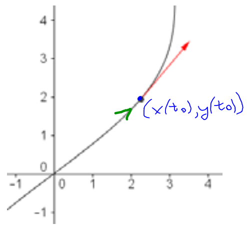
Review
Parametric Curves and Derivatives

param. for a curve.

$r(t) = (x(t), y(t))$

$r'(t) = x'(t)\hat{i} + y'(t)\hat{j}$

derivative $\qquad \underbrace{x'(t)\hat{i} + y'(t)\hat{j}}_{\text{vector}}$



$r'(t_0)$ is tangent to the curve at $r(t_0)$, and $r'(t_0)$ points in the direction of orientation.

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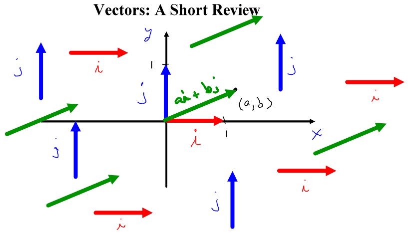
1. Give the slope of the tangent line to the curve parameterized by $(t + \cos(t), 2t - \sin(2t))$ at the point where $t = 1$.

2. Give the first component of the derivative vector at $t = 1$, associated with the parameterization given by $(t + \cos(t), 2t - \sin(2t))$.

3. Give the second component of the derivative vector at $t = 1$, associated with the parameterization given by $(t + \cos(t), 2t - \sin(2t))$.

Vectors: A Short Review



more generally: $ai + bj$, where a, b are real numbers.

Adding vectors:

$(ai + bj) + (ci + dj)$

$= (a+c)i + (b+d)j$.

Mult. by $\boxed{\text{scalars}}$: $\alpha(ai + bj)$

$= \alpha a i + \alpha b j$.

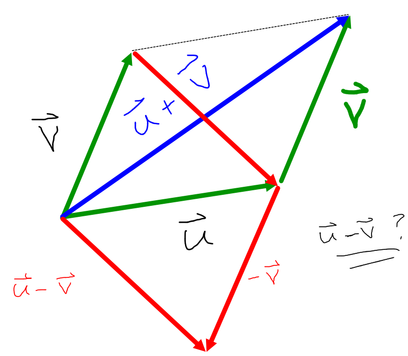
(Euclidean) length (or magnitude) of a vector:

$|ai + bj| = \sqrt{a^2 + b^2}$

$|\alpha(ai + bj)| = |\alpha a i + \alpha b j|$

$= \sqrt{(\alpha a)^2 + (\alpha b)^2}$

$= |\alpha|\ |ai + bj|$.



Relating Parametric Curves to Polar Curves

How: Polar curve $r = r(\theta)$.

Q: How can we param. this curve??

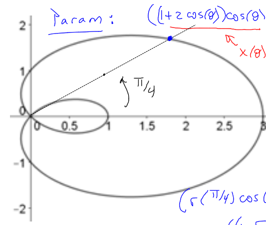
A: Use $x = r\cos(\theta)$, $y = r\sin(\theta)$

$(r(\theta)\cos(\theta), r(\theta)\sin(\theta))$

Is a param. in terms of the ind. var. θ.

Example: Graph the polar curve $r = 1 + 2\cos(\theta)$. Then find a parameterization for the tangent line to the curve at the points where $\theta = \pi/4$ and $\theta = \pi/2$.

Param: $(\underbrace{(1+2\cos(\theta))\cos(\theta)}_{x(\theta)}, \underbrace{(1+2\cos(\theta))\sin(\theta)}_{y(\theta)})$



For $\theta = \pi/4$:

Point:

$(r(\pi/4)\cos(\pi/4), r(\pi/4)\sin(\pi/4))$

$= ((1+\sqrt{2})\frac{\sqrt{2}}{2}, (1+\sqrt{2})\frac{\sqrt{2}}{2})$

$= (1+\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2})$

$x'(\theta) = \underbrace{(1+2\cos(\theta))}_{r(\theta)}(-\sin(\theta)) + \cos(\theta)\underbrace{(-2\sin(\theta))}_{r'(\theta)}$

$y'(\theta) = (1+2\cos(\theta))\cos(\theta) + \sin(\theta)(-2\sin(\theta))$

At $\pi/4$

$x'(\pi/4) = -1 - \frac{\sqrt{2}}{2} + (-1) = -2 - \frac{\sqrt{2}}{2}$

$y'(\pi/4) = 1 + \frac{\sqrt{2}}{2} + (-1) = \frac{\sqrt{2}}{2}$

Param of T.L. at $\theta = \pi/4$

$\left(1+\frac{\sqrt{2}}{2} + t\left(-2-\frac{\sqrt{2}}{2}\right), 1+\frac{\sqrt{2}}{2} + t\left(\frac{\sqrt{2}}{2}\right)\right)$

for $-\infty < t < \infty$

Note: Slope of T.L. at $\theta = \pi/4$?

$\frac{y'(\pi/4)}{x'(\pi/4)} = \frac{\sqrt{2}/2}{-2-\sqrt{2}/2}$

you can do $\theta = \pi/2$.

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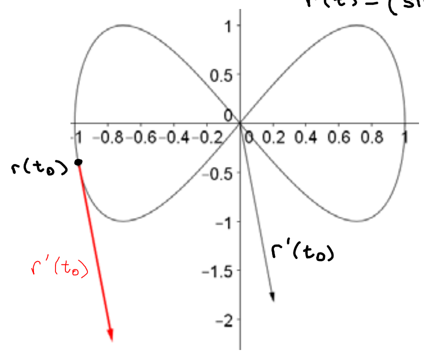
4. Consider the polar curve $r = 1 - 2\cos(\theta)$. Give the slope of the tangent line to the curve at the point where $\theta = \pi/2$.

5. Give the value of y where the tangent line in #4 intersects the y axis.

Example: Plot the parametric curve $(\sin(t), \sin(2t))$ and discuss the relationship between the curve and the derivative.

$r(t) = (\sin(t), \sin(2t))$



New Material

Position, velocity, speed and acceleration of a particle.

Next Time...

$$r(t) = (x(t), y(t))$$

$$\vec{r}(t) = x(t)i + y(t)j$$

$$\vec{v}(t) = x'(t)i + y'(t)j$$

$$\left|\vec{v}(t)\right| = \sqrt{(x'(t))^2 + (y'(t))^2}$$

Acceleration?

Falling Bodies
(neglecting friction)

Example: An object is launched from a height of 10 ft, at an angle of $\pi/4$ radians to horizontal. If the initial speed is 30 ft/sec, when will the object strike the ground, and what will the velocity of the object be at impact?