Info...
We will finish polar coordinates and start parametric equations.

More Area Examples

Example: Give the area of the region that is in Q4 and inside the outer loop of the polar graph $r = 1 - 2 \cos(\theta)$.

\[
\begin{align*}
1 - 2 \cos(\theta) & = 0 \\
\cos(\theta) & = \frac{1}{2} \\
\theta & = \pm \frac{\pi}{3} \\
\theta & = -\frac{\pi}{3} \\
Area & = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (1 - 2 \cos(\theta))^2 \, d\theta \\
& = \text{...}
\end{align*}
\]

Popper 11

1. Give the area inside the inner loop of $r = 1 + 2\cos(\theta)$.

2. Give the number of petals for the flower $r = 3 \sin(4\theta)$.

3. $\frac{1}{2}$
4. 5
Parametric Curves
(an introduction)

Parametric curves are given by
\((x(t), y(t))\), \(a \leq t \leq b\)

where \(x(t)\) and \(y(t)\) are given functions.

Example:
\((\sin(2t), \sin(t))\), \(0 \leq t \leq 2\pi\)

Example:
\((\cos(t), \cos(2t))\)

Circle of radius 1
Centered at \((0,0)\).

Note: Parametric Curves Can Be Complex!!

\((\sin(3t), \cos(7t)e^{\cos(2t)})\)

for \(0 \leq t \leq 2\pi\)

Note: A parametric curve has an orientation given by the parameterizing variable.

Example: Plot \((\cos(t), \sin(t))\) for \(0 \leq t \leq 2\pi\).

The manner in which \((x(t), y(t))\) changes as \(t\) increases gives the orientation on the curve.
Example: Give a parameterization of the portion of the line \(y = -2x + 5\) between \((1,3)\) and \((-2,9)\).

\[
\begin{align*}
x &= t \\
y &= -2t + 5 \\
-2 &\leq t \leq 1
\end{align*}
\]

Let's describe the general mechanism for parameterizing a line segment from \((a,b)\) to \((c,d)\).

\[
\begin{align*}
x &= a + t(c-a) \\
y &= b + t(d-b) \\
0 &\leq t \leq 1
\end{align*}
\]

Example: Give a parameterization for the line segment from \((3,6)\) to \((-2,5)\).

\[
\begin{align*}
x &= 3 + t(-2-3) \\
y &= 6 + t(5-6) \\
o &\leq t \leq 1
\end{align*}
\]

\[
\begin{align*}
x &= 3 - 5t \\
y &= 6 - t \\
o &\leq t \leq 1
\end{align*}
\]

Let's describe the general mechanism for parameterizing a line through the points \((a,b)\) and \((c,d)\).

\[
\begin{align*}
x &= a + t(c-a) \\
y &= b + t(d-b) \\
-\infty &< t < \infty
\end{align*}
\]

\[
\begin{align*}
x &= a + t(c-a) \\
y &= b + t(d-b) \\
0 &\leq t \leq 1
\end{align*}
\]
Example: Give a parameterization for the line through the points (3,6) and (-2,5).

\[ x = 2 + t(3 - 2) \]
\[ y = 5 + t(6 - 5) \]

\[ x = -2 + 5t \]
\[ y = 5 + t \]

Other parametric examples...

Example: Plot \((t, t^2)\) for \(-1 \leq t \leq 2\).

\[ y = x \]
\[ -1 \leq x \leq 2 \]

Example: Plot \((t, t^3)\) for \(-1 \leq t \leq 1\).

\[ y = x^3 \]
\[-1 \leq x \leq 1 \]

Example: Plot \((2\cos(t), 3\sin(t))\) for \(0 \leq t \leq 2\pi\).

\[ \frac{x^2}{4} + \frac{y^2}{9} = 1 \]

Ellipse