Test 3 is almost here!

Practice Test 3 is posted!

Review Videos will be posted.

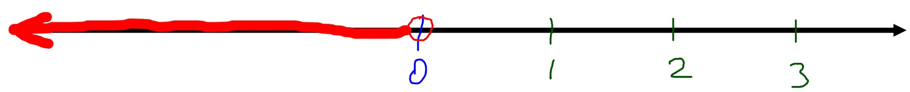
An Online Live Review will be held.

No Office Hours Today!!

Quick Questions...

There are many. $0, \frac{1}{2}, \frac{7}{8}, 1,$ etc...

Give an upper bound for the set of negative real numbers. What is the LUB of this set?



smallest upper bound $= 0$

Give a lower bound fo rthe set of negative real numbers. What is the GLB of this set?

DNE

DNE

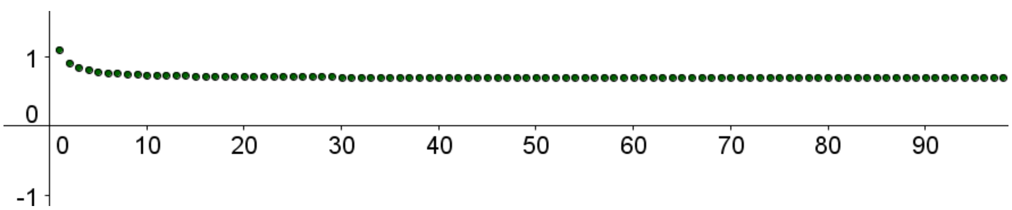
Give the LUB and GLB for the sequence $\{(-1)^n\}_{n=2}^{\infty}$.

$1, -1, 1, -1, 1, -1, 1, -1, \ldots$

LUB $= 1$

GLB $= -1$

Example: Give the limit (if it exists) of $\{\ln(2n+1) - \ln(n)\}_{n=1}^{\infty}$.



Note: $\ln(2n+1) - \ln(n) = \ln\left(\frac{2n+1}{n}\right)$

$$\lim_{n\to\infty} \ln\left(\frac{2n+1}{n}\right) = \ln(2)$$

2

Example: Give the limit (if it exists) of $\underbrace{\left\{\left(\frac{2}{n}\right)^n\right\}_{n=1}^{\infty}}$.

Note: When numbers close to 0 are raised to large powers, they get even closer to 0.

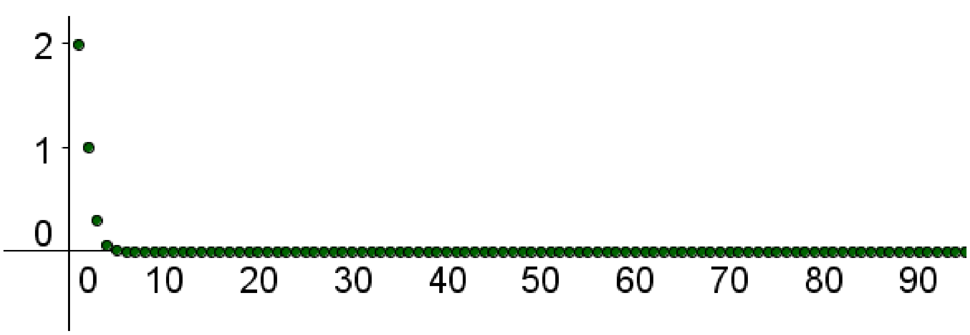
$$2,\ 1,\ \left(\frac{2}{3}\right)^3,\ \left(\frac{1}{2}\right)^4,\ \left(\frac{2}{5}\right)^5,\ \left(\frac{1}{3}\right)^6,\ \ldots$$

$\frac{2}{4}$ $\frac{2}{6}$

Note: for $n \geq 3$, $\frac{2}{n} \leq \frac{2}{3}$

$$\Rightarrow \quad 0 \leq \underbrace{\left(\frac{2}{n}\right)^n}_{\text{pinched}} \leq \left(\frac{2}{3}\right)^n \to 0 \quad \text{as } n \to \infty$$

$$\therefore \lim_{n\to\infty} \left(\frac{2}{n}\right)^n = 0.$$



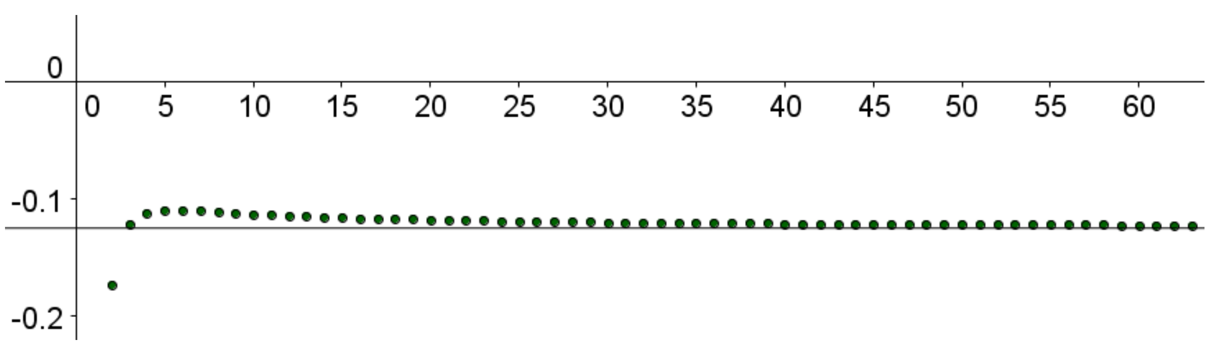
Example: Give the limit (if it exists) of $\left\{ \dfrac{2n^2 - 3n + 6}{3n - 16n^2 + 12} \right\}_{n=1}^{\infty}$.

$\hookrightarrow$ rational function

$$\lim_{n\to\infty} \frac{2n^2 - 3n + 6}{3n - 16n^2 + 12} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(2 - \underbrace{\frac{3}{n}}_{\to 0} + \underbrace{\frac{6}{n^2}}_{\to 0}\right)}{\cancel{n^2}\left(\underbrace{\frac{3}{n}}_{\to 0} - 16 + \underbrace{\frac{12}{n^2}}_{\to 0}\right)}$$

$$= \frac{2}{-16}$$

$$= -\frac{1}{8}$$



Example: Give the limit (if it exists) of $\left\{n^n\right\}_{n=1}^{\infty}$.

DNE

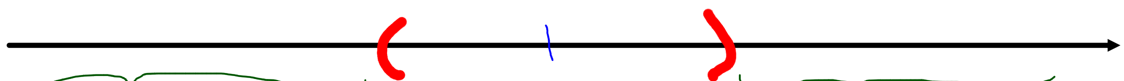
$$\underbrace{1, 4, 27, 256, \ldots}_{\text{Growing rapidly!}}$$

$$\lim_{n\to\infty} n^n = \infty$$

Recall: If a sequence has a limit, then it is bounded.

(why?)

all but the first "few" terms are here



a "few" terms here

L

a "few" terms here

Spse the limit is L.

Take the smallest of these.

lower bound

Take the largest of these.

upper bound

Recall: $\{ a_n \}$

1. Increasing Sequence: $a_k < a_{k+1}$ for all k

2. Non-decreasing Sequence:

$a_k \leq a_{k+1}$ for all k

3. Decreasing Sequence

$a_k > a_{k+1}$ for all k

4. Non-increasing Sequence:

$a_k \geq a_{k+1}$ for all k

Recall:

What tool can be used to help determine whether a sequence is increasing or decreasing?

Derivative

A sequence is **monotone** if and only if it is either increasing, nondecreasing, decreasing or nonincreasing.

Recall:

If a sequence has a limit, then it is bounded. Warning: The converse is not necessarily true!!

If a sequence is increasing, then the GLB is the first term and the LUB is the limit (if it exists).

If a sequence is decreasing, then the GLB is the limit (if it exists) and the LUB is the first term.

Example: Determine whether the sequence $\left\{1+\dfrac{3n+1}{n+2}\right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists). ← start here.

$$\lim_{n\to\infty}\left(1+\frac{3n+1}{n+2}\right) = \underline{\underline{4}}$$

$\frac{3n+1}{n+2} \to 3$

$\therefore$ the sequence is bounded.

?: $f(x) = 1 + \dfrac{3x+1}{x+2}$, $x \geq 1$.

$$f'(x) = \frac{(x+2)\cdot 3 - (3x+1)\cdot 1}{(x+2)^2} = \frac{5}{(x+2)^2} > 0 \text{ for } \underline{\underline{x \geq 1}}$$

$\Rightarrow$ f is increasing for $x \geq 1$.

$\Rightarrow$ the sequence is increasing.

$\Rightarrow$ the sequence is monotone.

Example: Determine whether the sequence $\left\{ \dfrac{3n+(-1)^n}{n+2} \right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists). ← start here

$$\lim_{n\to\infty} \frac{3n+(-1)^n}{n+2} = \lim_{n\to\infty} \frac{\cancel{n}\cdot\left(3+\frac{(-1)^n}{n}\right)}{\cancel{n}\cdot\left(1+\frac{2}{n}\right)}$$

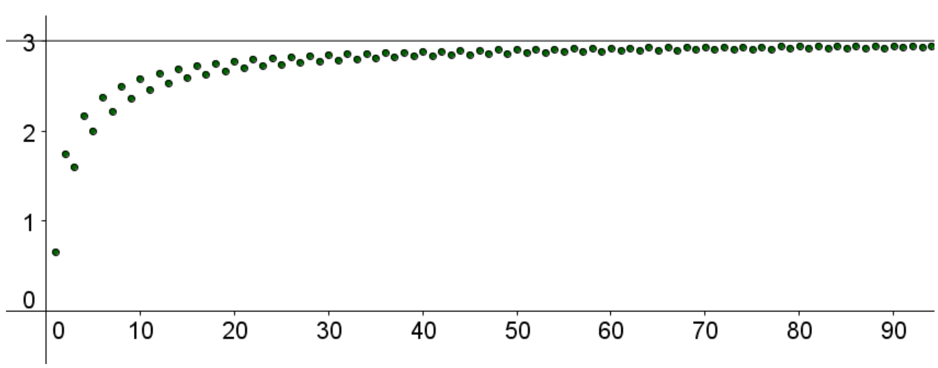
$$= \lim_{n\to\infty} \frac{3+\overbrace{\frac{(-1)^n}{n}}^{\to 0}}{1+\underbrace{\frac{2}{n}}_{\to 0}} = 3$$

∴ the sequence is bounded.

Look at some terms.

$$\left\{ \frac{3n+(-1)^n}{n+2} \right\}_{n=1}^{\infty} \qquad \underset{n=1}{\frac{2}{3}} \;<\; \underset{n=2}{\frac{7}{4}} \;>\; \underset{n=3}{\frac{8}{5}}$$

Not monotone!



Example: Determine whether the sequence $\left\{\dfrac{3n+n(-1)^n}{2n+5}\right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists). ← start here

$$\lim_{n\to\infty} \frac{3n+n(-1)^n}{2n+5} = \lim_{n\to\infty} \frac{\cancel{n}\cdot\left(3+(-1)^n\right)}{\cancel{n}\cdot\left(2+5/n\right)}$$

$$= \lim_{n\to\infty} \frac{3+(-1)^n}{2+\frac{5}{n}}, \quad \frac{5}{n} \to 0$$

$$= \text{DNE}$$

This will bounce back and forth between values close to 4/2 and 2/2.

So, there is no definite value that the sequence tends towards.

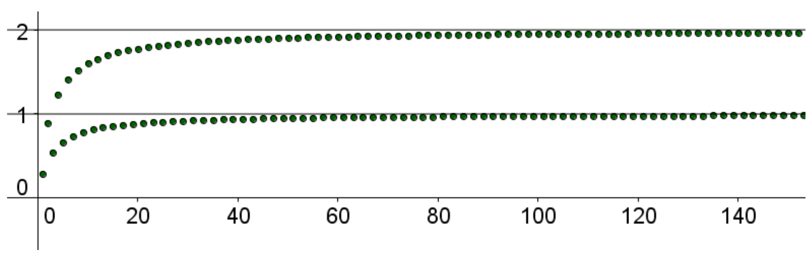
$n \geq 1$

$$0 < \frac{2n}{2n+5} \leq \frac{3n+n(-1)^n}{2n+5} \leq \frac{4n}{2n+5} < \frac{4n}{2n} = 2$$

lower bound

upper bound

$\therefore$ the sequence is bounded.



We can also see from the graph, that the sequence is not monotone.

This can be verified algebraically as well, by looking at the first few terms.

$$\left\{\frac{3n+n(-1)^n}{2n+5}\right\}_{n=1}^{\infty} \qquad \underset{n=1}{\frac{2}{7}} < \underset{n=2}{\frac{8}{9}} > \underset{n=3}{\frac{6}{11}}$$

Not monotone.

Example: Determine whether the sequence $\left\{\dfrac{\sqrt{n+1}}{\sqrt{n}}\right\}_{n=1}^{\infty}$ is

a. Bounded

b. Monotone

Then, give the limit (if it exists). ← Start here

$$\lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} = \lim_{n\to\infty} \sqrt{\frac{n+1}{n}} \quad \frac{n+1}{n} \to 1$$

$$= 1.$$

$\therefore$ the sequence is bounded.

Recall: If a sequence has a limit, then it is bounded!!!

define $f(x) = \dfrac{\sqrt{x+1}}{\sqrt{x}}$, $x \geq 1$

i.e. $f(x) = \sqrt{\dfrac{x+1}{x}}$, $x \geq 1$

$$f'(x) = \frac{1}{2}\left(\frac{x+1}{x}\right)^{-\frac{1}{2}} \cdot \frac{x \cdot 1 - (x+1)\cdot 1}{x^2}$$

$$= \underbrace{\frac{1}{2}\left(\frac{x+1}{x}\right)^{-\frac{1}{2}}}_{>0} \cdot \underbrace{\left(\frac{-1}{x^2}\right)}_{<0}, \quad x \geq 1$$

$< 0 \Rightarrow f$ is decreasing for $x \geq 1$.

$\Rightarrow$ the sequence is decreasing.

$\Rightarrow$ the sequence is monotone