Review
Parametric Curves and Derivatives

param, for a curve:

\[ r(t) = (x(t), y(t)) \]

\[ r'(t) = x'(t)i + y'(t)j \]

dervative

\[ r'(t_0) \] is tangent to the curve at \( r(t_0) \), and \( r'(t_0) \) points in the direction of orientation.
Popper 13

1. Give the slope of the tangent line to the curve parameterized by \(( t + \cos(t) , 2t - \sin(2t) )\) at the point where \( t = 1 \).

2. Give the first component of the derivative vector at \( t = 1 \), associated with the parameterization given by \(( t + \cos(t) , 2t - \sin(2t) )\).

3. Give the second component of the derivative vector at \( t = 1 \), associated with the parameterization given by \(( t + \cos(t) , 2t - \sin(2t) )\).
Vectors: A Short Review

More generally: \( ai + bj \), where \( a, b \) are real numbers.

Adding vectors:

\[
(a_i + b_j) + (c_i + d_j) = (a+c) i + (b+d) j.
\]

Mult. by scalars:

\[
\alpha (ai + bj) = \alpha ai + \alpha bj.
\]

(\text{Euclidean}) length (or magnitude) of a vector:

\[
|ai + bj| = \sqrt{a^2 + b^2}
\]

\[
|\alpha (ai + bj)| = |\alpha ai + \alpha bj| = \sqrt{(\alpha a)^2 + (\alpha b)^2} = |\alpha| |ai + bj|.
\]
Relating Parametric Curves to Polar Curves

**Q:** How can we parametrize this curve?

**A:** Use $x = r \cos(\theta)$, $y = r \sin(\theta)$.

$(r(\theta) \cos(\theta), r(\theta) \sin(\theta))$ is a param. in terms of the ind. var. $\theta$. 

Polar curve $r = f(\theta)$. 

How: Polar curve $r = f(\theta)$. 

How can we parametrize this curve?
**Example:** Graph the polar curve \( r = 1 + 2\cos(\theta) \). Then find a parameterization for the tangent line to the curve at the points where \( \theta = \pi/4 \) and \( \theta = \pi/2 \).

\[
\begin{align*}
\text{Param:} & \quad (1 + 2\cos(\theta))\cos(\theta), (1 + 2\cos(\theta))\sin(\theta) \\
\text{For } \theta = \pi/4: & \\
\text{Point:} & \\
& (\sqrt{2}/2, \sqrt{2}/2) \\
x'(\theta) & = (1 + 2\cos(\theta))\cos(\theta) - \sin(\theta) + \cos(\theta)(-2\sin(\theta)) \\
y'(\theta) & = (1 + 2\cos(\theta))\sin(\theta) + \sin(\theta)(-2\cos(\theta)) \\
\theta = \pi/4: & \\
x'(\pi/4) & = -1 - \sqrt{2}/2 + (-1) = -2 - \sqrt{2}/2 \\
y'(\pi/4) & = 1 + \sqrt{2}/2 + (-1) = \sqrt{2}/2 \\
\text{Param of T.L. at } \theta = \pi/4 & \\
(1 + \sqrt{2}/2 + t(-2 - \sqrt{2}/2), 1 + \sqrt{2}/2 + t(-2 - \sqrt{2}/2)) \\
\text{for } -\infty < t < \infty \\
\text{Note: Slope of T.L. at } \theta = \pi/4? \\
y'(\pi/4) & = \frac{\sqrt{2}/2}{-2 - \sqrt{2}/2} \\
\text{you can do } \theta = \pi/2.
Popper 13

4. Consider the polar curve $r = 1 - 2\cos(\theta)$. Give the slope of the tangent line to the curve at the point where $\theta = \frac{\pi}{2}$.

5. Give the value of $y$ where the tangent line in #4 intersects the $y$ axis.
Example: Plot the parametric curve $(\sin(t), \sin(2t))$ and discuss the relationship between the curve and the derivative.

$r(t) = (\sin(t), \sin(2t))$
New Material
Position, velocity, speed and acceleration of a particle.

\[ r(t) = (x(t), y(t)) \]

\[ \vec{r}(t) = x(t)i + y(t)j \]

\[ \vec{v}(t) = x'(t)i + y'(t)j \]

\[ |\vec{v}(t)| = \sqrt{(x'(t))^2 + (y'(t))^2} \]

Acceleration?
Falling Bodies
(neglecting friction)
Example: An object is launched from a height of 10 ft, at an angle of $\pi/4$ radians to horizontal. If the initial speed is 30 ft/sec, when will the object strike the ground, and what will the velocity of the object be at impact?