Example: Parametric curves can be interesting.

\[
\begin{align*}
\left( \cos(t), \sin(t) \right) & \quad \text{We saw this graph.} \\
(3\cos(3t)\cos(t), (4 - 5\cos(t))\sin(t)) &
\end{align*}
\]

\[x'(t) = 3\cos(3t)\cos(t) + \cos(t)(-9\sin(t))\]
\[y'(t) = (4 - 5\cos(t))\cos(t) + 3\sin(t) + 5\sin(t)\]

The derivative of the parameterization is
\[
\begin{bmatrix}
x'(t) \\
y'(t)
\end{bmatrix}
\]

\[\frac{dx}{dt} = x'(t)\]
\[\frac{dy}{dt} = y'(t)\]

Note: If we have a parametric curve \((x(t), y(t))\) where \(x(t)\) and \(y(t)\) are differentiable functions, then we can compute \(x'(t)\) and \(y'(t)\).

Question: What do these derivatives represent?

1. See the previous page.

2. Slope:
\[
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}
\]

The sign of the curve at \((x(t_0), y(t_0))\) is \(\frac{y'(t_0)}{x'(t_0)}\) provided \(x'(t_0) \neq 0\).

Derivatives - The Complete Story...

1. If \(x'(t_0) \neq 0\) then \(\frac{y'(t_0)}{x'(t_0)}\) is the slope of the tangent line to the curve at \((x(t_0), y(t_0))\).

2. If \(x'(t_0) \neq 0\) and \(y'(t_0) \neq 0\) are not both zero, then the vector \(\left( x'(t_0), y'(t_0) \right)\) is tangent to the curve at \((x(t_0), y(t_0))\), and it points in the direction of orientation along the curve.
Example: Give an equation for the tangent line to the curve parameterized by \((\cos(t), t + \sin(2t))\) at the point \((0, \pi/2)\).

\[
\begin{align*}
x(t) &= \cos(t) \\
y(t) &= t + \sin(2t)
\end{align*}
\]

What value of \(t\) places us at \((0, \pi/2)\)?

\[
\omega : \cos(t) = 0 \\
\Rightarrow t + \sin(2t) = \frac{\pi}{2}
\]

\[
\Rightarrow t = \frac{\pi}{2}
\]

The slope of T.L. = \(\frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}
\]

\[
x'(t) = -\sin(t), \quad y'(t) = 1 + 2\cos(2t)
\]

\[
\therefore \text{ The T.L. is } y - \frac{\pi}{2} = \frac{x-0}{1} \quad \therefore y = x + \frac{\pi}{2}.
\]

Example: Give a parameterization of the tangent line to the graph of \((\cos(\theta)/\theta, \sin(\theta)/\theta)\) at the point \((\pi/2, 2)\), and show the relationship between the vector \(x'(\pi/2)\hat{i} + y'(\pi/2)\hat{j}\) and the graph of the parametric curve.

\[
\begin{align*}
x(\theta) &= \cos(\theta) \\
y(\theta) &= \sin(\theta)
\end{align*}
\]

\[
\begin{align*}
x'(\pi/2) &= \frac{\sin(\pi/2)}{\pi/2} = \frac{1}{\pi/2} \\
y'(\pi/2) &= \frac{\cos(\pi/2)}{\pi/2} = \frac{0}{\pi/2} = 0
\end{align*}
\]

\[
E = \left(\frac{1}{\pi/2}, 0\right)
\]

\[
\begin{align*}
x'(\pi/2) &= -2\sin(\pi/2), \quad y'(\pi/2) = 3\cos(\pi/2) \\
x'(\pi/2) &= -\frac{2}{\pi} \\
y'(\pi/2) &= 3\frac{\pi}{2}
\end{align*}
\]

\[
\text{Formula for T.L.: } \left(\frac{\pi}{2}, -\frac{\pi}{2}\right) \frac{2\pi\hat{j}}{\pi} + \frac{3\pi}{2}\hat{j}
\]

\[
\text{Direction Vector: } -\frac{\pi}{2} \hat{i} + \frac{3\pi}{2} \hat{j}
\]

Peppe 12

1. Give the slope of the tangent line to the curve given by the parametrization \((2t^2 - 1, \sin(t))\), at the point where \(t = 1\).

2. Give the \(y\)-intercept of the tangent line to the curve given in the problem above at the point where \(t = 1\).
3. Give the slope of the tangent line to the curve parameterized by
\((\cos(t)+2, \sin(2t))\) at the point where \(t = \pi/2\).

4. Give the slope of the normal line to the curve parameterized by
\((r^2-2^2, 3r+1)\) at the point \((4,7,1)\).

3. \(\frac{1}{2}\)

4. 0