EMCF Keys are posted.

## Information

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|---|---|---|---|---|---|---|
| 10 Spring break | 11 Spring break | 12 Spring break | 13 Spring break | 14 Spring break | 15 Spring break | 16 **Test 3 Scheduler Opens!!** Spring break |
| 17 | 18 * **EMCF23 due at 9am** Homework 8 due | 19 | 20 | 21 | 22 | 23 **Quiz 8 closes** (9.6-9.8) |
| 24 | 25 | 26 | 27 **Last day to drop with a W** | 28 | 29 | 30 **Quiz 9 closes** (10.1-10.3) **Test 3 starts** **Check the dates on CourseWare** |
| 31 | April 1 | 2 | 3 | 4 | 5 | 6 **Quiz 10 closes** (10.4-10.5) |

**Length of a Curve**
(final comments)

**Question:** How do we find the length of a polar curve?

A: Spse $r = r(\theta)$, $a \le \theta \le b$.

(Assuming one-to-one)

1. Param the curve.

$$(\underbrace{r(\theta)\cos(\theta)}_{x(\theta)},\ \underbrace{r(\theta)\sin(\theta)}_{y(\theta)})$$

$a \le \theta \le b$.

2. Formula

$$\int_a^b \sqrt{(x'(\theta))^2 + (y'(\theta))^2}\, d\theta \boxed{= \int_a^b \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta}$$

$$x'(\theta) = r(\theta)(-\sin(\theta)) + r'(\theta)\cos(\theta)$$

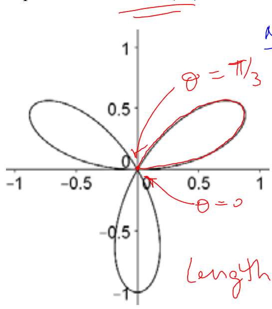
$$y'(\theta) = r(\theta)\cos(\theta) + r'(\theta)\sin(\theta)$$

$$(x'(\theta))^2 = \boxed{r(\theta)^2\sin^2(\theta)} - 2r(\theta)r'(\theta)\sin(\theta)\cos(\theta) + \boxed{r'(\theta)^2\cos^2(\theta)}$$

$$(y'(\theta))^2 = \boxed{r(\theta)^2\cos^2(\theta)} + 2r(\theta)r'(\theta)\sin(\theta)\cos(\theta) + \boxed{r'(\theta)^2\sin^2(\theta)}$$

ADD $x'(\theta)^2 + y'(\theta)^2 = r(\theta)^2 + r'(\theta)^2$

**Example:** Give an integral that represents the circumference of one petal of the polar flower $r = \sin(3\theta)$.




Note: $r = 0$ when

$\sin(3\theta) = 0$

$\boxed{\theta = 0,\ \pi/3}$

others

$$\text{Length} = \int_0^{\pi/3} \sqrt{\underbrace{\sin^2(3\theta) + 9\cos^2(3\theta)}_{r(\theta)^2 + r'(\theta)^2}}\, d\theta$$

$r(\theta) = \sin(3\theta)$

$r'(\theta) = 3\cos(3\theta)$

**Sequences - Chapter 10**
(new material)

A **sequence** is a list of numbers.

This is different from a set of numbers!!

first

third

**Example:** 2, 12, -3, 5, 3, -1, 0. is a sequence of numbers.

second

**Note:** A sequence is more than just the numbers in the sequence!

There is a first item, a second item, etc...

**Note:** Sometimes sequences of numbers have a pattern or nice behavior, and sometimes they don't.

**Example:** Consider the sequence generated by randomly flipping a coin (forever) and recording 1 for each head, and 0 for each tail.

No pattern here

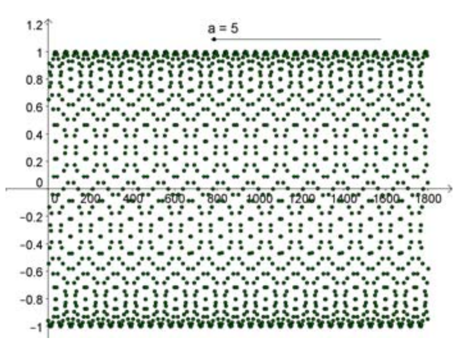
Although we have introduced sequences as lists of numbers, sequences can also be lists of objects.

**Example:** $\boxed{\text{Suppose } a \text{ is a given positive numbers}}$ and consider the *sequence* of points

$$(1,\sin(a)),\ (2,\sin(2a)),\ (3,\sin(3a)),\ \ldots$$

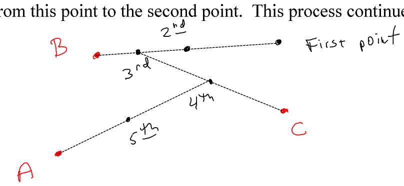
list of points

(Geogebra Demo)

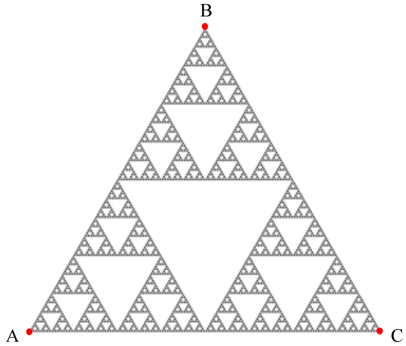



Sometimes random behavior can lead to emerging patterns!

**Example:** 3 non-colinear points named A, B and C are fixed in the *xy*-plane. Then a sequence of points is created as follows. The first point is chosen at random. To create the second point in the sequence, we select one of A, B or C (at random), and then create the midpoint of the line segment from this point to the first point. To create the third point in the sequence, we select one of A, B or C (at random), and then create the midpoint of the line segment from this point to the second point. This process continues forever.



**Sierpinski Triangle**
This pattern always emerges after
the first 100 or so points!!



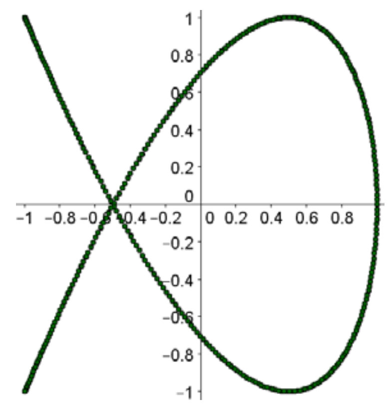

...and sometimes it does not...

**Example:** Consider the sequence of points given by $(\cos(n), \sin(\sqrt{2}\,n))$ for $n = 1, 2, 3, 4, 5, \ldots$

If you plot enough terms, you will essentially "paint" the square from -1 to 1 in $x$ and -1 to 1 in $y$.

...and sometimes it does...

**Example:** Consider the sequence of points given by $(\cos(8n), \sin(12n))$ for $n = 1, 2, 3, 4, 5, \ldots$




Mar 8-7:19 AM

**Infinite Sequences**

**Example:** 1, 1/2, 1/3, 1/4, 1/5, ...

sequences with infinitely many terms.

**Example:** -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, ...

**Popper 15**

1. Give the 10th term in the sequence 1, -1, 1, -1, 1, -1, 1, -1, ...

2. A sequence is given by $a_0, a_1, a_2, a_3, \ldots$ where $a_n = 1/(n+1)$. Give the 20th term in the sequence. **Be careful!!!**

Most of the sequences that we deal with will have a **pattern** and reasonably **nice behavior**.

The **pattern** will come from a generating formula.

The **nice behavior** will come in the form of "**limiting behavior**."

**Notes:**

1. Formally, a an infinite sequence of numbers is a function from a set of the form {k, k+1, k+2, ... } to the set of real numbers.
In many cases k = 0 or k = 1.

2. One of the most important aspects (from our perspective) will be something called the "**limit of a sequence**."

If we know the function $a(n)$, then we use the notation $a_n$ to stand for $a_n$, and we write one of

$a_n$ ← e.g. if $a_n = \frac{1}{n}$

$\{a_n\}$ $\qquad \frac{1}{n}$

or

$\{a_n\}_{\underline{n=1}}^{\infty}$ $\qquad \{\frac{1}{n}\}$ $\qquad \{\frac{1}{n}\}_{n=1}^{\infty}$

to stand for the sequence

$$\boxed{a_1, a_2, a_3, a_4, \ldots}$$

Note: $\{a_n\}_{n=-2}^{\infty}$ stands for

$$a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$$

↑ first term.

**Examples:** Give the "function" associated with each of the following sequences.

$\frac{1}{n}$ with $n=1,2,3,\ldots$ or $\frac{1}{n+1}$ with $n=0,1,2,\ldots$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$$

$(-1)^n$ with $n=1,2,3,\ldots$ or $(-1)^{n+1}$ with $n=0,1,2,\ldots$

$$-1, 1, -1, 1, -1, 1, -1, 1, \ldots$$

or $\{\cos(n\pi)\}_{n=1}^{\infty}$

$$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots$$

$\left\{\frac{(-1)^{n+1}}{n}\right\}_{n=1}^{\infty}$

**Popper 15**

3. Find the function associated with the sequence 1, 1/4, 1/9, 1/16, ... and then give the 10th term in the sequence.

**Example:** $a_n = \frac{1}{n+2}$, $n = 3, 4, 5, \ldots$

$\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \ldots$

tend towards 0.

The limit of a sequence is a number that the sequence values $a_n$ tend towards as $n \to \infty$.

(if such a number exists)

The limit of a sequence might not dependent upon the first $10$, $10^2$, $10^3$, $10^4$, or $10^{20}$ values.

**Example:** Suppose

$$a_n = \begin{cases} 2^n & \text{for } n < 10^{50} \\ \frac{1}{n} & \text{for } n \geq 10^{50} \end{cases}$$

"The sequence converges"
$\Updownarrow$
There is a limit.

If there is not a value, then we say the limit does not exist

"The sequence diverges" $\equiv$ No limit.

**Example:** Give the limit of the sequence

$$\left\{ \frac{2n-1}{3n+2} \right\}_{n=1}^{\infty} \to \frac{2}{3}$$

Converges.

$$\lim_{n \to \infty} \frac{2n-1}{3n+2} = \frac{2}{3}$$

**Popper 15**

4. Give the limit of $\left\{ \dfrac{2n-3}{n^2+6n+1} \right\}_{n=0}^{\infty}$

**Example:** Give the limit of the sequence

$$\left\{ (-1)^n \right\}_{n=1}^{\infty} \qquad \textbf{DNE}$$

**-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, ...**

**The sequence diverges. There is no limit.**

**Example:** Give the limit of the sequence

$$\left\{ \left(1+\frac{1}{n}\right)^n \right\}_{n=1}^{\infty} \longrightarrow e$$

**See Chapter 7. The limit of this sequence is how *e* is defined.**

**This sequence converges to *e*.**

**Example:** Give the limit of the sequence

$$\left\{ \frac{1}{2^n} \right\}_{n=1}^{\infty} \longrightarrow 0$$

**1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, ...**

**The sequence converges to 0.**