EMCF Keys are posted.

Information

10 Spring break	11 Spring break	12 Spring break	13 Spring break	14 Spring break	15 Spring break	16 **Test 3 Scheduler Opens!!** Spring break
17	18 **EMCF23 due at 9am** Homework 8 due	19	20	21	22	23 **Quiz 8 closes** (9.6-9.8)
24	25	26	27 **Last day to drop with a W**	28	29	30 **Quiz 9 closes** (10.1-10.3) **Test 3 starts** **Check the dates on CourseWare**
31	April 1	2	3	4	5	6 **Quiz 10 closes** (10.4-10.5)

Length of a Curve
(final comments)

Question: How do we find the length of a polar curve?

A: Spse $r = r(\theta)$, $a \le \theta \le b$.

(Assuming one-to-one)

1. Param the curve.

$$\left(\underbrace{r(\theta)\cos(\theta)}_{x(\theta)},\ \underbrace{r(\theta)\sin(\theta)}_{y(\theta)} \right)$$

$a \le \theta \le b$.

2. Formula

$$\int_a^b \sqrt{(x'(\theta))^2 + (y'(\theta))^2}\, d\theta \boxed{= \int_a^b \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta}$$

$$x'(\theta) = r(\theta)(-\sin(\theta)) + r'(\theta)\cos(\theta)$$

$$y'(\theta) = r(\theta)\cos(\theta) + r'(\theta)\sin(\theta)$$

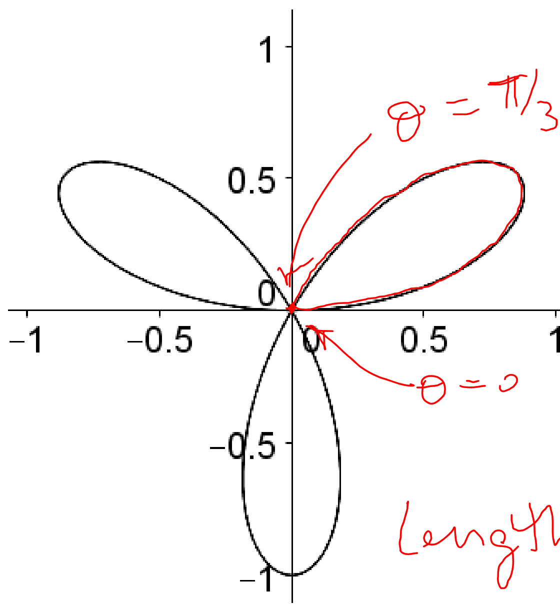
$$(x'(\theta))^2 = r(\theta)^2\sin^2(\theta) - 2r(\theta)r'(\theta)\sin(\theta)\cos(\theta) + r'(\theta)^2\cos^2(\theta)$$

$$(y'(\theta))^2 = r(\theta)^2\cos^2(\theta) + 2r(\theta)r'(\theta)\sin(\theta)\cos(\theta) + r'(\theta)^2\sin^2(\theta)$$

ADD

$$x'(\theta)^2 + y'(\theta)^2 = r(\theta)^2 + r'(\theta)^2$$

Example: Give an integral that represents the circumference of one petal of the polar flower $r = \sin(3\theta)$.



Note: $r = 0$ when

$\sin(3\theta) = 0$

$\boxed{\theta = 0, \ \pi/3}$

others

$$\text{Length} = \int_0^{\pi/3} \sqrt{\underbrace{\sin^2(3\theta) + 9\cos^2(3\theta)}_{r(\theta)^2 + r'(\theta)^2}}\, d\theta$$

$r(\theta) = \sin(3\theta)$

$r'(\theta) = 3\cos(3\theta)$

Sequences - Chapter 10

(new material)

A **sequence** is a list of numbers.

This is different from a set of numbers!!

first

third

Example: 2, 12, -3, 5, 3, -1, 0. is a sequence of numbers.

second

Note: A sequence is more than just the numbers in the sequence!

There is a first item, a second item, etc...

Note: Sometimes sequences of numbers have a pattern or nice behavior, and sometimes they don't.

Example: Consider the sequence generated by randomly flippling a coin (forever) and recording 1 for each head, and 0 for each tail.

No pattern here

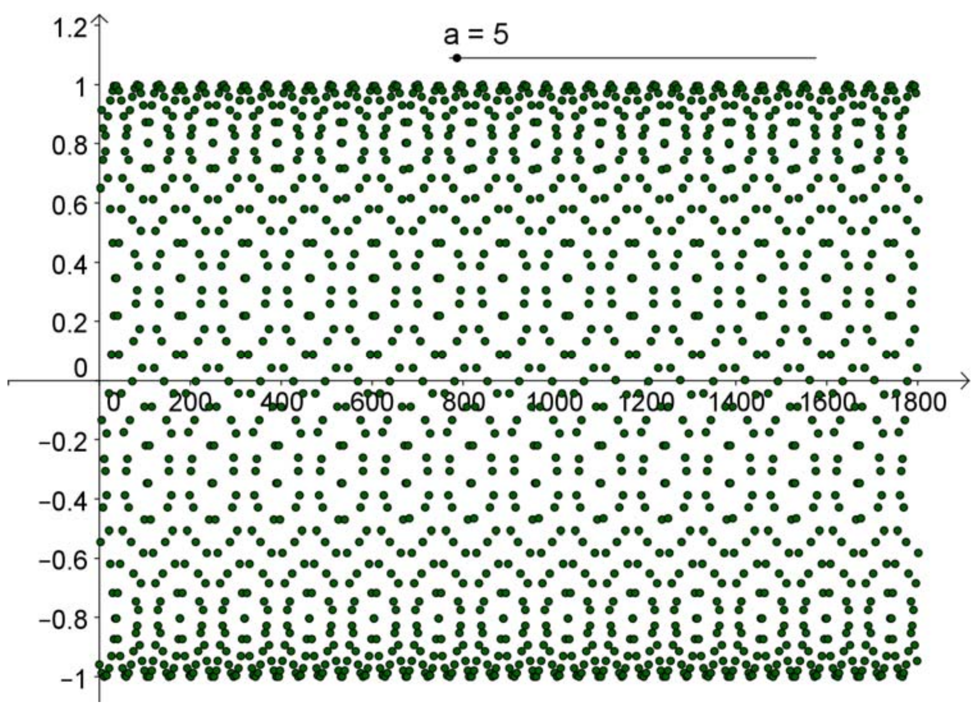
Although we have introduced sequences as lists of numbers, sequences can also be lists of objects.

Example: Suppose a is a given positive numbers, and consider the *sequence* of points

$$(1,\sin(a)),\ (2,\sin(2a),\ (3,\sin(3a),\ \ldots$$

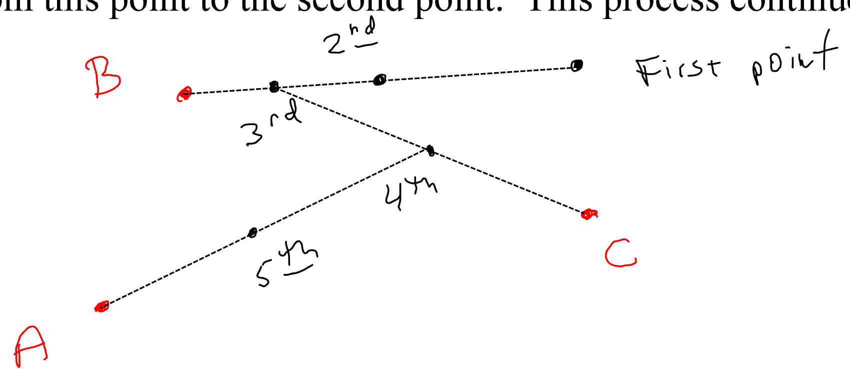
list of points

(Geogebra Demo)



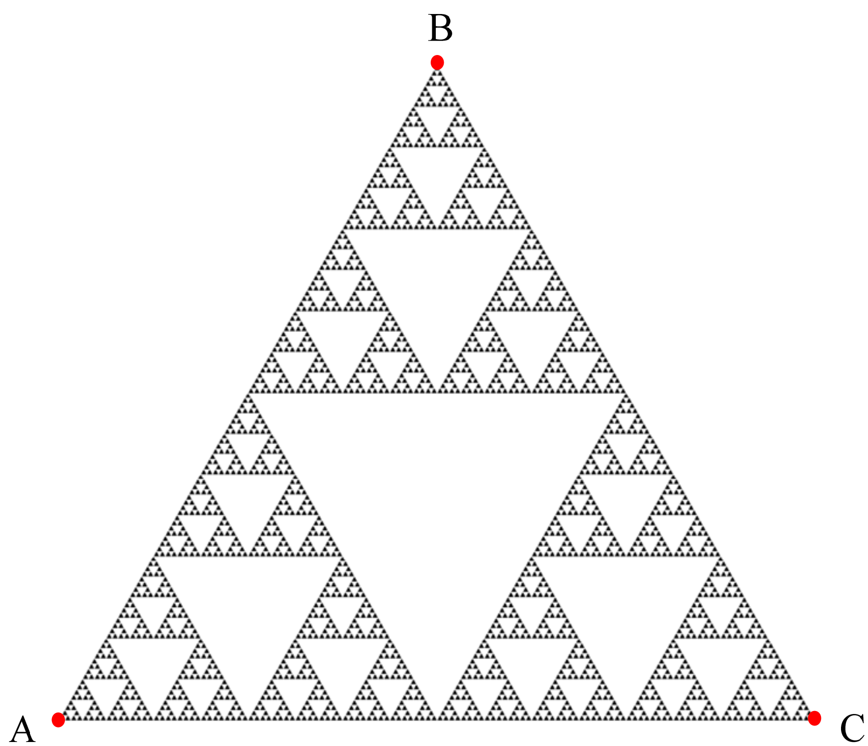
Sometimes random behavior can lead to emerging patterns!

Example: 3 non-colinear points named A, B and C are fixed in the *xy*-plane. Then a sequence of points is created as follows. The first point is chosen at random. To create the second point in the sequence, we select one of A, B or C (at random), and then create the midpoint of the line segment from this point to the first point. To create the third point in the sequence, we select one of A, B or C (at random), and then create the midpoint of the line segment from this point to the second point. This process continues forever.



Sierpinski Triangle

This pattern always emerges after
the first 100 or so points!!



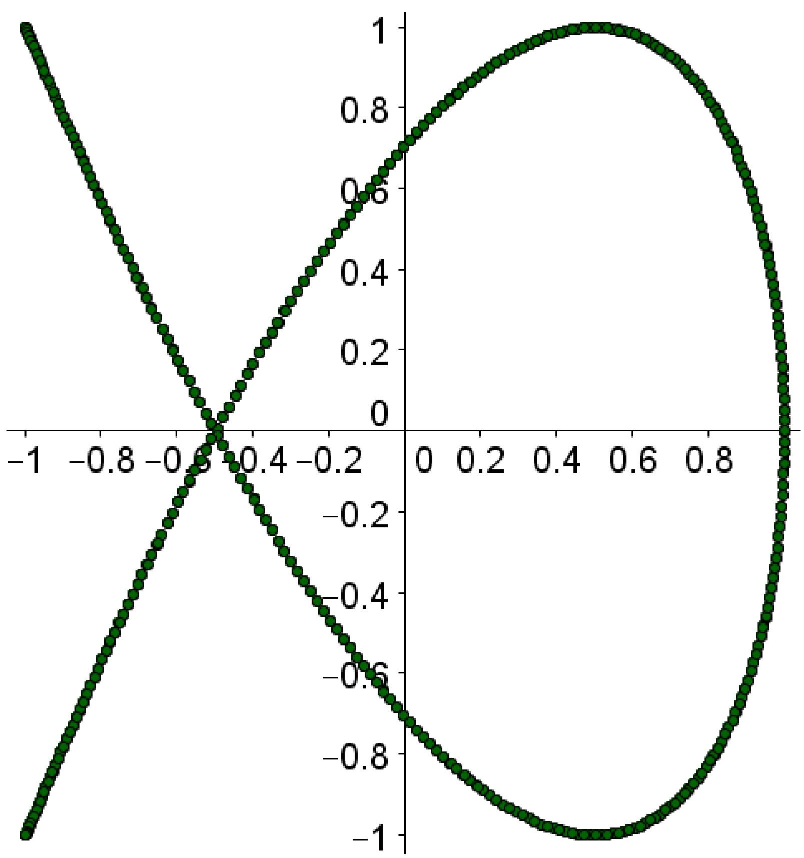
...and sometimes it does not...

Example: Consider the sequence of points given by $(\cos(n), \sin(\sqrt{2}\, n))$ for $n = 1, 2, 3, 4, 5, \ldots$

If you plot enough terms, you will essentially "paint" the square from -1 to 1 in x and -1 to 1 in y.

...and sometimes it does...

Example: Consider the sequence of points given by $(\cos(8n), \sin(12n))$ for $n = 1, 2, 3, 4, 5, \ldots$



Infinite Sequences

Example: 1, 1/2, 1/3, 1/4, 1/5, ...

sequences with infinitely many terms.

Example: -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, ...

Popper 15

1. Give the 10^{th} term in the sequence 1, -1, 1, -1, 1, -1, 1, -1, ...

2. A sequence is given by $a_0, a_1, a_2, a_3, \ldots$ where $a_n = 1/(n + 1)$. Give the 20th term in the sequence. **Be careful!!!**

Most of the sequences that we deal with will have a **pattern** and reasonably **nice behavior**.

The **pattern** will come from a generating formula.

The **nice behavior** will come in the form of "**limiting behavior**."

Notes:

1. Formally, a an infinite sequence of numbers is a function from a set of the form $\{k, k+1, k+2, \ldots\}$ to the set of real numbers.
In many cases $k = 0$ or $k = 1$.

2. One of the most important aspects (from our perspective) will be something called the "**limit of a sequence**."

If we know the function $a(n)$, then we use the notation a_n to stand for a_n, and we write one of

$$a_n$$

e.g. if $a_n = \frac{1}{n}$

$$\{a_n\}$$

$\frac{1}{n}$

or

$$\{a_n\}_{n=1}^{\infty}$$

$\left\{\frac{1}{n}\right\}$

$\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

to stand for the sequence

$$\boxed{a_1, a_2, a_3, a_4, \ldots}$$

__Note:__ $\{a_n\}_{n=-2}^{\infty}$ stands for

$$a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$$

first term.

Examples: Give the "function" associated with each of the following sequences.

$$1, \ \frac{1}{2}, \ \frac{1}{3}, \ \frac{1}{4}, \ \frac{1}{5}, \ \dots$$

$\frac{1}{n}$ with $n = 1, 2, 3, \dots$

or

$\frac{1}{n+1}$ with $n = 0, 1, 2, \dots$

$$-1, \ 1, \ -1, \ 1, \ -1, \ 1, \ -1, \ 1, \ \dots$$

$(-1)^n$ with $n = 1, 2, 3, \dots$

or $(-1)^{n+1}$ with $n = 0, 1, 2, \dots$

or $\left\{\cos(n\pi)\right\}_{n=1}^{\infty}$

$$1, \ -\frac{1}{2}, \ \frac{1}{3}, \ -\frac{1}{4}, \ \frac{1}{5}, \ -\frac{1}{6}, \ \dots$$

$$\left\{\frac{(-1)^{n+1}}{n}\right\}_{n=1}^{\infty}$$

Popper 15

3. Find the function associated with the sequence 1, 1/4, 1/9, 1/16, ... and then give the 10th term in the sequence.

Example: $a_n = \dfrac{1}{n+2}, \quad n = 3, 4, 5, \ldots$

$\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \ldots$

tend towards 0.

The limit of a sequence is a number that the sequence values a_n tend towards as $n \to \infty$.

(if such a number exists)

The limit of a sequence might not dependent upon the first $10,\ 10^2,\ 10^3,\ 10^4,$ or 10^{20} values.

Example: Suppose

$$a_n = \begin{cases} 2^n & \text{for } n < 10^{50} \\ \frac{1}{n} & \text{for } n \geq 10^{50} \end{cases}$$

"The sequence converges"

$\parallel$

There is a limit.

If there is not a value, then we say the limit does not exist

"The sequence diverges" $\equiv$ No limit.

Example: Give the limit of the sequence

$$\left\{\frac{2n-1}{3n+2}\right\}_{n=1}^{\infty} \longrightarrow \frac{2}{3}$$

Converges.

$$\lim_{n\to\infty} \frac{2n-1}{3n+2} = \frac{2}{3}$$

Popper 15

4. Give the limit of $\left\{ \dfrac{2n-3}{n^2+6n+1} \right\}_{n=0}^{\infty}$

Example: Give the limit of the sequence

$$\left\{(-1)^n\right\}_{n=1}^{\infty} \qquad \textbf{DNE}$$

-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, ...

The sequence diverges. There is no limit.

Example: Give the limit of the sequence

$$\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty} \longrightarrow e$$

See Chapter 7. The limit of this sequence is how e is defined.

This sequence converges to e.

Example: Give the limit of the sequence

$$\left\{ \frac{1}{2^n} \right\}_{n=1}^{\infty} \longrightarrow 0$$

1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, ...

The sequence converges to 0.