Test 3 is almost here!

Practice Test 3 is posted!

Review Videos will be posted.

An Online Live Review will be held.

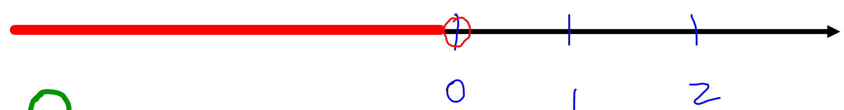
No Office Hours Today!!

Quick Questions...

Plenty: 0, 1, 2, $\frac{1}{2}$, .6713, π, ...

Give an upper bound for the set of negative real numbers. What is the LUB of this set?

0



Give a lower bound fo rthe set of negative real numbers. What is the GLB of this set?

DNE

DNE

Give the LUB and GLB for the sequence $\{(-1)^n\}_{n=2}^{\infty}$.

1, -1, 1, -1, 1, -1, ...

LUB = 1

GLB = -1

Popper 17

1. Give the GLB for the sequence $\{\, 2 - 1/n \,\}_{n=3}^{\infty}$.

2. Give the LUB for the sequence $\{\, 2 - 1/n \,\}_{n=3}^{\infty}$.

3. **Give the limit of** $\left\{ \dfrac{2n-6}{3n^2+2} \right\}_{n=1}^{\infty}$

Example: Give the limit (if it exists) of $\{\ln(2n+1)-\ln(n)\}_{n=1}^{\infty}$.

"$\infty - \infty$"

$$\lim_{n\to\infty}\left(\ln(2n+1)-\ln(n)\right)$$

$$=\lim_{n\to\infty}\ln\left(\frac{2n+1}{n}\right) \quad \left(\frac{2n+1}{n}\to 2\right)$$

$$=\ln(2)$$

Example: Give the limit (if it exists) of $\left\{ \left(\frac{2}{n} \right)^n \right\}_{n=1}^{\infty}$.

Note: When numbers close to 0 are raised to large powers, they get even closer to 0.

$2, 1, \left(\frac{2}{3}\right)^3, \left(\frac{2}{4}\right)^4, \left(\frac{2}{5}\right)^5, \left(\frac{2}{6}\right)^6, \ldots$

Tend to 0.

or For $n \geq 3$

$$0 < \left(\frac{2}{n}\right)^n \leq \left(\frac{2}{3}\right)^n \to 0$$

Gets pinched

$$\lim_{n \to \infty} \left(\frac{2}{n}\right)^n = 0.$$

Example: Give the limit (if it exists) of $\left\{ \dfrac{2n^2 - 3n + 6}{3n - 16n^2 + 12} \right\}_{n=1}^{\infty}$.

rational function.

$$\lim_{n\to\infty} \frac{2n^2 - 3n + 6}{3n - 16n^2 + 12} = \lim_{n\to\infty} \frac{n^2 \cdot \left(2 - \frac{3}{n} + \frac{6}{n^2}\right)}{n^2 \cdot \left(\frac{3}{n} - 16 + \frac{12}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{2 - \frac{3}{n} + \frac{6}{n^2}}{\frac{3}{n} - 16 + \frac{12}{n^2}} = -\frac{1}{8}$$

($\frac{3}{n} \to 0$, $\frac{6}{n^2} \to 0$, $\frac{3}{n} \to 0$, $\frac{12}{n^2} \to 0$)

Example: Give the limit (if it exists) of $\{n^n\}_{n=1}^{\infty}$. DNE

1, 4, 27, 256, 1225, . . .

zoom!

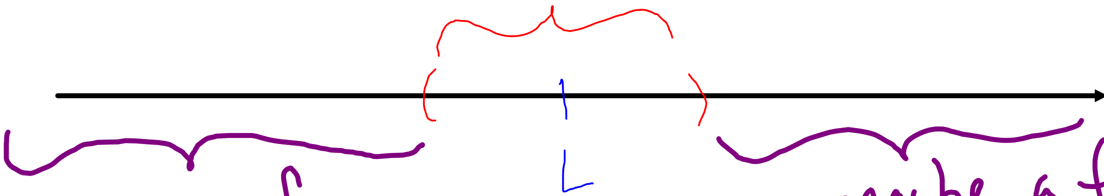
$$\lim_{n \to \infty} n^n = \infty$$

Recall: If a sequence has a limit, then it is bounded.

(why?)

Spse the limit is L.

All but a "few" terms are here.



maybe a few here

maybe a few here

Take the smallest ← Lower bound

Take the largest ← upper bound

∴ The sequence is bounded.

Recall: $\{ a_n \}$

1. Increasing Sequence:

$a_k < a_{k+1}$ for all k

2. Non-decreasing Sequence:

$a_k \leq a_{k+1}$ for all k

3. Decreasing Sequence

$a_k > a_{k+1}$ for all k

4. Non-increasing Sequence:

$a_k \geq a_{k+1}$ for all k

Recall:

What tool can (sometimes) be used to help determine whether a sequence is increasing or decreasing?

Derivative

A sequence is **monotone** if and only if it is either increasing, nondecreasing, decreasing or nonincreasing.

Recall:

(*) If a sequence has a limit, then it is bounded. Warning: The converse is not necessarily true!!

If a sequence is increasing, then the GLB is the first term and the LUB is the limit if it exists.

If a sequence is decreasing, then the GLB is the limit if it exists and the LUB is the first term.

Example: Determine whether the sequence $\left\{1+\dfrac{3n+1}{n+2}\right\}_{n=1}^{\infty}$ is

a. Bounded ✓

b. Monotone

Then, give the limit (if it exists). ← start here

$$\lim_{n\to\infty}\left(1+\frac{3n+1}{n+2}\right) = 4.$$

($\frac{3n+1}{n+2} \to 3$)

∴ the sequence is bounded

$$f(x) = 1 + \frac{3x+1}{x+2}, \quad x \geq 1$$

$$f'(x) = 0 + \frac{(x+2)\cdot 3 - (3x+1)}{(x+2)^2}$$

$$= \frac{3}{(x+2)^2}, \quad x \geq 1$$

← Positive

⇒ f is increasing for $x \geq 1$.

⇒ the sequence is increasing.

⇒ the sequence is monotone.

Example: Determine whether the sequence $\left\{\dfrac{3n+(-1)^n}{n+2}\right\}_{n=1}^{\infty}$ is

a. Bounded ✓

b. Monotone

Then, give the limit (if it exists). ← start here.

$$\lim_{n\to\infty} \frac{3n+(-1)^n}{n+2}$$

$$= \lim_{n\to\infty} \frac{n\left(3+\frac{(-1)^n}{n}\right)}{n\left(1+\frac{2}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{3+\frac{(-1)^n}{n}}{1+\frac{2}{n}} = 3$$

($\frac{(-1)^n}{n} \to 0$, $\frac{2}{n} \to 0$)

∴ the sequence is bounded.

$\left\{\dfrac{3n+(-1)^n}{n+2}\right\}_{n=1}^{\infty}$ monotone?

Terms:

$$\frac{2}{3} < \frac{7}{4} > \frac{8}{5}$$

($n=1$, $n=2$, $n=3$)

⋮

Not monotone!

Example: Determine whether the sequence $\left\{\frac{3n+n(-1)^n}{2n+5}\right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists). ⟵ Start here.

$$\lim_{n\to\infty} \frac{3n+n(-1)^n}{2n+5} = \lim_{n\to\infty} \frac{n\left(3+(-1)^n\right)}{n\left(2+\frac{5}{n}\right)}$$

$$= \lim_{n\to\infty} \frac{3+(-1)^n}{2+\frac{5}{n}} \quad \left(\frac{5}{n} \to 0\right)$$

When n gets big, the values oscillate between 4/2 and 2/2.

There is no limit!!

$= \text{DNE}$

Not monotone b/c ⟶ ↑

Bounded? Yes

Example: Determine whether the sequence $\left\{ \frac{\sqrt{n+1}}{\sqrt{n}} \right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists).