$$\ln(n) \ll \underset{p>0}{n^p} \ll \underset{\alpha>1}{\alpha^n} \ll n! \ll n^n$$

The following problems were given as poppers.
Let's work through these.

"$\frac{\infty}{\infty}$" ind. form

1. $\lim_{x\to\infty} \frac{100x^2}{e^x} = 0$

"$\frac{0}{0}$" ind form

4. $\boxed{\lim_{x\to 0} \frac{x-\tan(x)}{x-\sin(x)} =}$ -2

"$\frac{\infty}{\infty}$" ind form

2. $\lim_{x\to\infty} \frac{100\ln(x)}{x^2} = 0$

5. $\lim_{x\to 0} \frac{1-\sec(x)}{x^2} =$

3. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$

$x \to \infty$, $\frac{1}{x} \to 0$

Recall: $\lim_{u\to 0} \frac{\sin(u)}{u} = 1$

Try L-H: $\lim_{x\to 0} \frac{1-\sec^2(x)}{1-\cos(x)} = -2$

"$\frac{0}{0}$" ind form

Try L-H: $\lim_{x\to 0} \frac{-2\sec(x)\cdot\sec(x)\tan(x)}{\sin(x)} = \lim_{x\to 0} \frac{-2\sec^2(x)}{\cos(x)}$

$= -2$

$$\lim_{x\to 0}\frac{1-\sec(x)}{x^2} = -\frac{1}{2}$$

"$\frac{0}{0}$" ind. form

Try L-H: $\lim_{x\to 0}\frac{-\sec(x)\tan(x)}{2x} = \lim_{x\to 0}\frac{-\sec(x)\sin(x)}{2x\cos(x)}$ $\to 1$

$= \frac{-1}{2}$

Recall:

L'Hospital's Rule: $(0/0)$ Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0,$ and suppose*

$$f(x) \to 0 \text{ and } g(x) \to 0$$

$$\boxed{\text{“}\tfrac{0}{0}\text{” ind. form}}$$

as $x \to c^+, c^-, c, \infty, -\infty.$ Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

and...

L'Hospital's Rule: (∞/∞) Case.

Theorem *Suppose that f and g are differentiable functions with $g'(x) \neq 0$, and suppose*

$$f(x) \to \pm\infty \text{ and } g(x) \to \pm\infty$$

"$\frac{\infty}{\infty}$" ind. form

as $x \to c^+, c^-, c, \infty, -\infty$. Let L be a real number or $\pm\infty$.

$$\text{If } \frac{f'(x)}{g'(x)} \to L \text{ then } \frac{f(x)}{g(x)} \to L$$

Recall: $\lim_{x\to 0^+}\left(e^x+2x\right)^{1/x} = e^3$

L-H

Example: $\lim_{x\to\infty}\left(e^x+2x\right)^{1/x} = e$ "∞^0" ind form

$\boxed{\text{Guess } e}$

$$(e^x+2x)^{1/x} = e^{\ln\left((e^x+2x)^{1/x}\right)} = e^{\boxed{\frac{\ln(e^x+2x)}{x}}} \to e^{\boxed{1}} = e$$

$$\lim_{x\to\infty}\frac{\ln(e^x+2x)}{x} = 1$$

"$\frac{\infty}{\infty}$" ind. form. Try L-H: $\lim_{x\to\infty}\frac{\frac{e^x+2}{e^x+2x}}{1}$

$$= \lim_{x\to\infty}\frac{e^x+2}{e^x+2x} = 1$$

Example: $\lim_{x\to\infty}\left(\underbrace{x^2+2x}_{\to\infty}\right)^{1/x} = 1$ "∞^0" ind. form

$1/x \to 0$

Note:

$(x^2+2x)^{1/x} = e^{\ln\left(\left(x^2+2x\right)^{1/x}\right)} = e^{\boxed{\frac{\ln(x^2+2x)}{x}}} \to e^{\boxed{0}} = 1$

$\lim_{x\to\infty} \frac{\ln(x^2+2x)}{x}$

$\boxed{\text{Guess } 0}$

"$\frac{\infty}{\infty}$" ind. form

Try L-H: $\lim_{x\to\infty} \frac{\frac{2x+2}{x^2+2x}}{1} = \lim_{x\to\infty} \frac{2x+2}{x^2+2x} = 0$