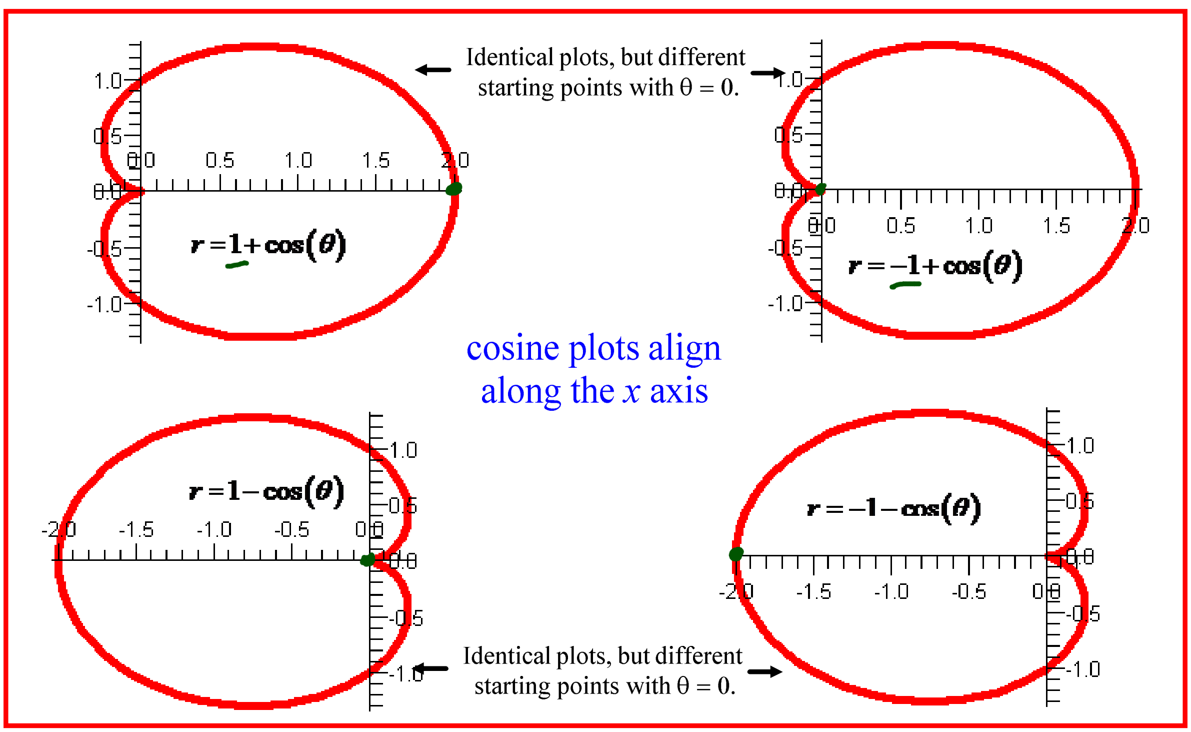




$$\boxed{\begin{array}{l} r = a + b\cos(\theta) \\ r = a + b\sin(\theta) \end{array} \quad |a| = |b|}$$

Some Cardioids

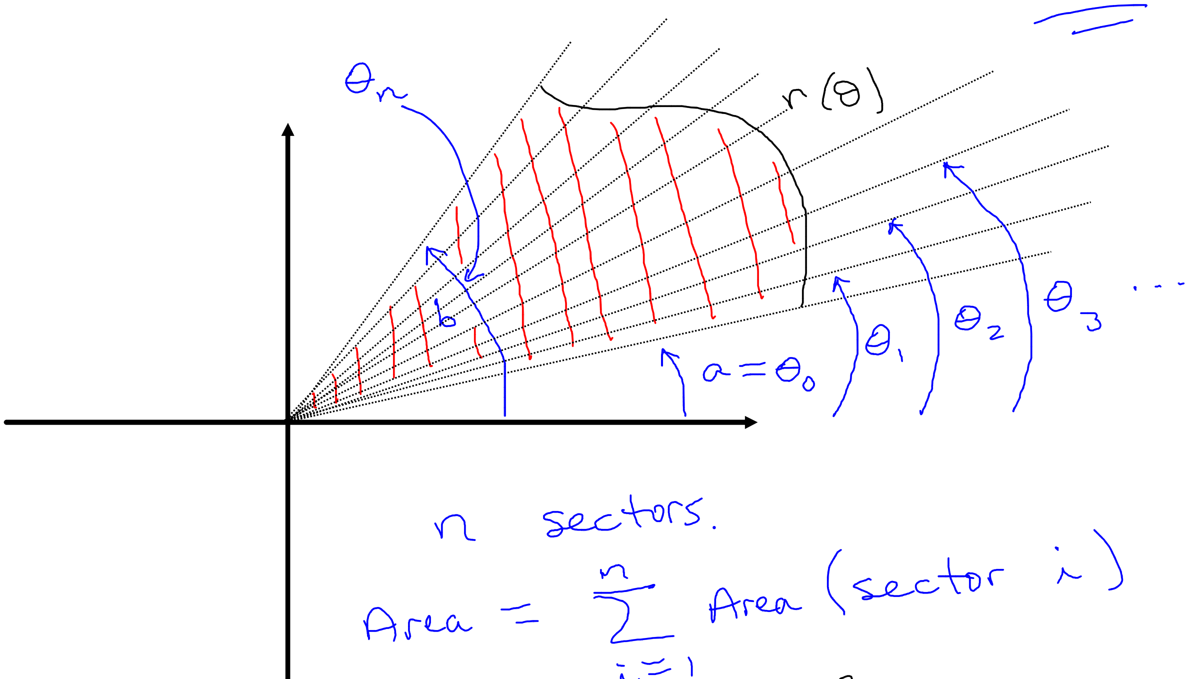




Area In Polar Coordinates

Our Goal: Find the area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b.

$\underline{\underline{a < b}}$

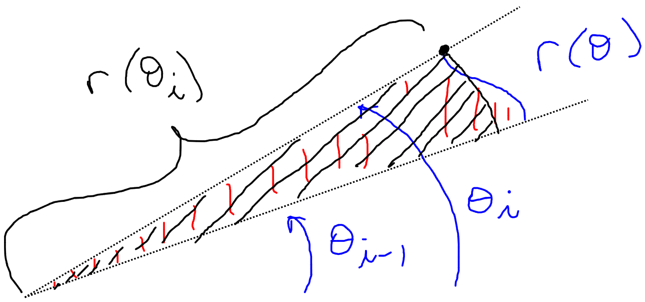


n sectors.

$$\text{Area} = \sum_{i=1}^{n} \text{Area}(\text{sector } i)$$

$$\approx \underbrace{\sum_{i=1}^{n} \tfrac{1}{2} r(\theta_i)^2 (\underbrace{\theta_i - \theta_{i-1}}_{\Delta\theta_i})}_{\text{R.S. for } \int_a^b \frac{1}{2} r(\theta)^2 \, d\theta}$$

Blown-up i^{th} sector



$$\text{Area}(\text{sector } i) \approx \text{Area}(\text{circular sector})$$

$$= \pi \, r(\theta_i)^2 \, \frac{\theta_i - \theta_{i-1}}{2\pi}$$

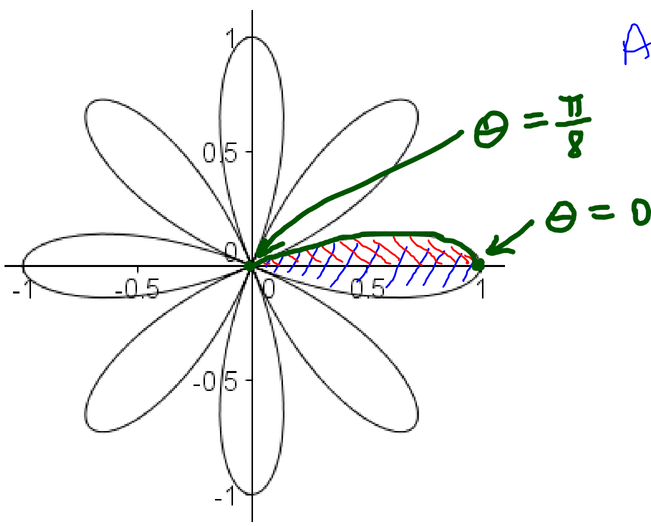
$$= \frac{1}{2} r(\theta_i)^2 (\theta_i - \theta_{i-1})$$

Area Formula: The area of the region between the origin and the polar graph of $r = r(\theta)$ for θ between a and b is given by

$\underline{\underline{a < b}}$

$$\frac{1}{2}\int_a^b r(\theta)^2 \, d\theta$$

Example: Find the area inside one petal of the flower given by $r = \cos(4\theta)$.



$$\text{Area} = \frac{1}{2}\int_a^b r(\theta)^2\, d\theta$$

$$= \text{Area}(|||||)$$

$$= 2\,\text{Area}(\backslash\backslash\backslash\backslash)$$

$$= 2 \cdot \frac{1}{2}\int_0^{\pi/8} \cos^2(4\theta)\, d\theta$$

$$= \int_0^{\pi/8} \left(\frac{1}{2} + \frac{1}{2}\cos(8\theta)\right) d\theta$$

$$= \left(\frac{1}{2}\theta + \frac{1}{16}\sin(8\theta)\right)\Bigg|_0^{\pi/8}$$

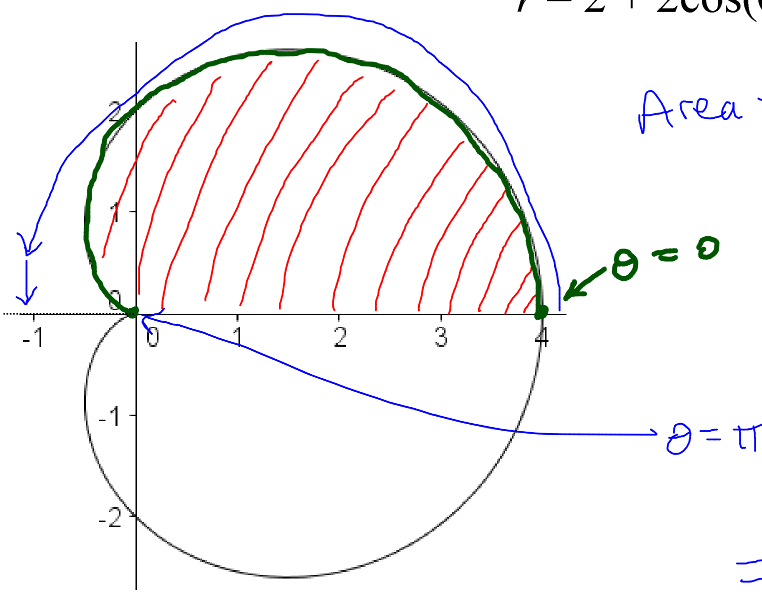
$$= \frac{\pi}{16} + 0 - 0 = \pi/16.$$

$$4\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{8}$$

Example: Find the area in the upper half of the cardioid

$$r = 2 + 2\cos(\theta)$$



$$\text{Area} = \frac{1}{2}\int_a^b r(\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \left(2 + 2\cos(\theta)\right)^2 d\theta$$

$2 + 2\cos(\theta) = 0$

$\cos(\theta) = -1$

$$= \frac{1}{2}\int_0^{\pi} \left(4 + 8\cos(\theta) + 4\cos^2(\theta)\right) d\theta$$

$$= \left(2\theta + 4\sin(\theta)\right)\Big|_0^{\pi} + 2\int_0^{\pi} \cos^2(\theta)\, d\theta$$

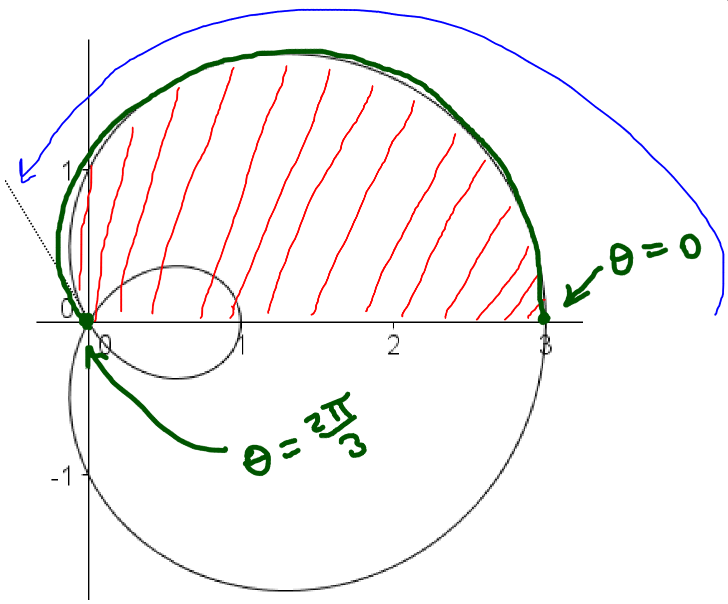
$$= 2\pi - 0 + 2\int_0^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos(2\theta)\right) d\theta$$

$$= 2\pi + \left(\theta + \frac{1}{2}\sin(2\theta)\right)\Big|_0^{\pi}$$

$$= 2\pi + \pi - 0 = \underline{\underline{3\pi}}$$

Example: Find the area inside the outer loop of

$$r = 1 + 2\cos(\theta)$$



$$\text{Area} = \frac{1}{2}\int_{0}^{2\pi/3} \left(1 + 2\cos(\theta)\right)^2 \, d\theta$$

You finish

$$1 + 2\cos(\theta) = 0$$

$$\left.\begin{array}{l} \cos(\theta) = -\frac{1}{2} \\ \frac{\pi}{2} < \theta < \pi \end{array}\right\} \theta = \frac{2\pi}{3}$$