Review

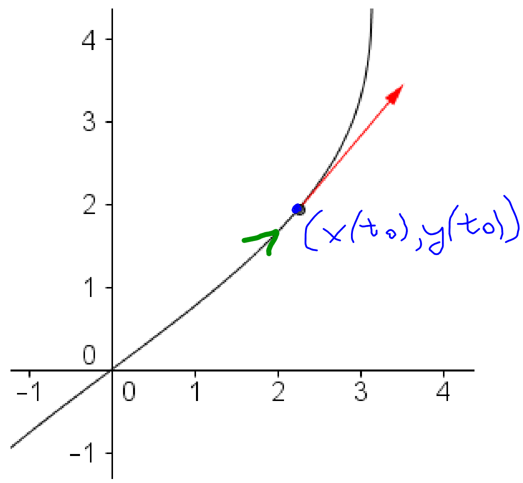
Parametric Curves and Derivatives

param. for a curve.

$$r(t) = (x(t), y(t))$$

$$r'(t) = \underbrace{x'(t)i + y'(t)\,j}_{\text{vector}}$$

derivative



$r'(t_0)$ is tangent to the curve at $r(t_0)$, and $r'(t_0)$ points in the direction of orientation.

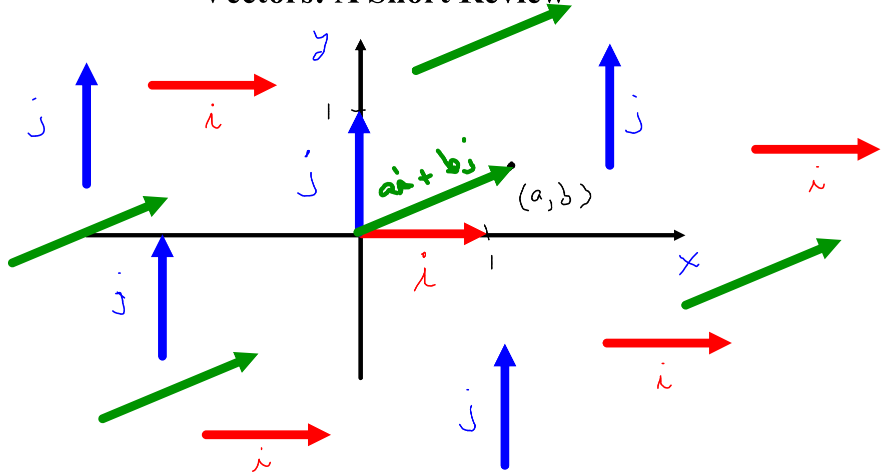
Popper 13

1. Give the slope of the tangent line to the curve parameterized by ($t + \cos(t)$, $2t - \sin(2t)$) at the point where t = 1.

2. Give the first component of the derivative vector at $t = 1$, associated with the parameterization given by ($t + \cos(t)$, $2t - \sin(2t)$).

3. Give the second component of the derivative vector at $t = 1$, associated with the parameterization given by ($t + \cos(t)$, $2t - \sin(2t)$).

Vectors: A Short Review



More generally: $ai + bj$, where a, b are real numbers.

Adding vectors:

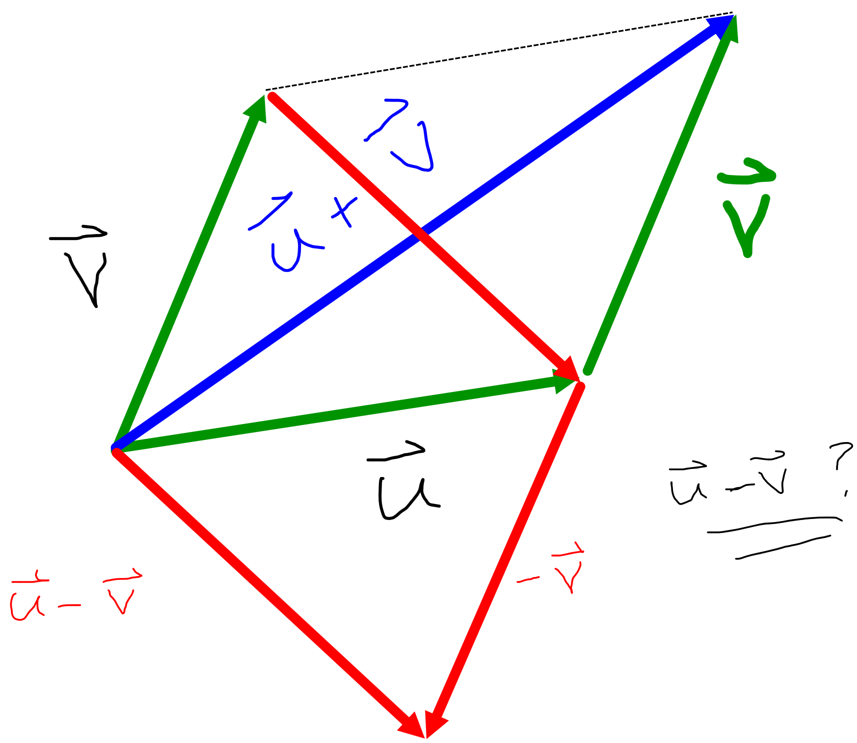
$$(ai + bj) + (ci + dj)$$
$$= (a+c)i + (b+d)j.$$

Mult. by $\boxed{\text{scalars}}$: $\alpha(ai + bj)$
$$= \alpha a i + \alpha b j.$$

(Euclidean) length (or magnitude) of a vector:

$$|ai + bj| = \sqrt{a^2 + b^2}$$

$$|\alpha(ai + bj)| = |\alpha a i + \alpha b j|$$
$$= \sqrt{(\alpha a)^2 + (\alpha b)^2}$$
$$= |\alpha|\,|ai + bj|.$$



$\vec{v}$
$\vec{u}+\vec{v}$
$\vec{v}$
$\vec{u}$
$\vec{u}-\vec{v}$?
$\vec{u}-\vec{v}$
$-\vec{v}$

Relating Parametric Curves to Polar Curves

How: Polar curve $r = r(\theta)$.

Q: How can we param. this curve??

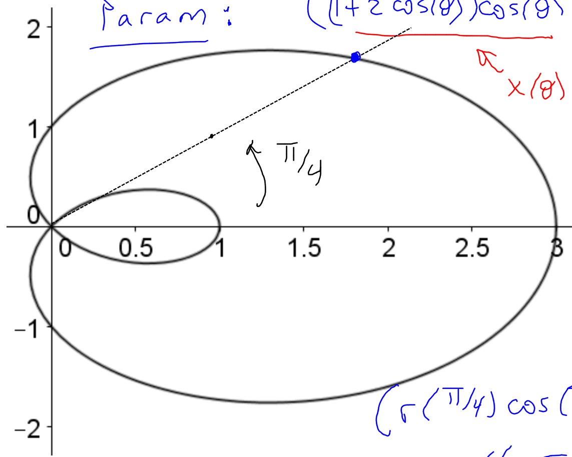
A: Use $x = r\cos(\theta)$, $y = r\sin(\theta)$

$$\big(r(\theta)\cos(\theta),\ r(\theta)\sin(\theta)\big)$$

Is a param. in terms of the ind. var. θ.

Example: Graph the polar curve $r = 1 + 2\cos(\theta)$. Then find a parameterization for the tangent line to the curve at the points where $\theta = \pi/4$ and $\theta = \pi/2$.

Param: $\left(\underbrace{(1+2\cos(\theta))\cos(\theta)}_{x(\theta)}, \underbrace{(1+2\cos(\theta))\sin(\theta)}_{y(\theta)}\right)$



For $\theta = \pi/4$:

Point:

$$\left(r(\pi/4)\cos(\pi/4), r(\pi/4)\sin(\pi/4)\right)$$
$$= \left((1+\sqrt{2})\frac{\sqrt{2}}{2}, (1+\sqrt{2})\frac{\sqrt{2}}{2}\right)$$
$$= \left(1+\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}\right)$$

$$x'(\theta) = \underbrace{(1+2\cos(\theta))}_{r(\theta)}(-\sin(\theta)) + \cos(\theta)\underbrace{(-2\sin(\theta))}_{r'(\theta)}$$

$$y'(\theta) = (1+2\cos(\theta))\cos(\theta) + \sin(\theta)(-2\sin(\theta))$$

At $\pi/4$

$$x'(\pi/4) = -1 - \frac{\sqrt{2}}{2} + (-1) = -2 - \frac{\sqrt{2}}{2}$$

$$y'(\pi/4) = 1 + \frac{\sqrt{2}}{2} + (-1) = \frac{\sqrt{2}}{2}$$

Param of T.L. at $\theta = \pi/4$

$$\left(1+\frac{\sqrt{2}}{2} + t\left(-2-\frac{\sqrt{2}}{2}\right), 1+\frac{\sqrt{2}}{2} + t\left(\frac{\sqrt{2}}{2}\right)\right)$$

for $-\infty < t < \infty$.

Note: Slope of T.L. at $\theta = \pi/4$?

$$\frac{y'(\pi/4)}{x'(\pi/4)} = \frac{\sqrt{2}/2}{-2-\sqrt{2}/2}$$

You can do $\theta = \pi/2$.

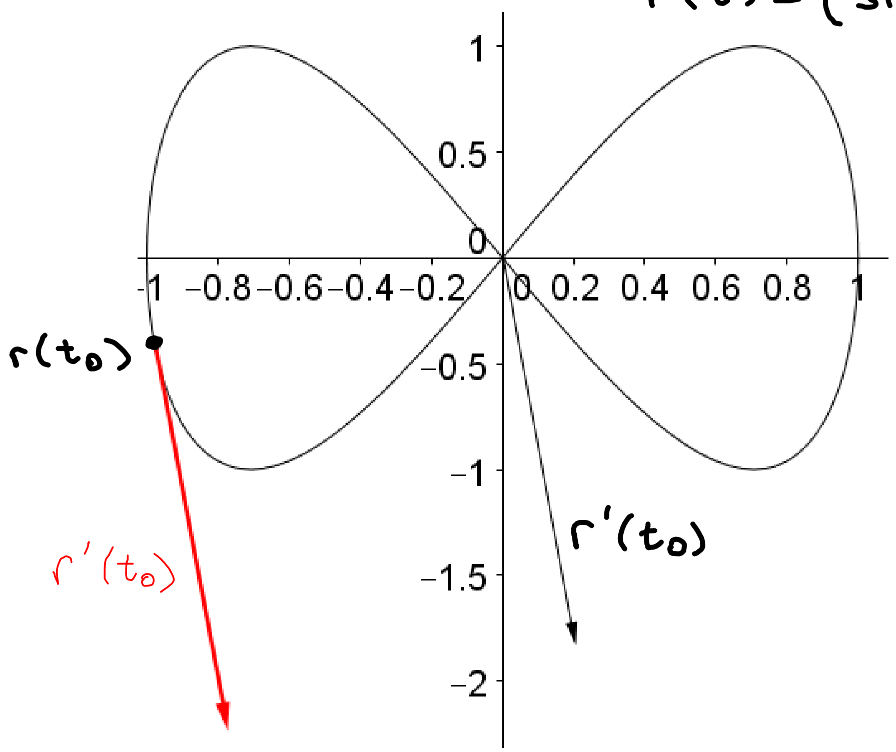
Popper 13

4. Consider the polar curve $r = 1 - 2\cos(\theta)$. Give the slope of the tangent line to the curve at the point where $\theta = \pi/2$.

5. Give the value of y where the tangent line in #4 intersects the y axis.

Example: Plot the parametric curve $(\sin(t), \sin(2t))$ and discuss the relationship between the curve and the derivative.

$r(t) = (\sin(t), \sin(2t))$



New Material

Position, velocity, speed and acceleration of a particle.

$$r(t) = (x(t), y(t))$$

$$\vec{r}(t) = x(t)i + y(t)j$$

Next Time...

$$\vec{v}(t) = x'(t)i + y'(t)j$$

$$\left|\vec{v}(t)\right| = \sqrt{(x'(t))^2 + (y'(t))^2}$$

Acceleration?

Falling Bodies
(neglecting friction)

Example: An object is launched from a height of 10 ft, at an angle of $\pi/4$ radians to horizontal. If the initial speed is 30 ft/sec, when will the object strike the ground, and what will the velocity of the object be at impact?