Info...

Test 3 review videos are posted!

We will have a review for Test 3 online on Friday from 4-6pm.

Examine the limits below.

0/0 $\lim_{x\to 0} \frac{\sin(x)-x}{x^3}$ a little more difficult

0/0 $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$

These are examples of
0/0 indeterminant forms.

Why is 0/0 called an indeterminant form?

0/0 $\lim_{x\to 1} \frac{17(x-1)}{x-1} = 17$

0/0 $\lim_{x\to 0} \frac{\sin(2x)}{5x} = \frac{2}{5}$

0/0 $\lim_{x\to 1} \frac{17(x-1)}{x^2-1} = \frac{17}{2}$, with $x^2-1 = (x-1)(x+1)$

0/0 $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

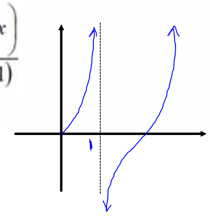
Bottom Line: "0/0" can lead to anything.

What is an (∞/∞) indeterminant form?

Example:

∞/∞ $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \frac{3}{2}$

$-\infty/-\infty$ $\lim_{x\to 1^+} \frac{\tan\left(\frac{\pi}{2}x\right)}{\ln(x-1)} = ??$



Why is (∞/∞) called an indeterminant form?

Answer: It can be almost anything!

$\frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{3x-2}{2x+1} = \frac{3}{2}$

$\frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{x^2}{x+1} = \infty$ DNE

$\frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{x^2}{x^3+1} = 0$

L'Hospital's Rule...

$$\lim_{x\to c}\frac{f(x)}{g(x)}$$

Two Cases:

1. (0/0)
2. (∞/∞)

These are both "indeterminant forms."

L'Hospital's Rule: (0/0) Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0$, and suppose*

$$f(x) \to 0 \text{ and } g(x) \to 0$$

as $x \to c^+, c^-, c, \infty, -\infty$. Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

i.e.

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

...when we have a 0/0 indeterminant form!!

Note: $\lim_{x\to 1}\frac{3x-7}{2x+6}$ This is NOT a 0/0 ind. form, AND the limit is $-\frac{1}{2}$.

BUT, if you are too smart to pay attention, you might try L'Hospital's rule (illegally). This leads to the WRONG ANSWER of $\frac{3}{2}$.

Why does this work? See the video.

Since $f(x) \to 0$ and $g(x) \to 0$ as $x \to c$, we can assume $f(c) = g(c) = 0$

Pick any value $b < c$, and define $h(x) = f(x) - \frac{f(b)}{g(b)}g(x)$.

Note: $h(b) = h(c) = 0$

L'Hospital's Rule: (∞/∞) Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0$, and suppose*

$$f(x) \to \pm\infty \text{ and } g(x) \to \pm\infty$$

as $x \to c^+, c^-, c, \infty, -\infty$. Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

i.e.

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

...provided we have an ∞/∞ indeterminant form!!!

Example: $\lim_{x\to 0}\frac{2\cos(x)-2+3x^2}{x^2} = 2$ "$\frac{0}{0}$" ind. form

diff. numerator + denom.

Try L-H: $\lim_{x\to 0}\frac{-2\sin(x)+6x}{2x} = 2$ New limit

OMG $\frac{0}{0}$ again!

Try L-H here

$\lim_{x\to 0}\frac{-2\cos(x)+6}{2} = 2$

Revisit...

$\lim_{x\to 0}\frac{\sin(x)-x}{x^3} = -\frac{1}{6}$ "$\frac{0}{0}$" ind. form

Try L-H: $\lim_{x\to 0}\frac{\cos(x)-1}{3x^2} = -\frac{1}{6}$

$\frac{0}{0}$ again!

Try L-H: $\lim_{x\to 0}\boxed{\frac{-\sin(x)}{6x}} = -\frac{1}{6}$ $\;\to 1$

Example: $\lim_{x\to\infty}\frac{x^2}{e^x} = 0$

we know this b/c e^x grows faster than any power of x as $x\to\infty$.

But, suppose you are brain dead.

$\frac{\infty}{\infty}$ ind. form. Try L-H: $\lim_{x\to\infty}\frac{2x}{e^x} = 0$

$\frac{\infty}{\infty}$ ind. form

Try L-H again: $\lim_{x\to\infty}\frac{2}{e^x} = 0.$

Example: $\lim_{x\to\infty} \frac{\ln(x)}{x} = 0$

we know this.

In fact, $\ln(x)$ grows slower than ANY positive power of x.

But, if you don't recall this, note that we have an $\frac{\infty}{\infty}$ ind. form.

Try L-H: $\lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

Example: $\lim_{x\to\infty} \frac{\ln(x)}{x^{0.0000001}} = 0$ b/c $\ln(x)$ grows slower (eventually) than any positive power of x.

But, if you don't recall this, note that it is in $\frac{\infty}{\infty}$ ind. form.

Try L-H: $\lim_{x\to\infty} \frac{\frac{1}{x}}{.0000001\, x^{-.9999999}}$

$= \lim_{x\to\infty} \frac{1}{.0000001\, x^{.0000001}}$

$= 0$

What is wrong with the following argument?

$$\lim_{x\to 0^+} \frac{x^2}{\sin(x)} = \lim_{x\to 0^+} \overbrace{\frac{2x}{\cos(x)}}^{=0} = \cancel{\lim_{x\to 0^+} \frac{2}{-\sin(x)}} = -\infty$$

Not an ind. form

Popper 19

1. **1**
2. **2**
3. **3**
4. **4**
5. **5**

Examples: $\boxed{\lim_{x\to 0^+}\left(e^x+2x\right)^{1/x}}$ $= e^3$

Different types of ind. forms.

$\lim_{x\to\infty}\left(x^2+2x\right)^{1/x}$

$(1)^\infty$

$1 \leftarrow$ Battle $\rightarrow \infty$

tlm:

$(e^x+2x)^{\frac{1}{x}} = e^{\ln\left((e^x+2x)^{1/x}\right)}$??

$= e^{\boxed{\frac{\ln(e^x+2x)}{x}}} \to e^3$

Look at $\lim_{x\to 0^+}\frac{\ln(e^x+2x)}{x}$

$\frac{0}{0}$ ind. form

Try L-H: $\lim_{x\to 0^+}\frac{\frac{e^x+2}{e^x+2x}}{1}$

$= \lim_{x\to 0^+}\frac{e^x+2}{e^x+2x} = 3$