Info...

Test 3 review videos are posted!

We will have a review for Test 3 online on Friday from 4-6pm.

Examine the limits below.

$\frac{0}{0}$ $$\lim_{x\to 0}\frac{\sin(x)-x}{x^3}$$ a little more difficult

$\frac{0}{0}$ $$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(\cancel{x-2})(x+2)}{(\cancel{x-2})} = 4$$

These are examples of
0/0 indeterminant forms.

Why is 0/0 called an indeterminant form?

$\frac{0}{0}$ $$\lim_{x\to 1}\frac{17(x-1)}{x-1} = 17$$

$\frac{0}{0}$ $$\lim_{x\to 0}\frac{\sin(2x)}{5x} = \frac{2}{5}$$

$\frac{0}{0}$ $$\lim_{x\to 1}\frac{17(x-1)}{x^2-1} = \frac{17}{2}$$

$(x-1)(x+1)$

$\frac{0}{0}$ $$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

Bottom Line: "0/0" can lead to anything.

What is an (∞/∞) indeterminant form?

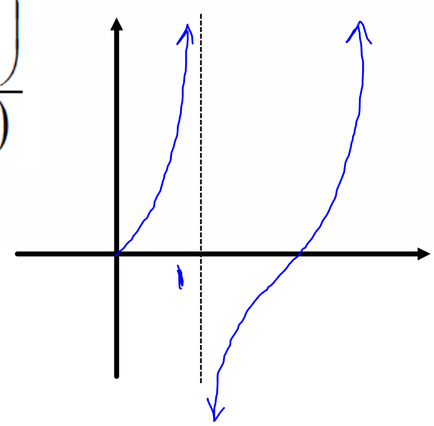
Example:

$\frac{\infty}{\infty}$ $$\lim_{x\to\infty}\frac{3x-2}{2x+1} = \frac{3}{2}$$

$\frac{-\infty}{-\infty}$ $$\lim_{x\to 1^+}\frac{\tan\left(\frac{\pi}{2}x\right)}{\ln(x-1)}$$

$=$

??



Why is (∞/∞) called an indeterminant form?

Answer: It can be almost anything!

$\frac{\infty}{\infty}$ $$\lim_{x\to\infty}\frac{3x-2}{2x+1} = \frac{3}{2}$$

$\frac{\infty}{\infty}$ $$\lim_{x\to\infty}\frac{x^2}{x+1} = \infty$$

DNE

$\frac{\infty}{\infty}$ $$\lim_{x\to\infty}\frac{x^2}{x^3+1} = 0$$

L'Hospital's Rule...

$$\lim_{x \to c} \frac{f(x)}{g(x)}$$

Two Cases:

1. $(0/0)$
2. (∞/∞)

These are both "indeterminant forms."

L'Hospital's Rule: (0/0) Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0$, and suppose*

$$f(x) \to 0 \text{ and } g(x) \to 0$$

as $x \to c^+, c^-, c, \infty, -\infty$. Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

i.e.

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

...when we have a 0/0 indeterminant form!!

Note: $\lim_{x \to 1} \frac{3x-7}{2x+6}$ This is NOT a 0/0 ind. form, AND the limit is $-\frac{1}{2}$.

BUT, if you are too smart to pay attention, you might try L'Hospital's rule (illegally). This leads to the WRONG ANSWER of $\frac{3}{2}$.

Why does this work? See the video.

Since $f(x) \to 0$ and $g(x) \to 0$ as $x \to c$, we can assume $f(c) = g(c) = 0$

Pick any value $b < c$, and define $h(x) = f(x) - \dfrac{f(b)}{g(b)} g(x)$.

Note: $h(b) = h(c) = 0$

L'Hospital's Rule: (∞/∞) Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0,$ and suppose*

$$f(x) \to \pm\infty \text{ and } g(x) \to \pm\infty$$

as $x \to c^+, c^-, c, \infty, -\infty.$ Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

i.e.

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

...provided we have an ∞/∞ indeterminant form!!!

Example: $\lim_{x \to 0} \frac{2\cos(x) - 2 + 3x^2}{x^2}$ $= 2$ "$\frac{0}{0}$" ind. form

diff. numerator + denom.

Try L-H: $\lim_{x \to 0} \frac{-2\sin(x) + 6x}{2x}$ $= 2$ New limit

OMG $\frac{0}{0}$ again!

Try L-H here

$\lim_{x \to 0} \frac{-2\cos(x) + 6}{2} = 2$

Revisit...

$$\lim_{x\to 0}\frac{\sin(x)-x}{x^3} = -\frac{1}{6}$$

" $\frac{0}{0}$ " ind. form

Try L-H:

$$\lim_{x\to 0}\frac{\cos(x)-1}{3x^2} = -\frac{1}{6}$$

$\frac{0}{0}$ again!

Try L-H:

$$\lim_{x\to 0}\frac{-\sin(x)}{6x} = -\frac{1}{6}$$

$\to 1$

Example: $\lim_{x\to\infty} \frac{x^2}{e^x}$ $= 0$ we know this b/c e^x grows faster than any power of x as $x \to \infty$.

But, suppose you are brain dead.

$\frac{\infty}{\infty}$ ind form. Try L-H: $\lim_{x\to\infty} \frac{2x}{e^x} = 0$

$\frac{\infty}{\infty}$ ind form

Try L-H again: $\lim_{x\to\infty} \frac{2}{e^x} = 0.$

Example: $\lim_{x\to\infty} \frac{\ln(x)}{x}$ $= 0$ we know this.

In fact, $\ln(x)$ grows slower than ANY positive power of x.

But, if you don't recall this, note that we have an $\frac{\infty}{\infty}$ ind. form.

Try L-H: $\lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

Example: $\lim_{x\to\infty} \frac{\ln(x)}{x^{0.0000001}} = 0$ b/c $\ln(x)$ grows slower (eventually) than any positive power of x.

But, if you don't recall this, note that it is in $\frac{\infty}{\infty}$ ind. form.

Try L-H: $\lim_{x\to\infty} \frac{\frac{1}{x}}{.0000001\, x^{-.9999999}}$

$= \lim_{x\to\infty} \frac{1}{.0000001\, x^{.0000001}}$

$= 0$

What is wrong with the following argument?

$$\lim_{x\to0^+}\frac{x^2}{\sin(x)} = \overbrace{\lim_{x\to0^+}\frac{2x}{\cos(x)}}^{=0} = \lim_{x\to0^+}\cancel{\frac{2}{-\sin(x)}} = -\infty$$

Not
an
ind.
form

Popper 19

1. **1** 2. **2** 3. **3** 4. **4** 5. **5**

Examples: $\boxed{\lim_{x\to 0^+}\left(e^x+2x\right)^{1/x}} = e^3$

$$\lim_{x\to\infty}\left(x^2+2x\right)^{1/x}$$

Different types of ind. forms.

$$\left(1\right)^{\infty}$$

$1 \leftarrow$ Battle $\rightarrow \infty$

tlm:

$$\left(e^x+2x\right)^{\frac{1}{x}} = e^{\ln\left(\left(e^x+2x\right)^{1/x}\right)}$$

??

$$= e^{\boxed{\frac{\ln(e^x+2x)}{x}}} \rightarrow e^3$$

Look at $\lim_{x\to 0^+}\frac{\ln(e^x+2x)}{x}$

$\frac{0}{0}$ ind. form

Try L-H: $\lim_{x\to 0^+}\dfrac{\frac{e^x+2}{e^x+2x}}{1}$

$$= \lim_{x\to 0^+}\frac{e^x+2}{e^x+2x} = 3$$