**Test 3** is almost here!

- Practice Test 3 is posted!
- Review Videos will be posted.
- An Online Live Review will be held.

**No Office Hours Today!!**

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**Quick Questions...**

Give an upper bound for the set of negative real numbers. What is the LUB of this set?

![Upper Bound Diagram]

Give a lower bound for the set of negative real numbers. What is the GLB of this set?

![Lower Bound Diagram]

Give the LUB and GLB for the sequence \((-1)^n, n = 1, 2, 3, \ldots\)

- LUB = 1
- GLB = -1

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**Popper 17**

1. Give the GLB for the sequence \(\{2 - 1/n\}_{n=1}^{\infty}\).

2. Give the LUB for the sequence \(\{2 - 1/n\}_{n=1}^{\infty}\).

3. **Give the limit of** \(\left\{\frac{2n-6}{3n^2+2}\right\}_{n=1}^{\infty}\).

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**Example:** Give the limit (if it exists) of \(\left\{\ln(2n+1) - \ln(n)\right\}_{n=1}^{\infty}\).

\[
\lim_{n \to \infty} \left(\ln\left(\frac{2n+1}{n}\right)\right) = \ln(2)
\]

---

\[\text{1}\]
Example: Give the limit (if it exists) of \( \left( \frac{2}{n} \right)^n \).

Note: When numbers close to 0 are raised to large powers, they get even closer to 0.

\[
\begin{align*}
2, 1, \left( \frac{2}{3} \right)^3, \left( \frac{2}{4} \right)^4, \left( \frac{2}{5} \right)^5, \left( \frac{2}{6} \right)^6, \ldots \\
\text{And so } 0.
\end{align*}
\]

\[
\text{for } n \geq 3 \quad 0 < \left( \frac{2}{n} \right)^n \leq \left( \frac{2}{3} \right)^3 \to 0
\]

\[
\text{Gets pinched}
\]

\[
\lim_{n \to \infty} \left( \frac{2}{n} \right)^n = 0.
\]

Example: Give the limit (if it exists) of \( \frac{2n^2 - 3n + 6}{3n - 16n^2 + 12} \).

\[
\begin{align*}
\text{rational function} \\
\lim_{n \to \infty} \frac{2n^2 - 3n + 6}{3n - 16n^2 + 12} &= \lim_{n \to \infty} \frac{2 - \frac{3}{n} + \frac{6}{n^2}}{3 - 16 + \frac{12}{n^2}} \\
&= \lim_{n \to \infty} \frac{\frac{2}{n^2} - \frac{3}{n} + \frac{6}{n^2}}{\frac{3}{n^2} - 16 + \frac{12}{n^2}} \\
&= -\frac{1}{8}.
\end{align*}
\]

Recall: If a sequence has a limit, then it is bounded.

(why?)

\[
\text{Spoe the limit is } L.
\]

All but a few terms are here.

\[
\text{maybe a few \hspace{1cm} L \hspace{1cm} maybe a few here}
\]

\[
\text{Take the smallest \hspace{1cm} Take the largest}
\]

\[
\text{lower bound \hspace{1cm} upper bound}
\]

* The sequence is bounded.
Recall: \( \{ a_n \} \)

1. Increasing Sequence:
\[ a_k < a_{k+1} \text{ for all } k \]

2. Non-decreasing Sequence:
\[ a_k \leq a_{k+1} \text{ for all } k \]

3. Decreasing Sequence
\[ a_k > a_{k+1} \text{ for all } k \]

4. Non-increasing Sequence:
\[ a_k \geq a_{k+1} \text{ for all } k \]

Recall:

What tool can be used to help determine whether a sequence is increasing or decreasing?

**Derivative**

A sequence is **monotone** if and only if it is either increasing, nondecreasing, decreasing or nonincreasing.

**Example:** Determine whether the sequence \( \left\{ \frac{1 + \frac{3n+1}{n+2}}{n+1} \right\}_{n=1}^{\infty} \) is

a. Bounded ✓

**Monotone**

Then, give the limit (if it exists).

\[ \lim_{n \to \infty} \left( 1 + \frac{3n+1}{n+2} \right)^{\frac{3}{n^3}} = 4. \]

\[ f(x) = 1 + \frac{2x+1}{x+2} \quad x \geq 1 \]
\[ f'(x) = 0 + \frac{(x+2)3 - (2x+1)}{(x+2)^2} \]
\[ = \frac{3}{(x+2)^2}, \quad x \geq 1 \]
\[ \Rightarrow f \text{ is increasing for } x \geq 1. \]
\[ \Rightarrow \text{ the sequence is increasing.} \]
\[ \Rightarrow \text{ the sequence is monotone.} \]
Example: Determine whether the sequence $\left\{ \frac{3n+(-1)^n}{n+2} \right\}_{n=1}^{\infty}$ is

a. Bounded ✓

b. Monotone

Then, give the limit (if it exists).

$$\lim_{n \to \infty} \frac{3n+(-1)^n}{n+2}$$

= $\lim_{n \to \infty} \frac{3 + \frac{(-1)^n}{n}}{1 + \frac{2}{n}}$

= $\lim_{n \to \infty} \frac{3 + 0}{1 + 0} = 3$

... the sequence is bounded.

Terms:

<table>
<thead>
<tr>
<th>n</th>
<th>$\frac{3n+(-1)^n}{n+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{3} = 1$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{7}{4}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{11}{5}$</td>
</tr>
</tbody>
</table>

... not monotone!

Example: Determine whether the sequence $\left\{ \frac{3n+n(-1)^n}{2n+5} \right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists).

$$\lim_{n \to \infty} \frac{3n+n(-1)^n}{2n+5}$$

= $\lim_{n \to \infty} \frac{3 + (-1)^n}{2 + \frac{5}{n}}$

= $\lim_{n \to \infty} \frac{3 + 0}{2 + 0} = \frac{3}{2}$

... not monotone, b/c

Bounded? No