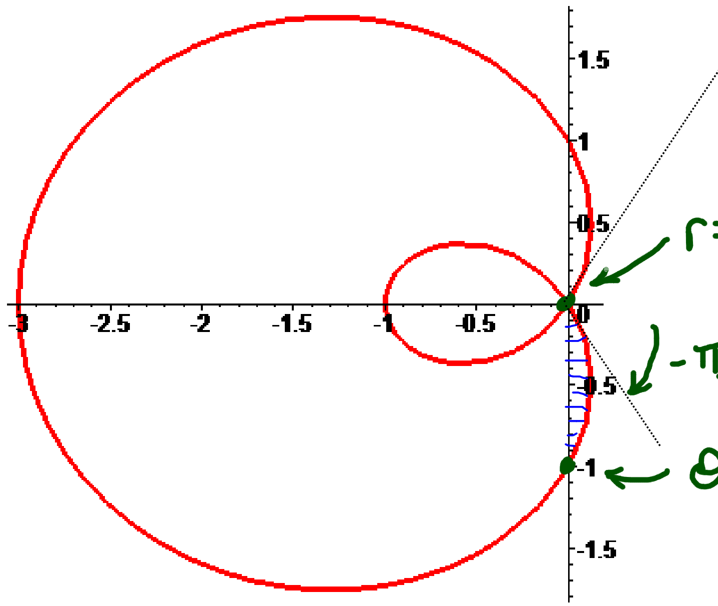
Info...

We will finish polar coordinates and start parametric equations.

More Area Examples

Example: **Give the area of the region that is in Q4 and insidc thc outcr loop of thc polar graph $r = 1 - 2\cos(\theta)$.**



$r = 0$

$-\pi/3$

$\theta = \frac{3\pi}{2}$ or $\boxed{\theta = \frac{-\pi}{2}}$

$1 - 2\cos(\theta) = 0$

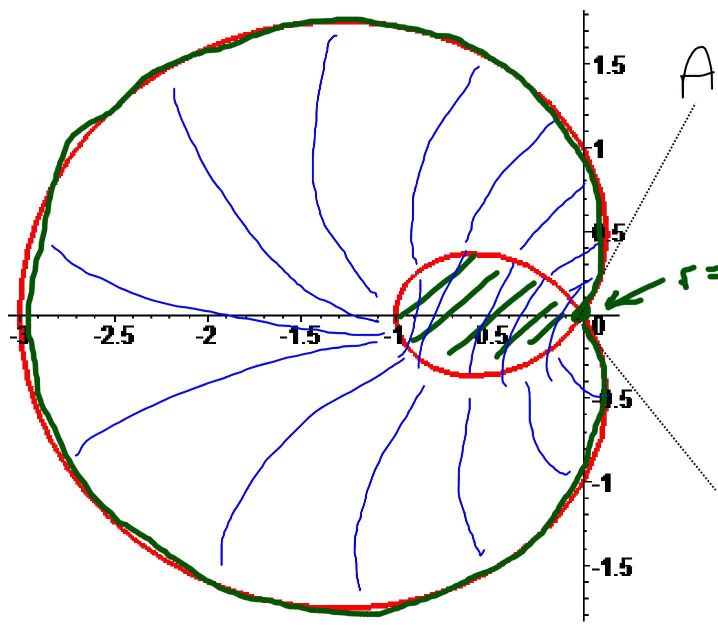
$\cos(\theta) = \frac{1}{2}$

$\boxed{\theta = -\pi/3}$

$$\text{Area} = \frac{1}{2}\int_{-\frac{\pi}{2}}^{-\pi/3} \left(1 - 2\cos(\theta)\right)^2 d\theta$$

$= \ldots$

Example: Give the area of the region that is inside the outer loop and outside the inner loop of the polar graph $r = 1 - 2\cos(\theta)$.



Area = Area(////) − Area(////)

$1 - 2\cos(\theta) = 0$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \; -\frac{\pi}{3}$

$-\frac{\pi}{3} \leftrightarrow \frac{5\pi}{3}$

$$= \frac{1}{2}\int_{\pi/3}^{5\pi/3} (1 - 2\cos(\theta))^2 \, d\theta \; - \; \frac{1}{2}\int_{-\pi/3}^{\pi/3} (1 - 2\cos(\theta))^2 \, d\theta$$

Popper 11

1. Give the area inside the inner loop of $r = 1 + 2\cos(\theta)$.

2. Give the number of petals for the flower
$r = 3\sin(4\theta)$.

8

Parametric Curves

(an introduction)

Parametric curves are given by

$$\big(x(t), y(t)\big),\ \ a \le t \le b$$

where $x(t)$ and $y(t)$ are given functions.

ex. (t, t^2) $\quad -\infty < t < \infty$

$x = t$, $y = t^2$ $\Rightarrow y = x^2$

$\therefore$ this parametric curve is the parabola $y = x^2$, and we say "this is a parameterization for the parabola."

ex. $(\cos(t), \sin(t))$

$x = \cos(t)$, $y = \sin(t)$

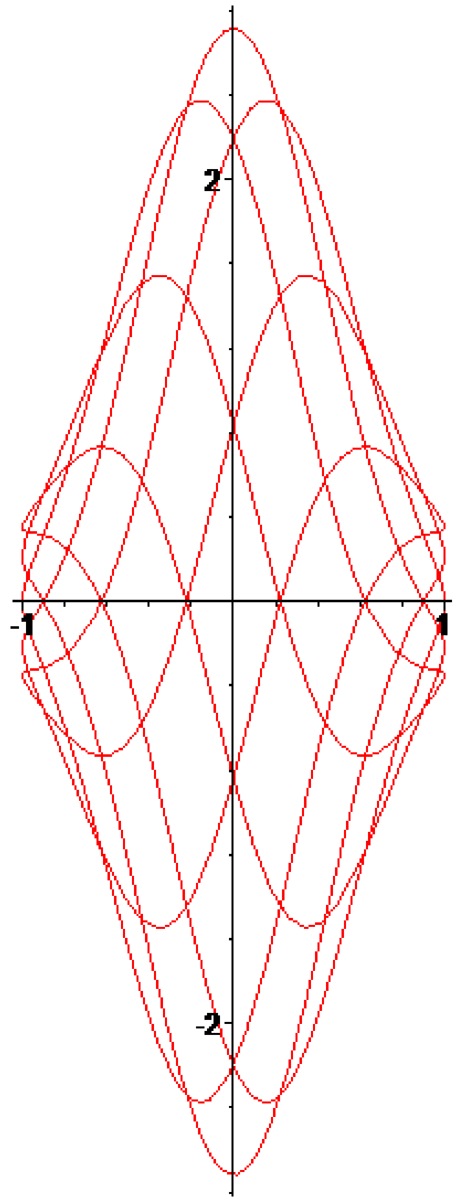
$x^2 + y^2 = 1$ circle of radius 1 centered at $(0,0)$.

Note: When t is not specified, it is assumed that all permissable values are used.

Note: Parametric Curves Can Be Complex!!

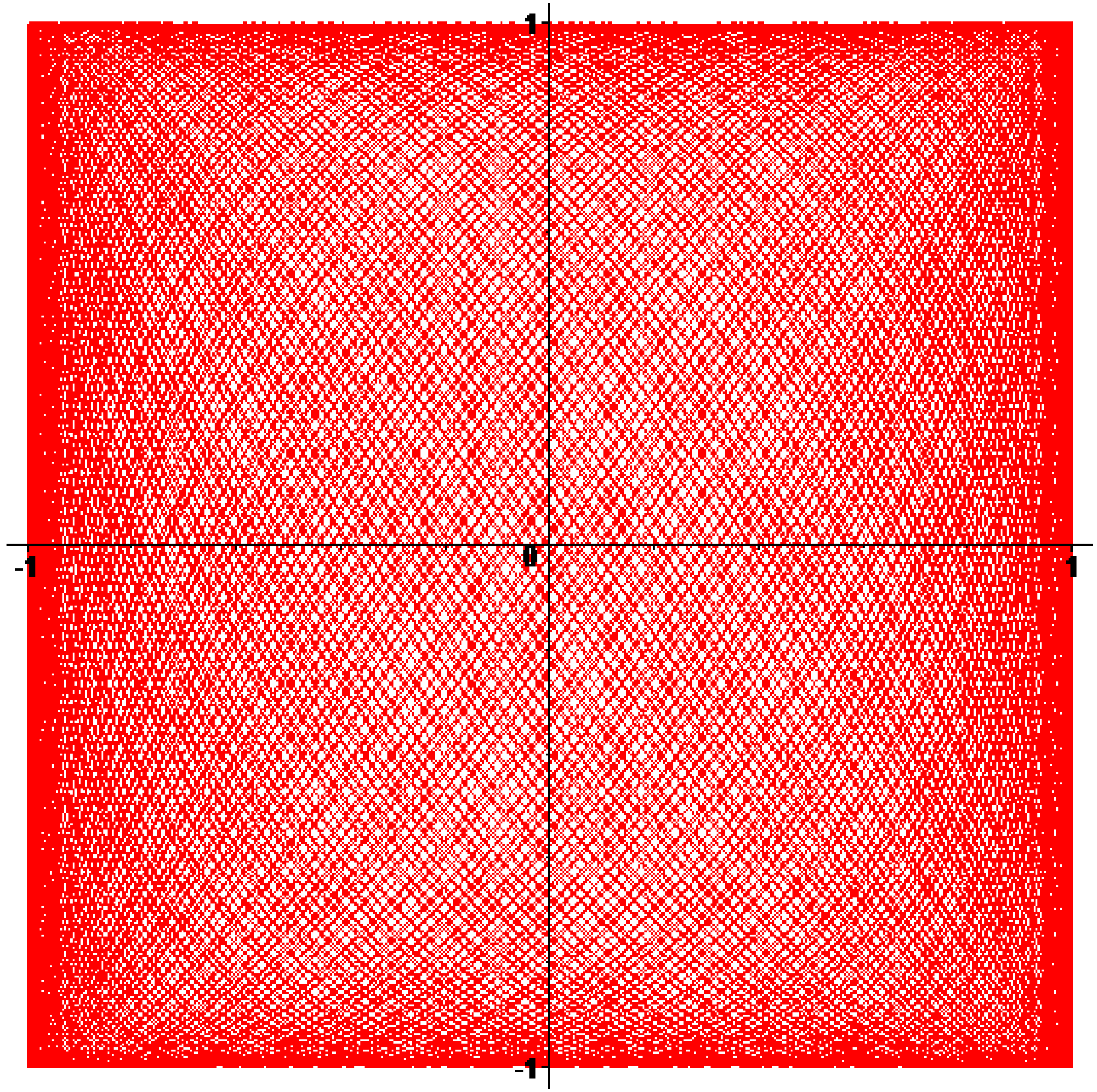
$$\left(\sin(5t), \cos(7t)e^{\cos(10t)}\right)$$

$$\text{for } 0 \le t \le 2\pi$$



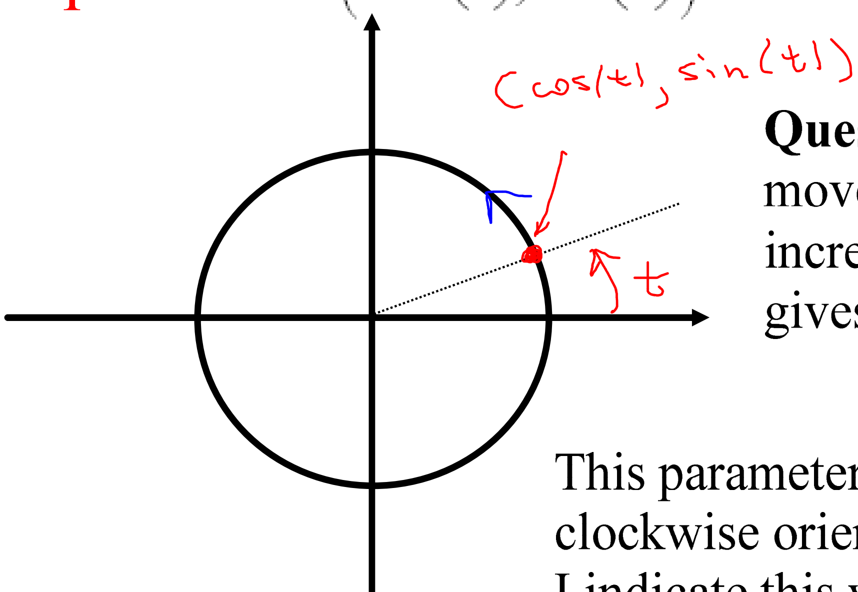
$$\left(\sin(t), \cos(\sqrt{2}t)\right)$$

$$\text{for } 0 \le t \le 1000$$



Note: A parametric curve has an *orientation* given by the parameterizing variable.

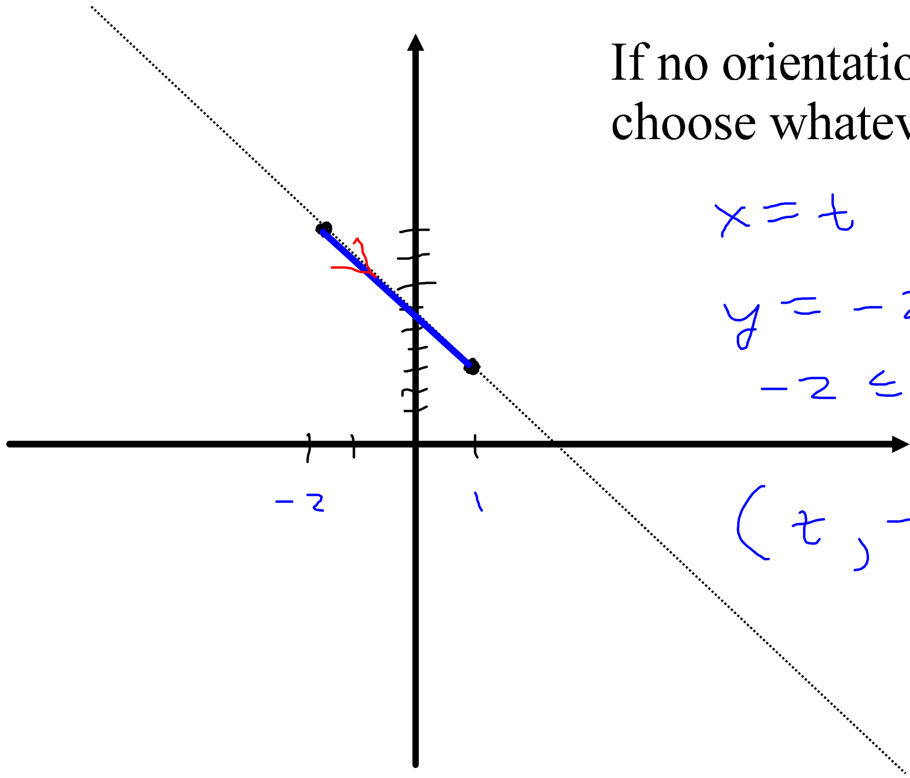
Example: Plot $(\cos(t), \sin(t))$ for $0 \leq t \leq 2\pi$.



Question: How do we move along the curve as t increases? This movement gives the orientation.

This parameterization gives a counter clockwise orientation of the unit circle. I indicate this with the blue arrow.

Example: **Give a parameterization of the portion of the line $y = -2x + 5$ between $(1,3)$ and $(-2,9)$.**



If no orientation is specified, we can choose whatever we like.

$x = t$

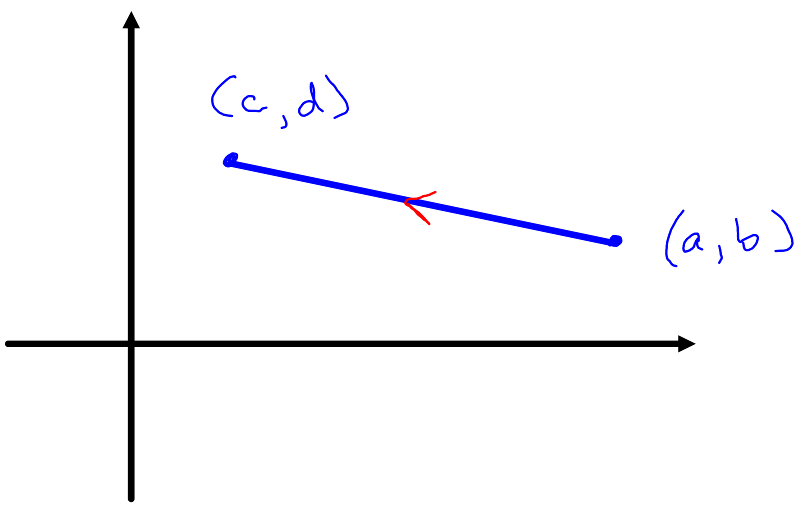
$y = -2t + 5$

$-2 \le t \le 1$

$(t, -2t+5)$ $\quad -2 \le t \le 1$

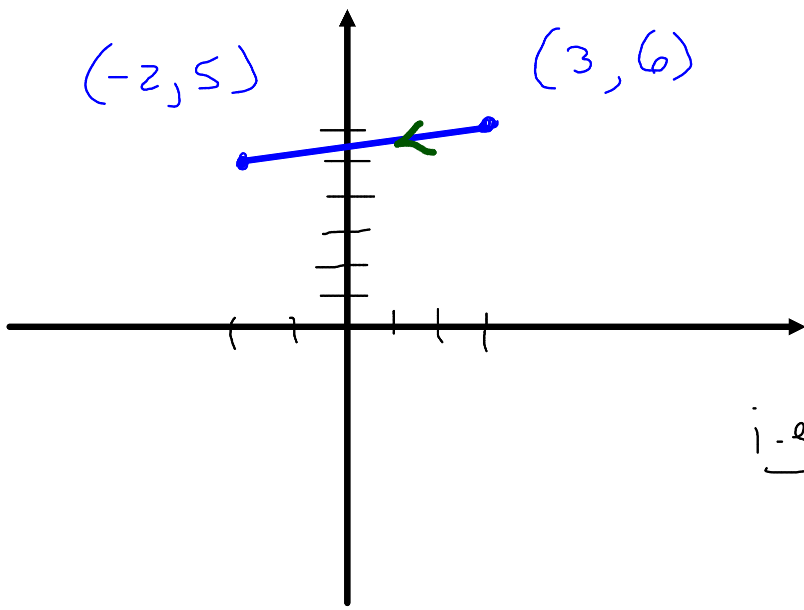
Let's describe the general mechanism for parameterizing a line segment from (a,b) to (c,d).

This wording indicates the desired orientation.



$$\boxed{\begin{aligned} x &= a + t(c-a) \\ y &= b + t(d-b) \\ 0 &\le t \le 1 \end{aligned}}$$

Example: Give a parameterization for the line segment from (3,6) to (-2,5).



$$x = 3 + t(-2-3)$$

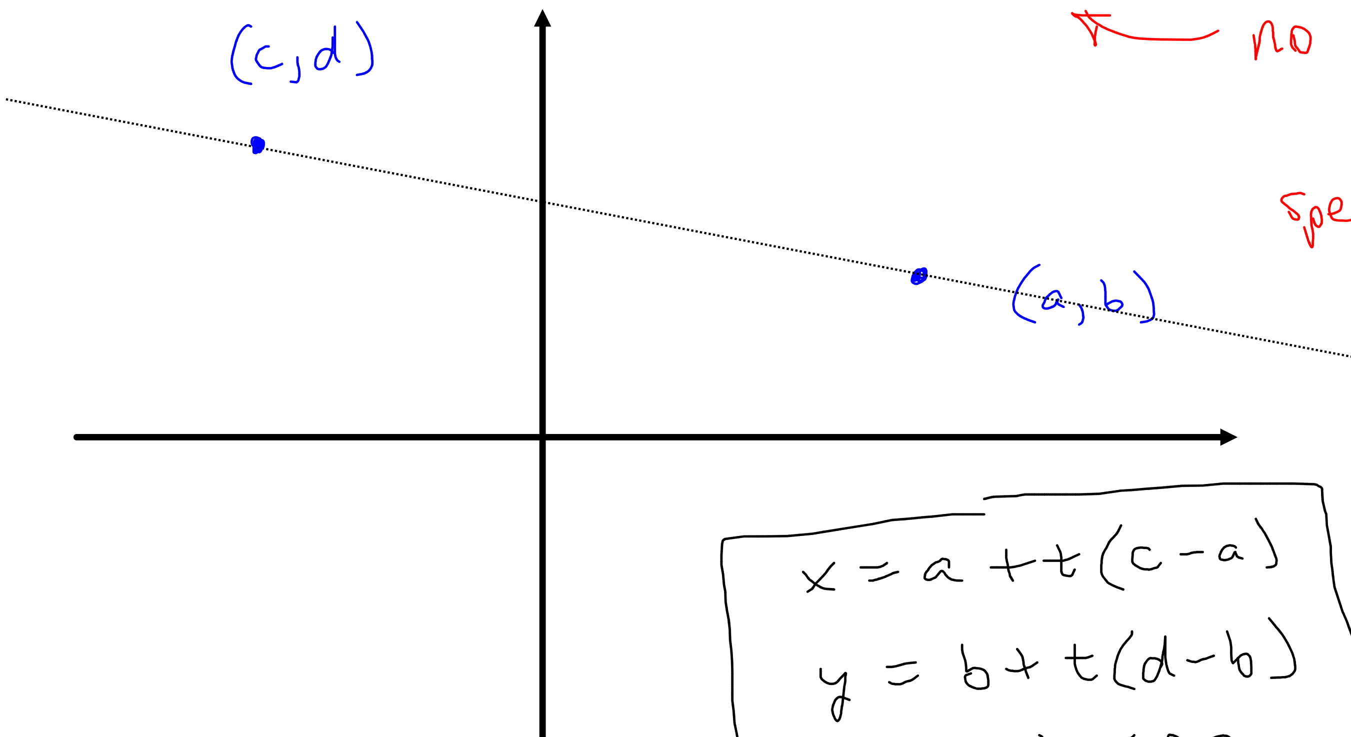
$$y = 6 + t(5-6)$$

$$0 \le t \le 1$$

i.e.

$$\boxed{\begin{aligned} x &= 3 - 5t \\ y &= 6 - t \\ 0 &\le t \le 1 \end{aligned}}$$

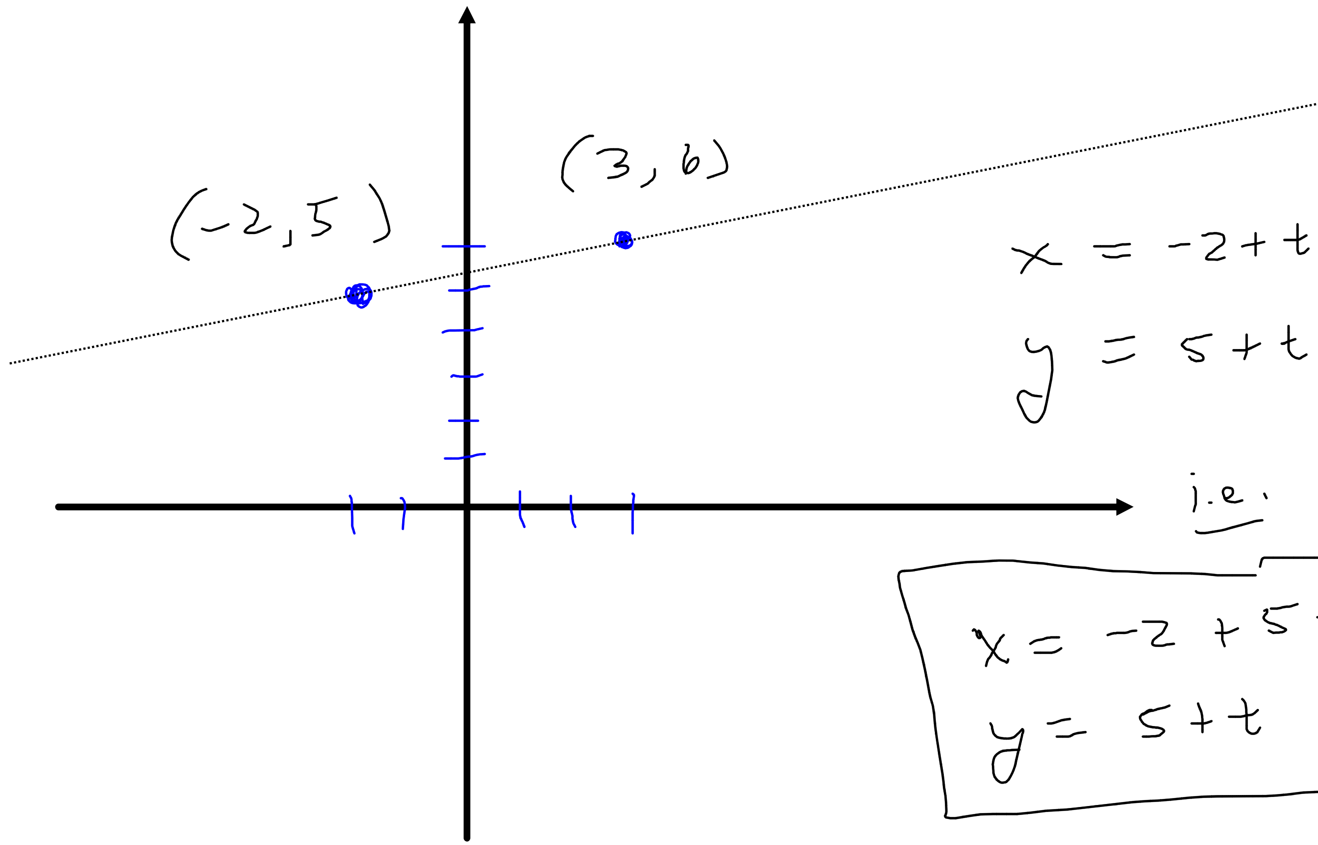
Let's describe the general mechanism for parameterizing a line through the points (a,b) and (c,d).



no orientation is specified

$$x = a + t(c-a)$$
$$y = b + t(d-b)$$
$$-\infty < t < \infty$$

Example: Give a parameterization for the line through the points (3,6) and (-2,5).



$x = -2 + t(3 - -2)$

$y = 5 + t(6 - 5)$

i.e.

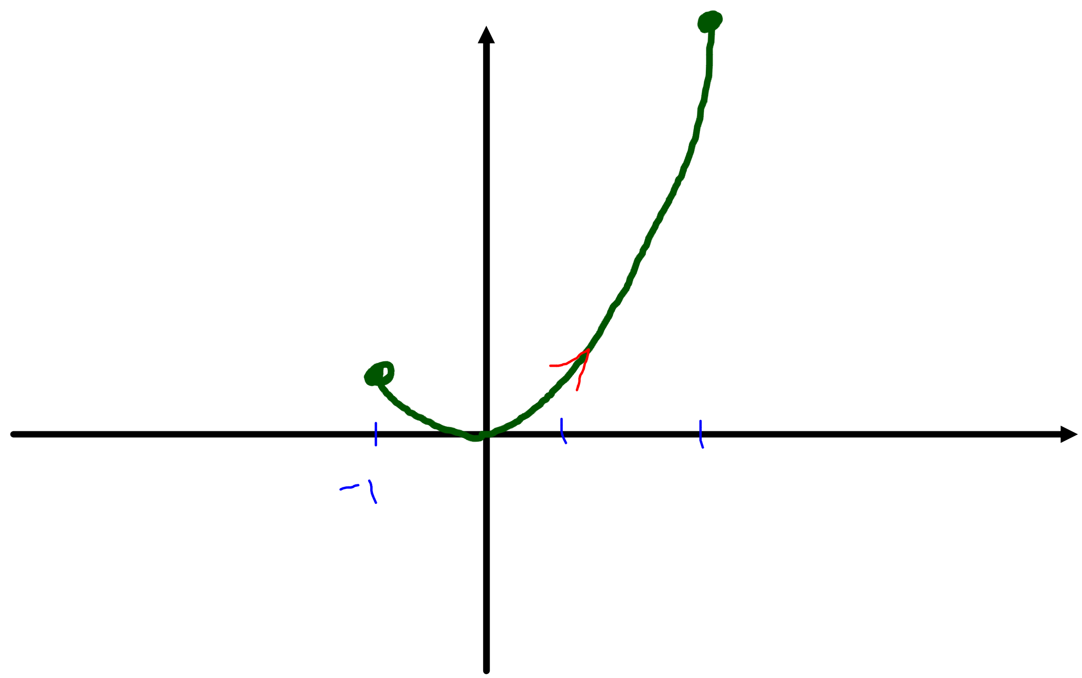
$$\boxed{\begin{aligned} x &= -2 + 5t \\ y &= 5 + t \end{aligned}}$$

Other parametric examples...

Example: Plot $\left(t, t^2\right)$ for $-1 \le t \le 2$.

x y

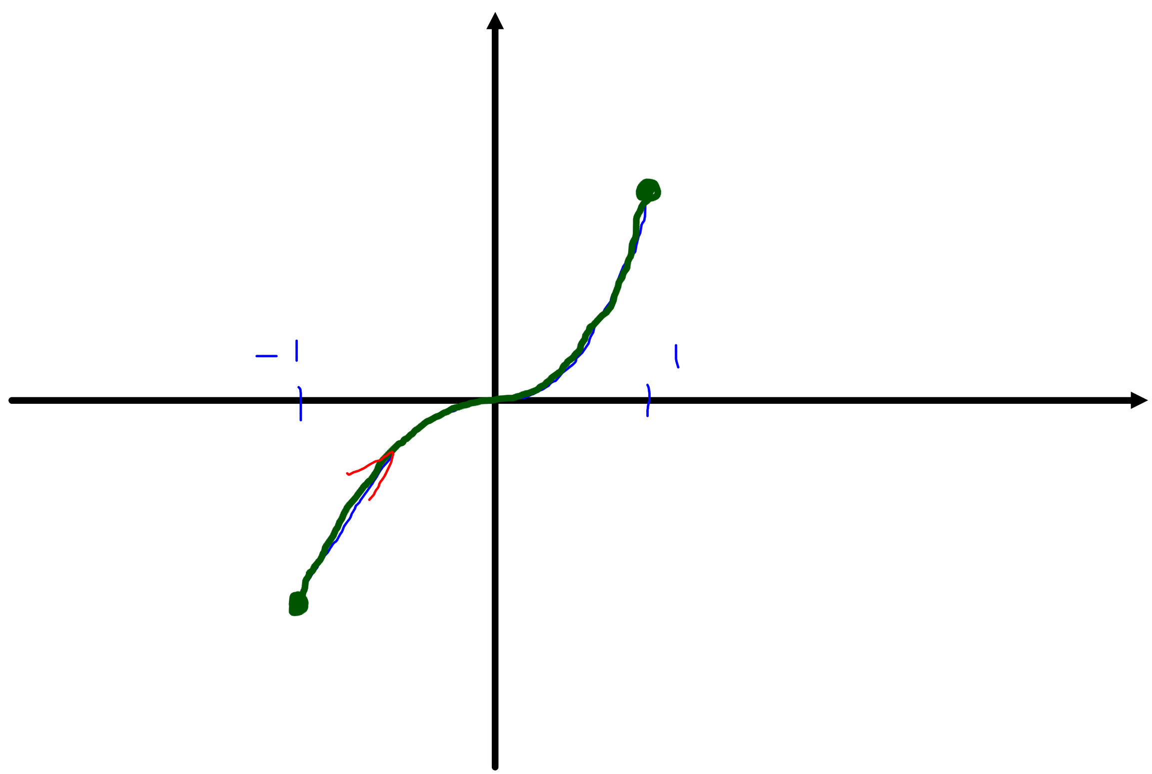
$y = x^2$, $-1 \le x \le 2$



Example: Plot $\left(t, t^3\right)$ for $-1 \leq t \leq 1$.

x y

$y = x^3$, $-1 \leq x \leq 1$



Example: Plot $\left(2\cos(t), 3\sin(t)\right)$ for $0 \le t \le 2\pi$.

$x \qquad y$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

ellipse