Info...

- New homework and EMCFs are posted.
- Video help is posted for selected problems in 9.4 and 9.5.
Review of Polar Coordinates:

\[ x = r \cos(\theta) \]

\[ y = r \sin(\theta) \]

Standard Representation for \( r = \sqrt{x^2 + y^2} \)

Standard Representation for \( \theta = \arctan \left( \frac{y}{x} \right) \)

\((x, y)\) in \(\mathbb{D}_1\) or \(\mathbb{D}_4\).
More Review:

Overview of Polar Graphs:

\[ r = \cos(3\theta) \text{ is a} \quad \text{3 petal flower} \]
\[ r = \sin(4\theta) \text{ is a} \quad \text{8 petal flower} \]
\[ r = 3 \cos(\theta) \text{ is a} \quad \text{circle of radius 3/2 centered at (3/2,0)} \]
\[ r = 4 \sin(\theta) \text{ is a} \quad \text{circle of radius 2 centered at (0,2)} \]
\[ r = a + b \cos(\theta) \text{ is a limacon, with the actual shape and placement dependent on } a \text{ and } b. \]
\[ r = a + b \sin(\theta) \text{ is a limacon, with the actual shape and placement dependent on } a \text{ and } b. \]
\[ |a| = |b| \quad \leftrightarrow \quad \text{cardiod} \]
\[ |a| > |b| \quad \leftrightarrow \quad \text{dent/dimple} \]
\[ |a| < |b| \quad \leftrightarrow \quad \text{inner loop} \]
\[ r = a + b \cos(\theta) \quad |a| < |b| \]
\[ r = a + b \sin(\theta) \]

Some Limacons with Inner Loops

- Identical plots, but different starting points with \( \theta = 0 \).
- Sine plots align along the y-axis.
- Identical plots, but different starting points with \( \theta = 0 \).
- Cosine plots align along the x-axis.
- Identical plots, but different starting points with \( \theta = 0 \).
\[ r = a + b \cos(\theta) \]
\[ r = a + b \sin(\theta) \quad |a| > |b| \]

Some Limacons with Dimples (Dents)

- **Identical plots, but different starting points with \( \theta = 0 \).**
  - \( r = 2 + \sin(\theta) \)
  - \( r = 2 - \sin(\theta) \)

- **Sine plots align along the y axis**

- **Cosine plots align along the x axis**
  - \( r = 2 + \cos(\theta) \)
  - \( r = 2 - \cos(\theta) \)

- **Very slight dent.**
  - \( r = 3 + \sin(\theta) \)
  - \( r = 3 - \sin(\theta) \)
  - \( r = -2 + \cos(\theta) \)
  - \( r = -2 - \cos(\theta) \)
\[ r = a + b \cos(\theta) \quad |a| = |b| \]

Some Cardioids

- \( r = 1 + \sin(\theta) \)
- \( r = 1 - \sin(\theta) \)
- \( r = -1 + \sin(\theta) \)
- \( r = -1 - \sin(\theta) \)

- \( r = 1 + \cos(\theta) \)
- \( r = 1 - \cos(\theta) \)
- \( r = -1 + \cos(\theta) \)
- \( r = -1 - \cos(\theta) \)

Sine plots align along the y-axis.

Cosine plots align along the x-axis.
Area In Polar Coordinates

**Our Goal:** Find the area of the region between the origin and the polar graph of $r = r(\theta)$ for $\theta$ between $a$ and $b$. $\alpha < b$

$n$ sectors.

\[
\text{Area} = \sum_{i=1}^{n} \text{Area (sector } i \text{)}
\]

Blown-up $i^{th}$ sector

\[
\approx \sum_{i=1}^{n} \frac{1}{2} r(\theta_i)^2 (\theta_i - \theta_{i-1})
\]

\[\Delta \theta_i\]

\[\frac{1}{2} \int_{a}^{b} r(\theta)^2 \, d\theta\]

\[\text{Area (sector } i \text{)} \approx \text{Area (circular sector)}
\]

\[= \pi r(\theta_i)^2 \frac{\theta_i - \theta_{i-1}}{2\pi}
\]

\[= \frac{1}{2} r(\theta_i)^2 (\theta_i - \theta_{i-1})\]
**Area Formula:** The area of the region between the origin and the polar graph of $r = r(\theta)$ for $\theta$ between $a$ and $b$ is given by

$$\frac{1}{2} \int_{a}^{b} r(\theta)^2 \, d\theta$$

$a < b$
Example: Find the area inside one petal of the flower given by \( r = \cos(4\theta) \).

\[
\text{Area} = \frac{1}{2} \int_{a}^{b} r(\theta)^2 \, d\theta
\]

\[
= \text{Area} \left( \frac{\pi}{4} \right)
\]

\[
= 2 \cdot \text{Area} \left( \frac{\pi}{8} \right)
\]

\[
= 2 \cdot \frac{1}{2} \int_{0}^{\pi/8} \cos^2(4\theta) \, d\theta
\]

\[
= \int_{0}^{\pi/8} \left( \frac{1}{2} + \frac{1}{2} \cos(8\theta) \right) \, d\theta
\]

\[
= \left. \left( \frac{1}{2} \theta + \frac{1}{16} \sin(8\theta) \right) \right|_{0}^{\pi/8}
\]

\[
= \frac{\pi}{16} + 0 - 0 = \frac{\pi}{16}.
\]
Example: Find the area in the upper half of the cardioid 
\[ r = 2 + 2\cos(\theta) \]

\[
\text{Area} = \frac{1}{2} \int_a^b r(\theta)^2 \, d\theta
\]

\[
= \frac{1}{2} \int_0^\pi (2 + 2\cos(\theta))^2 \, d\theta
\]

\[
= \frac{1}{2} \int_0^\pi (4 + 4\cos(\theta) + 4\cos^2(\theta)) \, d\theta
\]

\[
= \frac{1}{2} \left[ 2\theta + 4\sin(\theta) + 2\int_0^\pi \cos^2(\theta) \, d\theta \right]
\]

\[
= 2\pi - 0 + 2\left[ \frac{\pi}{2} + \frac{1}{2} \cos(2\theta) \right]_0^\pi
\]

\[
= 2\pi + (\theta + \frac{1}{2} \sin(2\theta)) \bigg|_0^\pi
\]

\[
= 2\pi + \pi - 0 = 3\pi
\]
Example: Find the area inside the outer loop of \( r = 1 + 2 \cos(\theta) \)

\[
\text{Area} = \frac{1}{2} \int_{0}^{\frac{2\pi}{3}} (1 + 2 \cos(\theta))^2 \, d\theta
\]

\[
1 + 2 \cos(\theta) = 0 \\
\cos(\theta) = -\frac{1}{2} \\
\frac{\pi}{2} < \theta < \pi \\
\theta = \frac{2\pi}{3}
\]