Info...

We will finish polar coordinates and start parametric equations.
More Area Examples

Example: Give the area of the region that is in Q4 and inside the outer loop of the polar graph $r = 1 - 2 \cos(\theta)$.

\[
\text{Area} = \frac{1}{2} \int_{-\pi/2}^{-\pi/3} (1 - 2 \cos(\theta))^2 \, d\theta
\]
Example: Give the area of the region that is inside the outer loop and outside the inner loop of the polar graph $r = 1 - 2 \cos(\theta)$.

The outer loop is traced for $\frac{\pi}{3} \leq \theta \leq \frac{5\pi}{3}$.

Note: Inner loop is traced for $-\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}$.

\[
\text{Area (1111)} = \text{Area (1110)} - \text{Area (1111)}
\]

\[
= \frac{1}{2} \int_{\pi/3}^{5\pi/3} (1 - 2 \cos(\theta))^2 d\theta - \frac{1}{2} \int_{-\pi/3}^{\pi/3} (1 - 2 \cos(\theta))^2 d\theta
\]
Popper 11

1. Give the area inside the inner loop of $r = 1 + 2\cos(\theta)$.

2. Give the number of petals for the flower $r = 3 \sin(4\theta)$.

3. $\frac{1}{2}$

4. 5
Parametric Curves
(an introduction)

Parametric curves are given by
\[(x(t), y(t)), \ a \leq t \leq b\]

where \(x(t)\) and \(y(t)\) are given functions.

Ex. \((t, \sin(t)), \ 0 \leq t \leq 2\pi\)

\(y = \sin(x), \ 0 \leq x \leq 2\pi\)

Ex. \((\cos(t), \sin(t))\)

\(x^2 + y^2 = 1\)
Circle of radius 1
Centered at \((0,0)\).
Note: Parametric Curves Can Be Complex!!

$$(\sin(5t), \cos(7t)e^{\cos(10t)})$$

for $0 \leq t \leq 2\pi$
\((\sin(t), \cos(\sqrt{2}t))\)

for \(0 \leq t \leq 1000\)
Note: A parametric curve has an orientation given by the parameterizing variable.

Example: Plot \((\cos(t), \sin(t))\) for \(0 \leq t \leq 2\pi\).

The manner in which \((x(t), y(t))\) changes as \(t\) increases gives the orientation on the curve.

Here, the orientation is counter-clockwise.
Example: Give a parameterization of the portion of the line $y = -2x + 5$ between $(1,3)$ and $(-2,9)$.

$$x = t$$

$$y = -2t + 5$$

$$-2 \leq t \leq 1$$
Let's describe the general mechanism for parameterizing a line segment from \((a,b)\) to \((c,d)\).

\[
\begin{align*}
\text{Orientation} \\
(c, d) & \quad (a, b)
\end{align*}
\]

\[
\begin{align*}
x &= a + t(c-a) \\
y &= b + t(d-b) \\
0 &\leq t \leq 1
\end{align*}
\]
Example: Give a parameterization for the line segment from (3,6) to (-2,5).

\[ x = 3 + t (-2 - 3) \]
\[ y = 6 + t (5 - 6) \]
\[ 0 \leq t \leq 1 \]

i.e.

\[ x = 3 - 5t \]
\[ y = 6 - t \]
\[ 0 \leq t \leq 1 \]
Let's describe the general mechanism for parameterizing a line through the points \((a,b)\) and \((c,d)\).

\[
\begin{align*}
x &= a + t(c-a) \\
y &= b + t(d-b)
\end{align*}
\]

\(-\infty < t < \infty\)
Example: Give a parameterization for the line through the points (3,6) and (-2,5).

\[ x = -2 + t (3 - -2) \]
\[ y = 5 + t (6 - 5) \]

i.e.

\[ x = -2 + 5t \]
\[ y = 5 + t \]
Other parametric examples...

Example: Plot \((t, t^2)\) for \(-1 \leq t \leq 2\).
Example: Plot \((t, t^3)\) for \(-1 \leq t \leq 1\).
Example: Plot \((2 \cos(t), 3 \sin(t))\) for \(0 \leq t \leq 2\pi\).

\[
\frac{x^2}{4} + \frac{y^2}{9} = 1
\]

ellipse