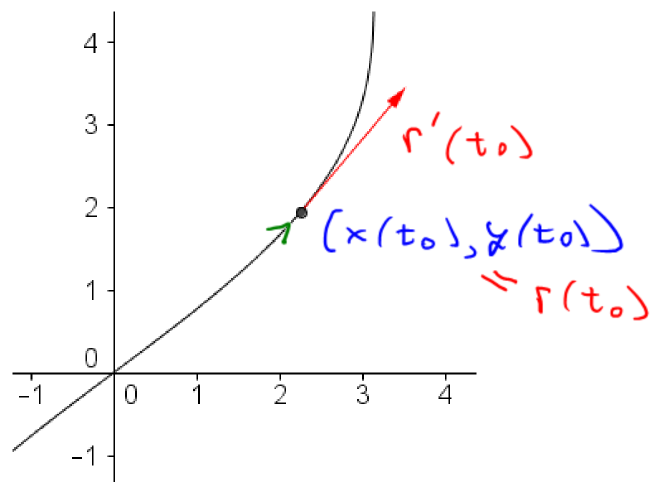
Review
Parametric Curves and Derivatives

param. for a curve

$$r(t) = (x(t), y(t))$$

$$r'(t) = \underbrace{x'(t)i + y'(t)\,j}_{\text{vector}}$$

derivative of $r(t)$

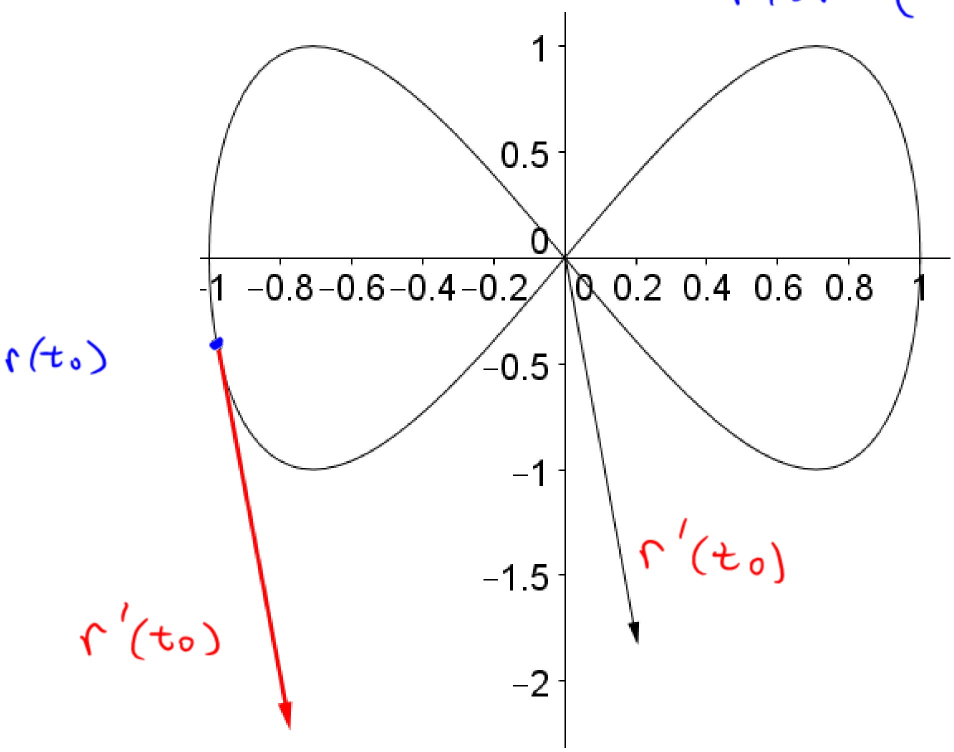


$r'(t_0)$ is tangent to the curve at $r(t_0)$,

<u>AND</u> it points in the direction of orientation.

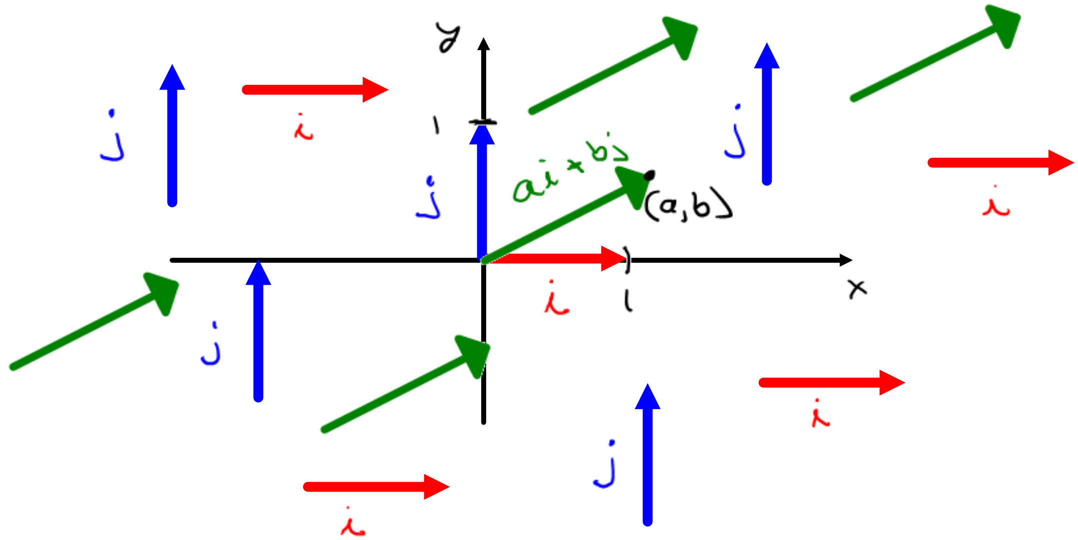
Example: Plot the parametric curve $(\sin(t), \sin(2t))$ and discuss the relationship between the curve and the derivative.

$r(t) = (\sin(t), \sin(2t))$



$r'(t) = \cos(t)\, i + 2\cos(2t)\, j$

Vectors: A Short Review



More generally: $\quad ai + bj$

Length of $ai+bj$ (Euclidean length) $\qquad |ai+bj| = \sqrt{a^2+b^2}$

$$\alpha(ai+bj) = (\alpha a)i + (\alpha b)j$$

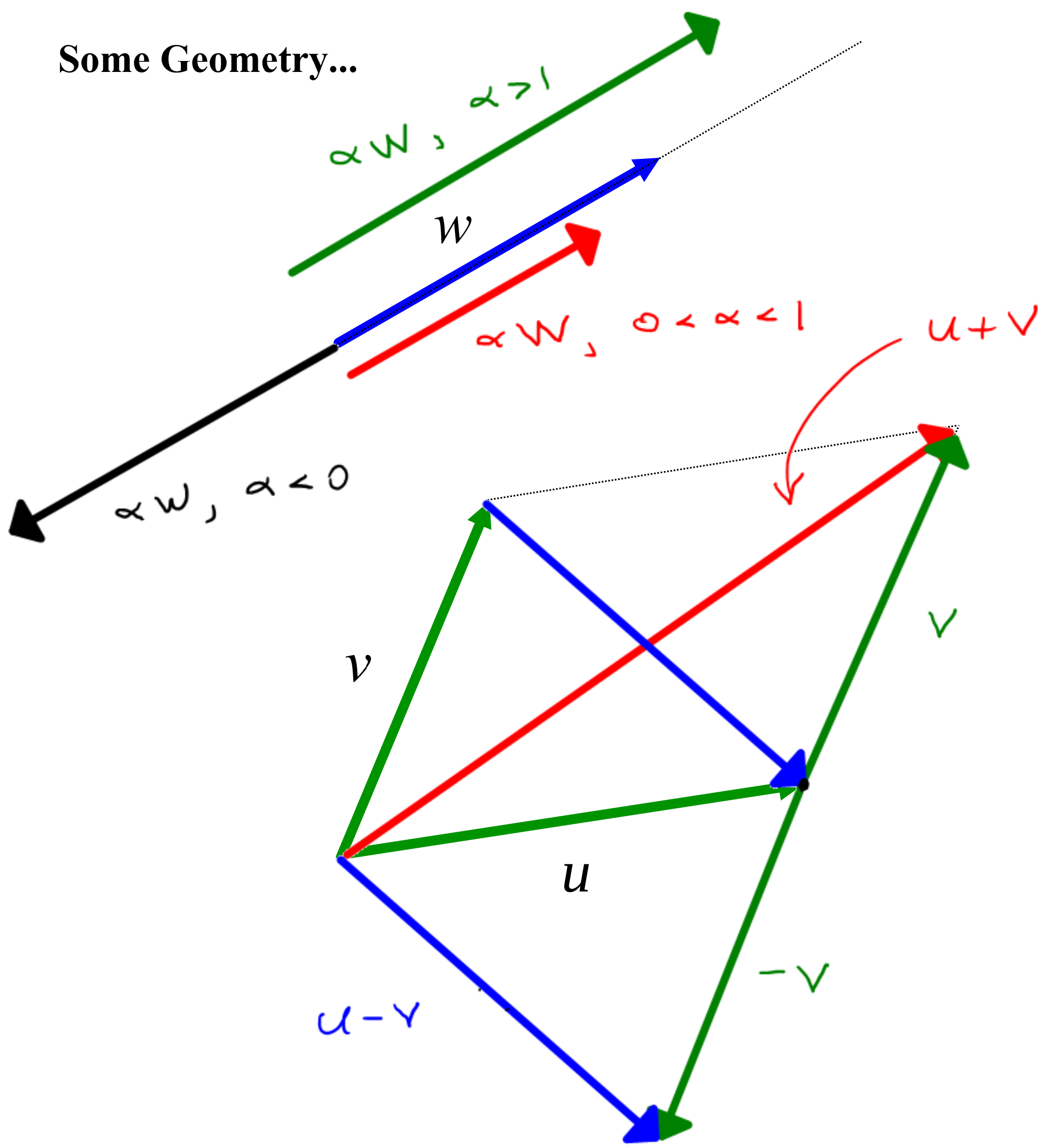
Note: $\quad |\alpha(ai+bj)| = \sqrt{\alpha^2 a^2 + \alpha^2 b^2}$

$$= |\alpha|\sqrt{a^2+b^2}$$

$$= |\alpha|\,|ai+bj|$$

$$(ai+bj)+(ci+dj) = (a+c)i + (b+d)j$$

Some Geometry...



Relating Parametric Curves to Polar Curves

Polar Curve: $r = \underline{\underline{r(\theta)}}$

Parametric: recall that $x = r\cos(\theta)$, $y = r\sin(\theta)$

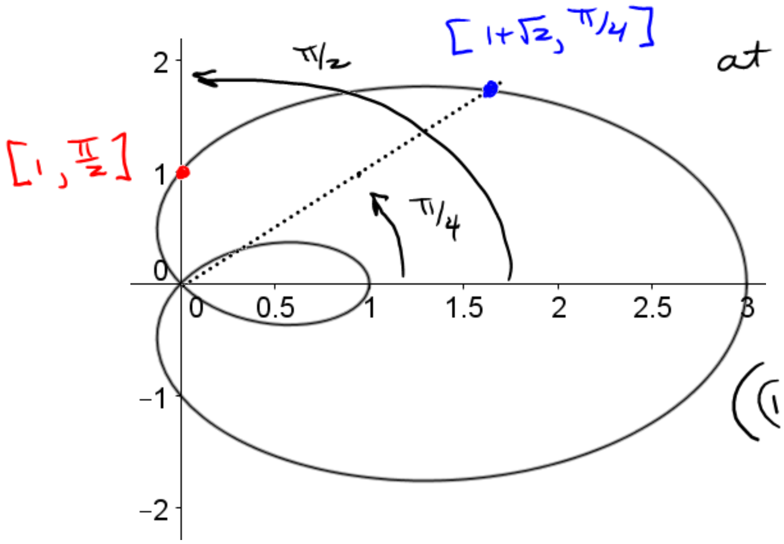
$$\left(r(\theta)\cos(\theta) \,,\, r(\theta)\sin(\theta) \right)$$

This parameterizes the curve in terms of the parameterizing variable θ.

Polar: $r = r(\theta)$

Leads to a parametrization of the form $(r(\theta)\cos(\theta) , r(\theta)\sin(\theta))$, where θ is the parameterizing variable.

Example: Graph the polar curve $r = 1 + 2\cos(\theta)$. Then find a parameterization for the tangent line to the curve at the points where $\theta = \pi/4$ and $\theta = \pi/2$.



at $\theta = \frac{\pi}{4}$, $r = 1+\sqrt{2}$

$\theta = \frac{\pi}{2} \Rightarrow r = 1$

Param. for curve.

$$\Big(\underbrace{(1+2\cos(\theta))\cos(\theta)}_{x(\theta)}, \underbrace{(1+2\cos(\theta))\sin(\theta)}_{y(\theta)}\Big)$$

$\underline{\theta = \frac{\pi}{4}}$: $\underline{\text{Point}}$: $\left(x(\pi/4), y(\pi/4)\right) = \left((1+\sqrt{2})\frac{\sqrt{2}}{2}, (1+\sqrt{2})\frac{\sqrt{2}}{2}\right)$

$$= \left(\frac{\sqrt{2}}{2}+1, \frac{\sqrt{2}}{2}+1\right)$$

$\underline{\text{Derivative vector}}$: $-\left(\frac{\sqrt{2}}{2}+2\right)i + \frac{\sqrt{2}}{2} j$

$$x'(\theta) = (1+2\cos(\theta))(-\sin(\theta)) + \cos(\theta)(-2\sin(\theta))$$

$$y'(\theta) = (1+2\cos(\theta))\cos(\theta) + \sin(\theta)(-2\sin(\theta))$$

$$x'(\pi/4) = (1+\sqrt{2})\left(-\frac{\sqrt{2}}{2}\right) + (-1) = -\frac{\sqrt{2}}{2} - 2$$

$$y'(\pi/4) = (1+\sqrt{2})\frac{\sqrt{2}}{2} - 1 = \frac{\sqrt{2}}{2}$$

$\underline{\text{Param for T.L.}}$: $\left(\frac{\sqrt{2}}{2}+1 + t\left(-\frac{\sqrt{2}}{2}-2\right), \frac{\sqrt{2}}{2}+1+t\frac{\sqrt{2}}{2}\right)$

$$-\infty < t < \infty.$$