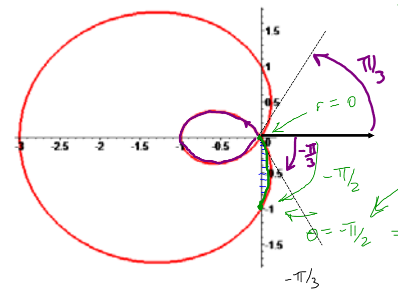
Info...

We will finish polar coordinates and start parametric equations.

More Area Examples

Example: **Give the area of the region that is in Q4 and inside the outer loop of the polar graph $r = 1 - 2\cos(\theta)$.**



$1 - 2\cos(\theta) = 0$

$\cos(\theta) = \frac{1}{2}$

use $\theta = -\frac{\pi}{3}$

b/c of

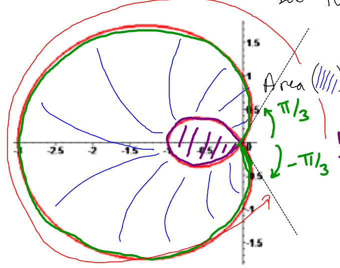
$\theta = -\pi/2 \Rightarrow r = 1$

$$\text{Area} = \frac{1}{2}\int_{-\pi/2}^{-\pi/3} (1 - 2\cos(\theta))^2 \, d\theta$$

$= \ldots$

Example: Give the area of the region that is inside the outer loop and outside the inner loop of the polar graph $r = 1 - 2\cos(\theta)$.

See the previous problem.



Area(////) = Area(////) − Area(///)

Note: Inner loop is traced for $-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$.

The outer loop is traced for $\frac{\pi}{3} \le \theta \le \frac{5\pi}{3}$.

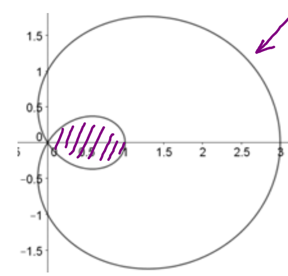
$\therefore$ Area(////) = Area(////) − Area(///)

$$= \frac{1}{2}\int_{\pi/3}^{5\pi/3} (1 - 2\cos(\theta))^2 \, d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3} (1 - 2\cos(\theta))^2 \, d\theta$$

Popper 11

1. Give the area inside the inner loop of $r = 1 + 2\cos(\theta)$.

2. Give the number of petals for the flower $r = 3\sin(4\theta)$.



3. $\frac{1}{2}$

4. 5

Parametric Curves

(an introduction)

Parametric curves are given by

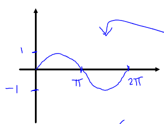
$(x(t), y(t)),\ a \le t \le b$ — parametrizing variable

where $x(t)$ and $y(t)$ are given functions.

Ex. $(t, \sin(t))$, $0 \le t \le 2\pi$

$\uparrow$ x $\uparrow$ y

$y = \sin(x)$, $0 \le x \le 2\pi$



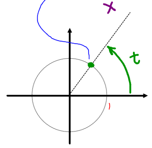
Ex. $(\cos(t), \sin(t))$

$\uparrow$ x $\uparrow$ y

$x^2 + y^2 = 1$

Circle of radius 1

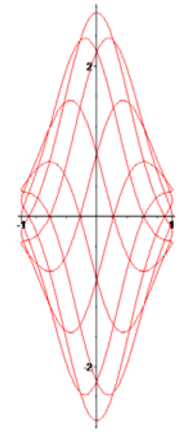
centered at $(0,0)$.



Note: Parametric Curves Can Be Complex!!

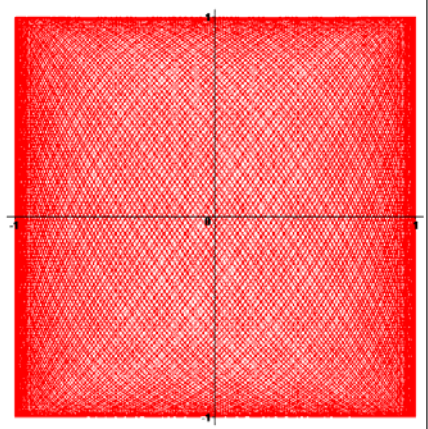
$$\left(\sin(5t), \cos(7t)e^{\cos(10t)}\right)$$

for $0 \le t \le 2\pi$



$$\left(\sin(t), \cos(\sqrt{2}t)\right)$$

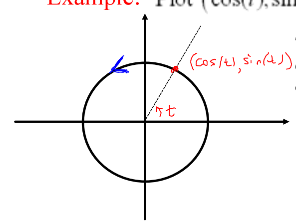
for $0 \le t \le 1000$



Note: A parametric curve has an *orientation* given by the parameterizing variable.

on the curve

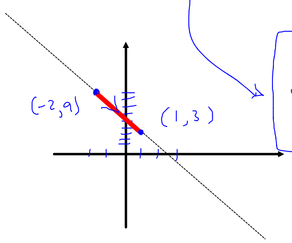
Example: Plot $(\cos(t), \sin(t))$ for $0 \le t \le 2\pi$.



The manner in which (x(t),y(t)) changes as t increases gives the orientation on the curve.

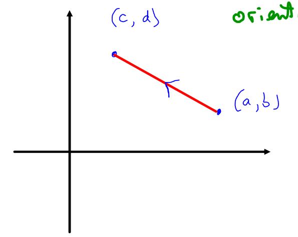
Here, the orientation is counter-clockwise.

Example: **Give a parameterization of the portion of the line $y = -2x + 5$ between $(1,3)$ and $(-2,9)$.**



$$\boxed{\begin{aligned} x &= t \\ y &= -2t + 5 \\ -2 \le\ & t \le 1 \end{aligned}}$$

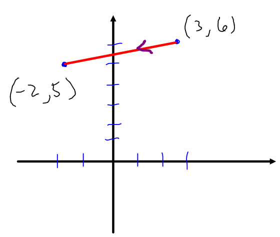
Let's describe the general mechanism for parameterizing a line segment $\underbrace{\text{from } (a,b) \text{ to } (c,d)}_{\text{orientation}}$.



$$\boxed{\begin{aligned} x &= a + t(c-a) \\ y &= b + t(d-b) \\ 0 \le\ & t \le 1 \end{aligned}}$$

Example: Give a parameterization for the line segment from (3,6) to (-2,5).

orientation



$x = 3 + t(-2-3)$

$y = 6 + t(5-6)$

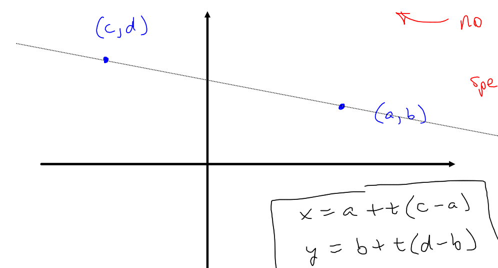
$0 \le t \le 1$

i.e.

$$\boxed{\begin{aligned} x &= 3 - 5t \\ y &= 6 - t \\ 0 \le\ & t \le 1 \end{aligned}}$$

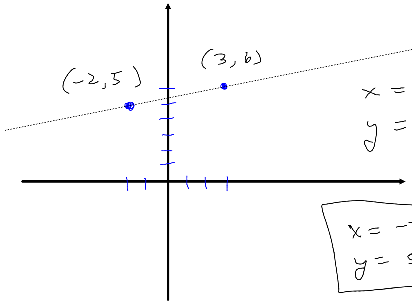
Let's describe the general mechanism for parameterizing a line through the points (a,b) and (c,d).

no orientation is specified



$$\boxed{\begin{aligned} x &= a + t(c-a) \\ y &= b + t(d-b) \\ -\infty <\ & t < \infty \end{aligned}}$$

Example: Give a parameterization for the line through the points (3,6) and (-2,5).



$x = -2 + t(3 - -2)$

$y = 5 + t(6-5)$

i.e.

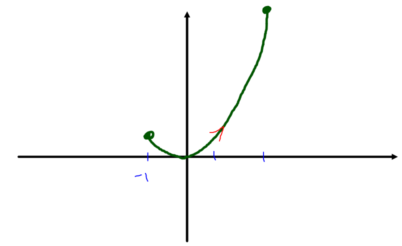
$$\boxed{\begin{aligned} x &= -2 + 5t \\ y &= 5 + t \end{aligned}}$$

Other parametric examples...

Example: Plot (t, t^2) for $-1 \le t \le 2$.

$x \quad y$

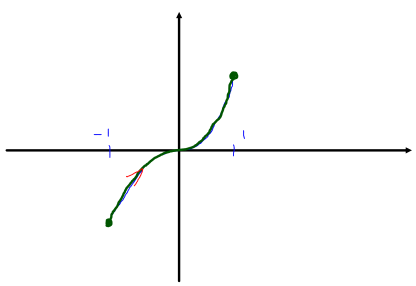
$y = x^2, \quad -1 \le x \le 2$



Example: Plot (t, t^3) for $-1 \le t \le 1$.

$x \quad y$

$y = x^3, \quad -1 \le x \le 1$



Example: Plot $(2\cos(t), 3\sin(t))$ for $0 \le t \le 2\pi$.

$x \quad y$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ ellipse